NASA
Reference
Publication
1253

1992

Reliability Training

Edited by
Vincent R. Lalli
Lewis Research Center
Cleveland, Ohio

Henry A. Malec
Siemens Stromberg-Carlson
Albuquerque, New Mexico

NASA
National Aeronautics and
Space Administration
Office of Management
Scientific and Technical
Information Program

Preface

What Does Reliability Mean?

Systems. . .

The word "reliability" applies to systems made up of people, machines, and written information.

A system is reliable—that is, has good reliability—if the people who need the system can depend on it over a reasonable period of time. People can depend on a system if it reasonably satisfies their needs. These statements are purposely somewhat vague because quantifying them for any particular situation is a big task in itself.

People. . .

Several kinds of people are involved in a system, and they have different views of it. Some people rely on the system, others help to keep the system reliable, and still others do both. For example, consider an automatic grocery checkout system. The people involved are

- The owners, who bought the system
- The store manager, who is responsible for the system's operation
- The clerk, who actually operates it
- The repair person, who keeps it working
- The customer, who is being waited on

Machines. . .

A system can comprise several kinds of machines. A grocery checkout system has mechanical parts, electrical parts, and electronic parts. An automobile has chemical parts (fuel), liquid parts (hydraulic fluid for brakes), mechanical parts (engine, transmission, wheels), electrical parts (wiring, lights), electronic parts (ignition system, radio, engine controls), structural parts (body, frame, wheels, seats), miscellaneous parts (windows, windshield wiper blades), and many parts that can be classified in several ways (e.g., the fuel).

Written Information. . .

Several kinds of written information are important to the way people rely on a system; for example,

- The sales literature that led the owner to buy the system
- The specifications for the system
- The detailed manufacturing drawings
- The software, programs, and procedures
- The operating instructions to the people who actually operate the system
- The repair instructions to the people who keep the system running and fix its parts when it fails
- The supply instructions so that people know what kind of repair parts should be made and stocked

- The instructions to the machine, especially computer programs, which are so vital to so many machines
- The inventory control to restock goods

Reliability. . .

People rely on systems

- To do useful or amusing things for them
- To do no unintentional harm to users, bystanders, property, or the environment
- To be reasonably economical to own and to fix
- To be safe to store or dispose of
- To accomplish their purposes without failure

What Does Reliability Engineering Mean?

Reliability engineering means doing special tasks while a system is being planned, designed and developed, manufactured, used, and improved. These special tasks are over and above the usual engineering and management tasks and are needed to ensure that the people involved in these usual tasks pay attention to all important details. These tasks ensure that the people who rely on the system will not be let down—not only when it is new, but also as the system gets older, worn, and repeatedly fixed.

Why Do We Need Reliability Engineering?

We, as users of technology, have always needed reliability engineering, but the separate discipline of reliability engineering has developed only since the 1940's. Before the industrial revolution most of the reliability details were handled by the individual workers for relatively simple machines, products, and tools. But shoddy goods were produced—wheels that broke too soon, farming implements that were not dependable, wood that rotted before its time.

Technology is changing rapidly. Systems are now large and complex. Companies that produce these systems must likewise be large and complex. In such situations, many important details—the kinds that affect reliability—slip by unnoticed in the press of getting things done on time and at an affordable cost. The telephone and electric power utilities and the military were among the first to see the need for a separate reliability discipline.

Acknowledgments

In 1963 the Orlando Division of the Martin Marietta Company recognized the need to provide its engineers, especially its design engineers, with a practical understanding of the principles and applications of reliability engineering. To this end, a short, informative reliability training program was prepared. The author of this company-sponsored effort was Richard B. Dillard, who was also the principal instructor.

In response to the student's enthusiasm, their quest for additional information, and the support of their supervisors and managers, Mr. Dillard researched and wrote chapters 2 to 6 and appendix A of this text.

Credit is also due to Mr. William L. Hadley, who stimulated many of the ideas presented, and to Dr. D.C. Schiavone and Mr. William P. Wood, who directed and supported the efforts that went into this material.

Thanks are extended to Mr. Frank E. Croxton and Prentice-Hall, Inc., for the use of two-tail and one-tail tables of the normal distribution, and to Mr. Arthur Wald and John Wiley & Sons, Inc., for the use of tables of the cumulative normal distribution.

In recognition of the need to help project managers better understand reliability and quality assurance activities, Mr. Frank J. Barber and Mr. Frank J. Barina prepared appendix B.

Mr. Kam L. Wong prepared chapter 1 using information and concepts from Mr. Charles R. Ryerson and Mr. Irwin Quart; and thanks are extended to North-Holland, Inc., for permission to reprint it. Mr. Henry A. Malec, presently the quality assurance evaluation/reliability manager at Siemens Stromberg-Carlson in Albuquerque, New Mexico, prepared chapters 7 to 9 and organized this text. Thanks are extended to the Digital Press of Digital Equipment Corporation for the software evaluation materials contained in chapter 7. Mr. Vincent R. Lalli, presently the electronics systems product assurance manager at the NASA Lewis Research Center in Cleveland, Ohio, prepared some of the new sections and appendix C, added some of the problems, and edited and cared for the final NASA printing of this text. Mr. E.A. Winsa served as the final NASA project office reviewer. His suggestions improved the usefulness of the text for flight projects.

The bibliography at the end of this manual will help you select other authoritative material on specific areas in reliability to supplement the material presented herein.

The editors, Vincent R. Lalli and Henry A. Malec, would like to thank the many members of the IEEE Reliability Society Administrative Committee for their help in the development of this text.

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Chapter 1
Introduction to Reliability

This perspective on the past, present, and future of reliability was prepared by Mr. Kam L. Wong. It was adapted from a keynote speech he gave at the 1982 European Conference on Electrotechnics.

Ever since the need for improved reliability in modern systems was recognized, it has been difficult to establish an identity for reliability engineering. Attempts to separate out an independent set of tasks for reliability engineering in the 1950's and 1960's resulted in the development of applied statistics for reliability and a large group of tasks for management. However, most of these tasks are in truth not reliability engineering tasks. Although much of the engineering work done in the name of reliability pertains to basic design, field failures in a well-designed system come from defects (flaws) that remain in the system after delivery and not from the basic design. Defect (flaw) control is the key to reliability. The traditional reliability tasks for a project are still important and should still be performed by reliability engineers. A new direction for system reliability engineers should be to act as dynamic synthesizing feedbacks—identifying and ranking flaws and stresses, determining flaw failure mechanisms, and explaining flaw control techniques to those responsible for design, manufacturing, and support planning. Reliability engineers and basic engineers must work closely together to create a synergistic effect for achieving ever higher reliability.

For the purpose of this chapter reliability engineering is defined as a branch of engineering devoted to preserving the required performance of a system operating under the stipulated conditions for the time period of interest within a set of constraints such as cost and weight. This formidable-sounding definition means that reliability engineering is a branch of engineering for making things work as advertised. Such a nebulous definition has made it difficult to establish an identity for reliability engineering.

This chapter identifies traditional reliability disciplines. The one reliability discipline excluded from this discussion is management. Although management is important, especially with the contemporary awareness of Japanese productivity and the continuing quality and reliability of Japanese products, it is outside of the scope of this discussion. We concentrate on the engineering and technical aspects of reliability. In 1979 a paper (ref. 1-1) was published that used the number of published papers and the number of their pages as measurement indexes to describe the development of reliability and maintainability disciplines. This chapter does not use the same types of indexes as the 1979 paper. Instead the rough magnitudes of published works in reliability areas will be used to estimate the relative emphasis. Furthermore, this chapter concentrates on reliability engineering and not on the broad field of reliability disciplines. When the term "reliability engineering" is used, it is understood to relate to systems engineering. To forecast for the future, we need to determine the what's and why's. If we cannot do that, then, at least, we need to establish a trend. Therefore, let us begin with what has been done in the name of reliability engineering in the last 40 years.

Era of Mechanical Designs

Before World War II, most equipment was mechanical. A failure could usually be isolated to a rather simple part. Of course, mechanical systems could be complex and contain many interacting parts, but the difficulty in assembling such products to sell at a reasonable price precluded building complex systems. Therefore, one generally needed to deal only with rather simple items.

Safety, which is closely related to reliability, was a critical factor in a piece of equipment. The key to reliable products then was safety margins in either stress-strength, wear, or fatigue conditions. Most of the efforts toward achieving good safety margins were simply considered good engineering practices. Therefore, calling a task a reliability effort was not meaningful. At times redundancy was used to ensure safety such as in multiengine aircraft and large structures. In effect, reliability in this era was implied in a product and was automatically expected by its users. Buyers usually bought only from manufacturers that were well known for producing reliable products. The only quantitative measure related to reliability and considered in equipment procurement and usage was the wearout life of the equipment. After something was designed and built, the only efforts expended for reliability were inspection and testing. Reliability engineering as such did not exist.

Era of Electron Tubes

The availability of electron tubes opened the way to rapidly increasing complexity of equipment, both in functions and parts counts. By the end of World War II the state of the art was growing much more quickly in electronics than in reliability engineering. The gap between technology and reliability in electronic equipment was beginning to be felt by the U.S. military. Why should electronic equipment present a greater reliability problem than earlier mechanical systems? First, the heart of electronic equipment then was the electron tube. An electron tube is a complex device in itself. It is an assemblage of many small parts. Material purity—glass, glass seal, and cathode—is critical. Thus, an electron tube is not highly reliable to begin with. Although it was good enough for use in a five-tube radio, the chance of failure went up exponentially when complexity increased. Therefore, complex digital or analog electronic equipment can have low reliability.

A complex function could be performed rather easily by a piece of electronic equipment constructed by repetitive standard assembly methods from a large number of mass-produced, reasonably priced electron tubes and passive parts. Using purely mechanical devices to perform such complex functions was not economically feasible. The economics of production that enabled economical manufacture of complex electronic equipment was also the major driving force for low reliability. Assume that a part will sell for a fixed price in the marketplace. If a company can spend 10 percent more money to gain 15 percent higher production yield, the additional spending will give the company more profit. However, if 15 percent more money will produce a yield gain of only 10 percent, it may not be profitable to spend the money. Thus, in mass production there is a point where the company should not put more money into improving production yield. Although a quantitative relationship between reliability and yield has not been established, low-yield parts probably have low reliability. If the number of visible defects (flaws), which cause rejects, is high, it is logical that invisible defects will also be high. Although these defects are invisible at the time of manufacture, they cause failures during equipment usage.

Total sales was another factor that kept manufacturers from improving reliability; if the parts were more reliable, fewer replacement parts would be sold. There is no fiscal incentive to improve reliability unless the customer complains or a competitor's product demonstrates much higher reliability for the same cost. From an economic viewpoint we really should not expect the reliability of electronic equipment to improve unless a basic improvement in manufacturing process is made at no increase in manufacturing cost. Fortunately, this does happen, so that reliability generally does improve with calendar time. However, the public usually does not wish to pay much more for additional reliability. A spare might still be the best method for achieving high reliability even in critical operations like broadcasting, where using redundant transmitters solves the problem. Because of their need to maintain strike capability and minimize logistic supplies, the military is most sensitive to the reliability problem.

During the 1940's the U.S. military promulgated the joint Army and Navy (JAN) standards for parts and established the Vacuum Tube Development Committee. By 1946 the airlines had set up a study for the development of better electron tubes. Later, Aeronautical Radio, Inc., and Cornell University did extensive analyses on defective electron tubes. About 1950, Vitro Laboratories and Bell Telephone Laboratories also pursued studies on failed parts, and the U.S. Department of Defense established an ad hoc committee on reliability that became the Advisory Group on the Reliability of Electronic Equipment (AGREE) in 1952. This group published its monumental report in 1957. In the meantime efforts directed toward reliability mushroomed. A few of the many noteworthy activities and publications during this explosive developmental period are listed in table 1-1. Each entry has some significance in the development of reliability engineering.

Not reflected in table 1-1 are military specifications, standards, and handbooks. Military specifications, standards, and handbooks were generated in the United States during the 1950's, primarily to improve the understanding of reliability. Much of the work that resulted in the publications shown in table 1-1 was funded by the U.S. Government. The military and the Government's pushing gave birth to reliability engineering. Their specifications required that various tasks be done (see fig. 1-1) by an independent system engineering group. Whether the product had been designed in a reliable manner was the important question. The greater emphasis at that time was on the need to make products more reliable by, for example, reliability prediction. Reliability can be predicted by counting parts or by analyzing part stress. Most proposed predictions are parts count predictions to provide a model for tradeoff studies.

Various reliability efforts have been grouped into a number of categories: manufacturing control, design control, reliability methods, failure cause detection, finished item reliability control, and flow control. Figure 1-1 depicts how these categories have been emphasized through the years. Admittedly, the construction of figure 1-1 is rather subjective; its purpose is to establish trends, not to classify efforts precisely. Note which specific quality and reliability effort emphasis is changing. Bear in mind that the amount of effort expended may not be proportional to the emphasis, although quite often it is the case. For example, wear life is always important. The decrease in the design control emphasis does not mean that wear life is unimportant. It only reflects that the importance of wear life has been well established and that wear life has become a standard design control task as part of the design process.

Era of Semiconductors

The invention of the transistor in 1948 opened up a new frontier for electronics. The simplicity of semiconducting

TABLE 1-1.—RECOGNIZED ACTIVITIES AND PUBLICATIONS DURING DEVELOPMENTAL PERIOD OF RELIABILITY ENGINEERING

Date	Event
July 1949	Formation of the Professional Group on Quality Control.
September 1951	Publication of "A Statistical Distribution Function of Wide Applicability" by W. Weibull in Journal of Applied Mechanics, vol. 18, no. 3, pp. 293-297.
July 1952	Publication of "An Analysis of Some Failure Data" by D.J. Davis in Journal of American Statistical Association, vol. 47, no. 258, pp. 113-150.
August 1952	Establishment of the Advisory Group on Reliability of Electronic Equipment (AGREE) by the U.S. Department of Defense. Publication of "A Survey of Current Status of the Electronic Reliability Problem," Rand Research Memorandum 1131, by R.R. Carhart.
May 1953	Publication of "Rudiments of Good Circuit Design," by N.H. Taylor.
September 1953	Publication of "Life Testing" by B. Epstein and M. Sobel in Journal of American Statistical Association, vol. 48, no. 263, pp. 486-502.
1954	Publication of monographs on "Electron Tube Life and Reliability" by M.A. Acheson.
March 1954	Publication of "NEL Reliability Design Handbook" by U.S. Navy Electronics Laboratory.
September 1954	Publication of "Truncated Life Tests in the Exponential Case," by B. Epstein in Annals of Mathematical Statistics, vol. 23, p. 639.
November 1954	First national symposium on quality control and reliability in electronics in United States.
March 1955	Publication of "RCA Reliability Program and Long Range Objective" by C.M. Ryerson.
May 1955	Publication of "Electronics Reliability: Definition of Terms of Interest in Study of Reliability" by G.R. Herd et al. of Aeronautical Radio, Inc., in Trans. Reliability Quality Control, vol. PGRQC-5.
May 1955	Publication of "Sequential Life Tests in the Exponential Case," by B. Epstein and M. Sobel in Annals of Mathematical Statistics, vol. 26, pp. 82-93
July 1955	Formation of the Reliability and Quality Control Group.
August 1955	Publication of "Systems Approach to Electronic Reliability" by W.F. Luebbert of U.S. Signal Corps.
September 1955	Publication of "Handbook of Preferred Circuits, Navy Aeronautical Electronic Equipment," by National Bureau of Standards for U.S. Navy, Naval Weapons Department, in Inst. Radio Eng. Proc., vol. 44, pp. 523-528.
October 1955	Publication of Vitro Laboratories Report No. 80, "Techniques for Reliability Measurements and Prediction, Based on Field Failure Data."
1956	Publication of "Reliability Factors for Ground Electronic Equipment," edited by K. Henney, McGraw-Hill, New York.
November 1956	Publication of TR1100, "Reliability Stress Analysis for Electronic Equipment," by J.A. Connor et al. of RCA in Trans. Reliability Quality Control, vol. PGRQC-9.
June 1957	Publication of AGREE report "Reliability of Military Electronic Equipment" by the Advisory Group on Reliability of Electronic Equipment.
October 1958	Publication of Technical Report No. 3, U.S. Navy, "Statistical Techniques in Life Testing," by B. Epstein.
September 1978	Formation of the IEEE Reliability Society.

devices held promise for much higher reliability. Indeed, semiconducting devices ultimately improved equipment reliability by one to two orders of magnitude over the electron tube equivalents. By the mid-1950's transistors became available in sufficient production quantities for use in electronic equipment. In the early 1960's integrated circuits (IC's) were invented and now dominate the electronic parts industry. During the 1960's reliability methods gained momentum. Design review then became a predominant element of reliability methods. The total reliability effort has been

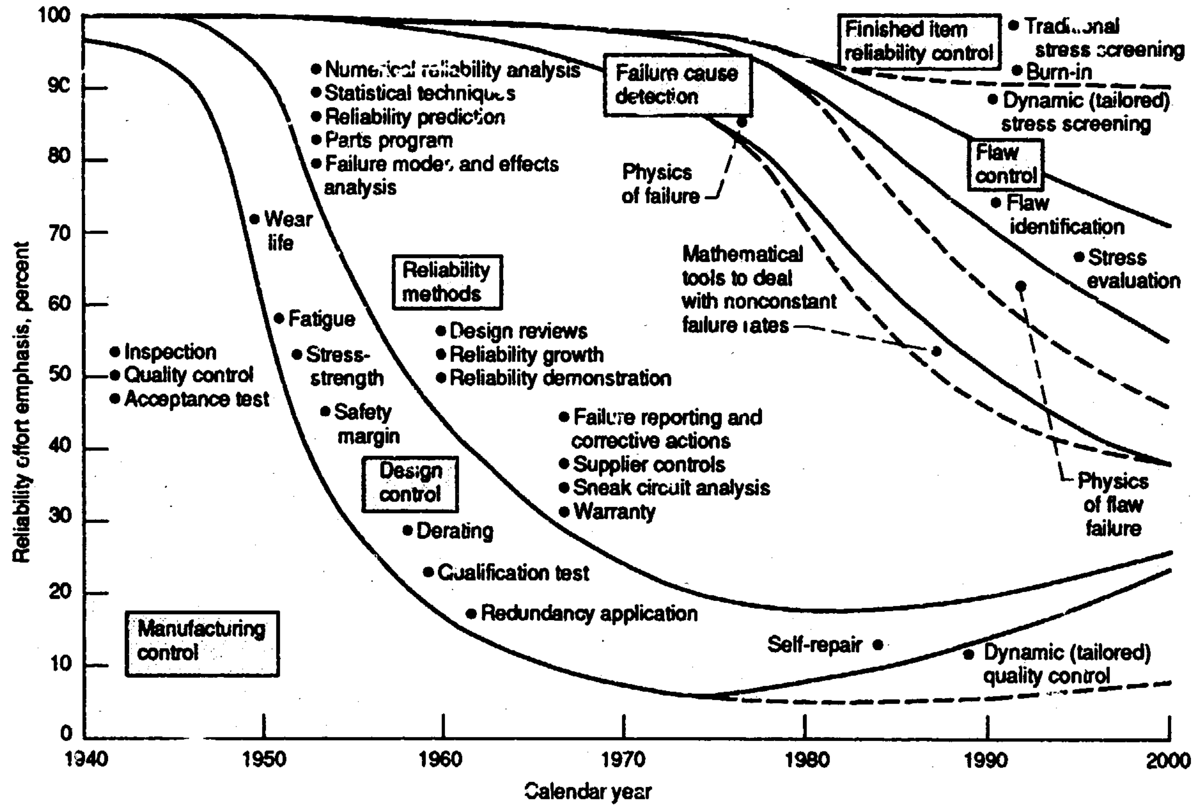


Figure 1-1.—Distribution of reliability emphasis with respect to calendar year.

increasing through the years, as shown in figure 1-2. Again, classifying tasks to be called reliability engineering is an inexact science. Do not attempt to read more than a trend indicator in figure 1-2. The launching of Sputnik in 1957 gave the world space program a tremendous push. The failure of Vanguard TV3 in the same year and many more U.S. satellite failures in 1958 forced the United States into high gear to strive for better reliability. Redundancy then became a life-saving tool. Without the application of redundancy in their design, many satellites, spacecraft, and to a certain extent boosters would have failed. The emphasis placed by the U.S. Government on reliability in the 1950's and early 1960's greatly improved equipment reliability.

While the equipment designers and manufacturers were making equipment more reliable, so were the parts suppliers. The improvements came from better material purity, better process controls, better designs, and new technology. One interesting phenomenon developed in semiconducting device technology when more complex devices were produced. People began to notice that the reliability of semiconducting devices was not inversely proportional to the complexity of the device, as intuition might have led them to believe. For example, a 100-transistor IC is more reliable than a circuit constructed from 100 individual transistors. Attempts were made to relate reductions in die bonds, wire bonds, seal length on the packages, etc., to reliability improvements. But the improvements were much greater than could be accounted for. Although not quantitatively proven, the mass production yield theory mentioned earlier can be used to explain this phenomenon. In effect, for simple devices the production yield has

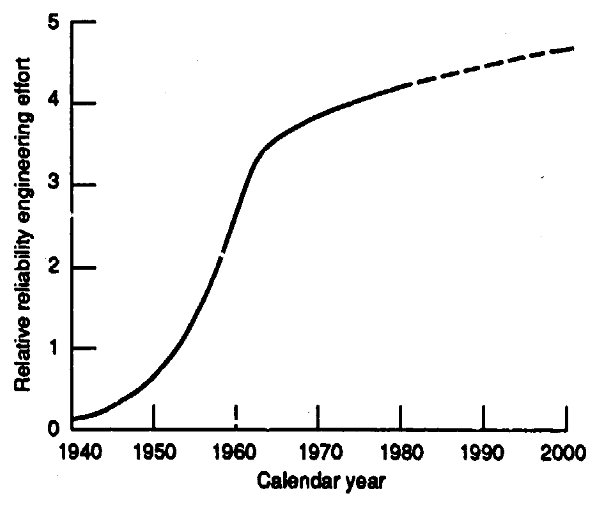


Figure 1-2.—Relative buildup of reliability effort in United States. (Related efforts such as environmental testing, structural and thermal analysis, parts and materials engineering, and standard quality controls are not grouped under reliability effort.)

reached a point where additional cost to improve yield would not bring sufficient income to increase profit. For complex devices tighter process control (tighter in-process inspection) pays off in profit because of the much higher rate of yield improvement. As was indicated, reliability is positively related to production yield. Therefore, when the yield of more complex devices is raised to approach that of simpler devices, their reliability also approaches that of simpler devices. As complex devices become more reliable, more of them will be used in equipment, causing the reliability of the equipment to need improvement.

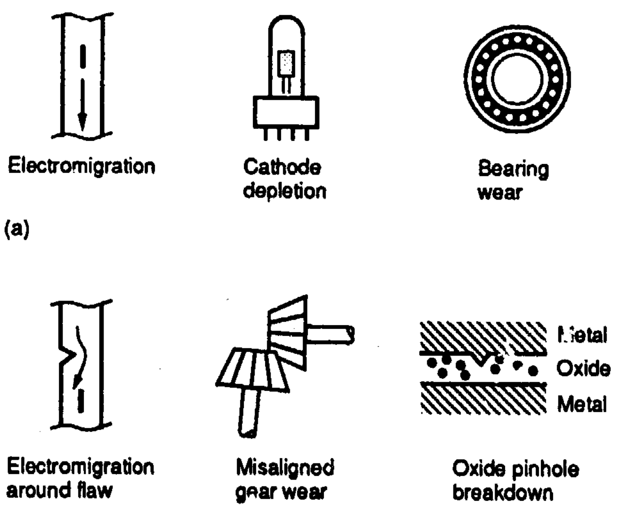
Many books on reliability statistics were written during this growth era of the 1950's and 1960's. Most of them were mathematical. In effect, they were books on how to apply statistical and probability theories to reliability work. In particular, small-sample statistics was the main field of application. Most of these books were written for applied mathematicians and not engineers. Physics uses much math, but applied mathematics used for physics is not physics. The same reasoning applies for reliability engineering. Throughout the 1960's most of the efforts in developing reliability engineering followed the classical method of trying to separate out an independent set of disciplines for reliability. In a mathematical analogy, people tried to break down the engineering function into orthogonal functions so that each orthogonal term could be dealt with individually in the hope of successfully recombining all the terms later. Through the years the reliability engineer provided a check on the design and process control engineers to improve the product's reliability. Like an electrical or mechanical engineer, the reliability engineer should perform an independent systems engineering function.

During the heyday of reliability activity a small group of engineers recognized that really improving reliability meant eliminating the source of failure. This led to the calling of the first physics of failure symposium in 1962 (ref. 1-3). Since then, much work has been done to investigate failure mechanisms. Papers have been presented every year in follow-on symposia on the subject. Also, parts screening was becoming a must. The issuance of MIL-STD-883 (ref. 1-4) in May 1968 set the tone for microcircuit screening to the present. Some real reliability engineering was being done.

Before proceeding further, consider "The Tale of Two Failures." A semiconductor diode developed a short. Analysis showed that a surge voltage was occurring occasionally that exceeded the breakdown voltage of the diode and was burning it up. It was a problem of stress exceeding strength. Let us call this a type I failure. A transistor suddenly stopped functioning. Analysis showed that aluminum metalization opened at an oxide step on the chip. The opening was accelerated by the neckdown of the metalization at the step. This failure was caused by a manufacturing flaw. In the classical terminology this is a random failure. Let us call this a type II failure. These two failure types are shown in figure 1-3. Until now, most of the design control efforts shown in figure 1-1 have been aimed at the type I failure (i.e., stress exceeding strength). Such design controls are important. For example, much equipment still has inadequate design, such as undercooling leading to overheating, even though cooling methods are well known. Designers need only to design according to standard methods to provide adequate designs. However, most equipment failures in the field bear no relation to the results of reasonable stress analyses during design. These

Period of Awakening

By the 1970's the implementation of reliability programs had become routine in developing equipment for the U.S. Government and the military. Basically, the reliability programs ensured that certain good engineering practices were carried out and provided a reliable product to the customer. However, equipment still continued to fail, although at a lower rate. Design reviews were helpful, but more was needed. With the tight funding situation, the benefits derivable from various reliability program elements were questioned. This encouraged tailoring the reliability program to the need; that is, doing only what gives a high return and not everything in the specification. A way to alleviate the customer's repair cost problem was to let the manufacturer share the burden. This led to the push for reliability improvement warranties (RIW's). There were many ways of implementing RIW's. Some were simply warranties such as those on car batteries and household appliances (ref. 1-2). These changes were mainly changes in management emphasis; there was really no engineering involved.



(a) Type I failures (a design margin problem on stress/strength, fatigue, and wear).

(b) Type II failures (a flaw problem).

Figure 1-3.—Two types of failure.

failures are type II (i.e., those caused by built-in flaws). It has become evident that flaws are what must be dealt with.

Flaws have long been recognized as the cause of early life failures. The parts screening practice was developed to remove such flaws. Equipment screening performed during the 1960's also attests to such recognition. In the early 1970's, Ryerson used defect or flaw as a parameter in his Cost Reduction Early Decision Information Techniques (CREDIT). In 1981, Quart presented some data and developed an equation relating failures resulting from screening to flaws (ref. 1-5), and later Wong extended the flaw theory to cover failures occurring during the normal operating period of the system (ref. 1-6). In essence, the combination of flaws and stresses causes most failures. In recognition of this fact, a large amount of energy was exerted in developing a screening technique in the late 1970's. Two national meetings on environmental stress screening of electronic hardware were held in the United States, in 1979 and 1981, under the sponsorship of the Institute of Environmental Sciences. The screening guidelines documents distributed at the 1981 meeting indicated that a number of systems experienced 20- to 90-percent reduction in field failure rate after the addition of environmental stress screening in manufacturing. Reliability engineers should concentrate on flaws and stresses and leave the basic design to the designers.

New Direction

The new direction in reliability engineering will be toward more realistic recognition of the causes and effects of failures from the system down to the microlevel. Instead of attempting to operate independently, reliability engineering should work interactively with other engineering functions. At the system level, critical environmental stresses must be identified and quantified. Design and manufacturing flaws, internal and external stresses, and failure mechanisms need to be classified and folded into the overall quantitative model of failure characteristics. The increasing emphasis on reliability physics has been bringing reliability engineering back toward the understanding and application of basic engineering principles. Although some work has been done, the different reliability technical areas have not been working together to provide a unified methodology. For example, although thermal cycling has been recognized as a key factor in inducing failures, MIL-HDBK-217E (ref. 1-7) does not take into account thermal cycling effects on failure rates. An attempt was started to bridge the gap between failure rate, thermal cycling, and fatigue failure mechanisms in 1981 (ref. 1-8). Stress/strength, wear, and fatigue will still be considered in this manual, but in reference to their effects on flaws rather than on the basic design.

Future emphasis should shift as indicated in figure 1-1. In several reliability efforts the words "dynamic" or "tailored" were used, signifying that flaws do not stay constant. They are very much human related as well as affected by the economic environment. Therefore, what is done to control or eliminate flaws must be flexible. There is no point in trying to eliminate something that is not there. Dynamic quality control will receive more emphasis, as shown at the lower right corner of figure 1-1 and discussed in appendix B, in achieving reliability, since it is a task for removing flaws. Although the investigation of failure physics will continue, the key now lies in the physics of flaw failures. For visibility and ease of system analysis, some quantitative measure of reliability will still be required. The flaw theory covers both nonconstant and constant failure rates (ref. 1-6). The mathematical tools developed with the assumption of constant failure rate will no longer be sufficient. An analysis published in 1988 (ref. 1-9) indicated that the failure rate of electronic systems generally decreases with system age, with failure humps along the way resembling the track of a roller coaster. It is, therefore, necessary now to deal with a roller-coaster curve, rather than a bathtub curve, for electronic system hazard rates. Fortunately, the advent of high-speed computers enables nonstationary failure rate models to be dealt with by simulation or Monte Carlo methods without requiring complicated closed-form mathematical expressions. A new set of mathematical tools is expected to be developed for use with the latest reliability models (ref. 1-10).

Software reliability, not shown in figure 1-1, requires increasing emphasis. However, software reliability is really a misnomer. It has an entirely different meaning from that of hardware reliability. Software reliability is a measure of software design adequacy. Therefore, it is a separate topic and is discussed in chapters 7 and 8.

It is proposed that new boundaries be defined for reliability engineering that exclude management, applied mathematics, and double-checking. Not that these functions are not important. In fact, they may still be performed by reliability engineers even though they are not classified as reliability engineering. Then, let us redefine reliability engineering in tighter boundaries as a synthesizing function devoted to flaw control. Figure 1-4 diagrams how this function interacts with others. Reliability engineering would act like a filter or synthesizer feedback loop, performing the following tasks:

(1) Identifying flaws and stresses and ranking them for priority actions

(2) Engaging the material technologists to determine the flaw failure mechanisms

(3) Developing flaw control techniques and feeding such information back to the engineers responsible for design, manufacture, and support planning

The types of output to be expected from reliability engineering are stress screening regimens, failure characteristics of parts and systems, effects of environmental stresses on flaws and failures, relationship of failure mechanisms such as electromigration to flaw failures, relationship of manufacturing yield to product reliability, flaw detection methods such as automated IC chip inspection and vibration signature monitor-

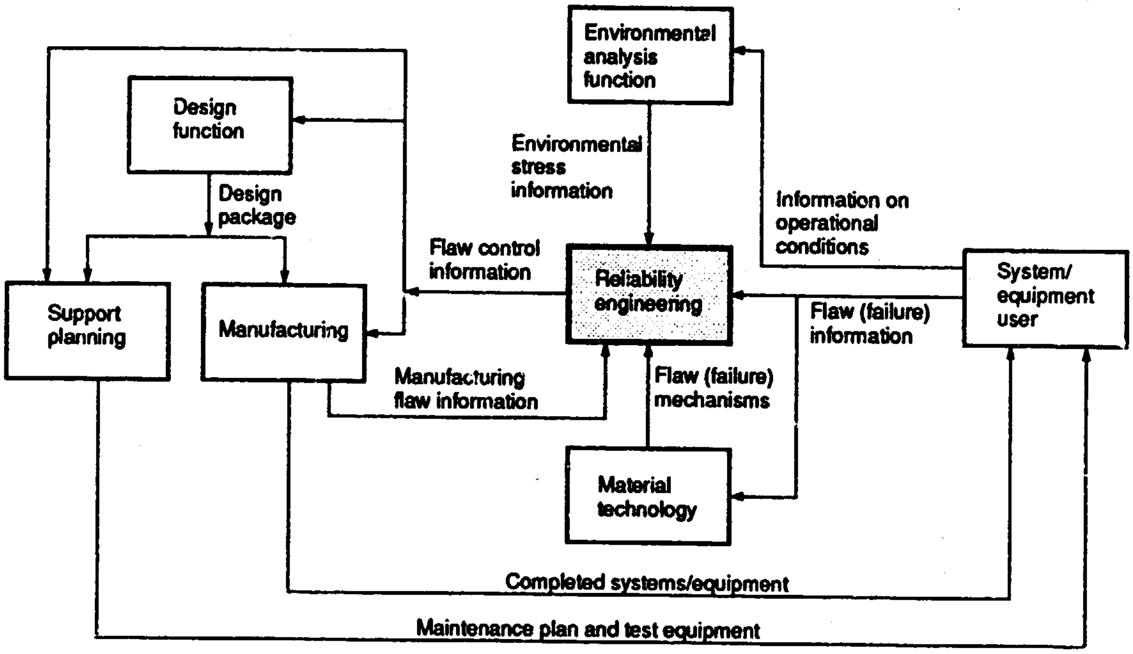


Figure 1-4.—Role of reliability engineering for the 1990's.

ing, and many more outputs than an engineering function should provide.

As mentioned earlier, flaws in an item depend on the design, manufacturing processes, quality control, parts, and materials. Therefore, the distribution of flaws does not stay constant. Reliability engineering must act dynamically to provide flaw control information to the proper functions for action on a timely basis. It is important that customers recognize this fact and allow proper controls to be tailored to the needs of the time instead of demanding a one-time negotiation on what should be done for the total contract period.

Concluding Remarks

Much of the reliability effort through the years has been aimed at increasing independent systems engineering and further refining basic design approaches. Now the time has come to direct our attention to flaw failures. These failures come from interaction of stresses and flaws. We must bring to bear on these flaws all the engineering techniques at our disposal in order to eliminate them. Reliability engineers are entering the era of interaction. Reliability engineering and basic engineering must work closely to create a synergistic effect for achieving ever higher reliability.

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1-7. Reliability Prediction of Electronic Equipment, MIL-HDBK-217E, Jan. 1990

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[1]A bibliography of other useful documents on reliability is given at the end of this manual.

Reliability Training[2]

1. Who has provided a large impetus toward safe and predictable products?

   A. Industry B. Universities C. Government

2. What brought on the reliability problem?

   A. Use of semiconductor devices
   B. Increased complexity of equipment
   C. Material shortages

3. How does production yield relate to reliability?

   A. There is no relationship.
   B. High yield correlates with low reliability.
   C. High yield correlates with high reliability.

4. What is the theme of this course?

   A. Nothing is learned from failures.
   B. Failures only need to be fixed.
   C. Each failure should be studied to see what can be done about it.

---

[2]Answers are given at the end of this manual.

Chapter 2

Reliability Mathematics and Failure Physics

Mathematics Review

Readers should have a good working knowledge of algebra and a slight knowledge of integral and differential calculus. However, for those who feel rusty in these subjects the following review includes solved examples for every mathematical manipulation used in this manual.

Notation

The Greek symbol Σ (sigma) means "take the sum of," and the notation

$$\sum_{i=1}^{n} x_i$$

means to take the sum of the x_i's from $i = 1$ to $i = n$.

The symbol $\sqrt[n]{x}$ means "take the n^{th} root of x." The square root, $\sqrt[2]{x}$, is usually written as $\sqrt{x}$ without the radicand (the 2).

The Greek symbol Π (pi) means "take the product of," and the notation

$$\prod_{i=1}^{n} x_i$$

means to take the product of the x_i's from $i = 1$ to $i = n$.

The notation $x!$ is referred to as a factorial; it is a shorthand method of writing $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times \ldots \times x$; or, in general: $x! = x(x-1)(x-2) \ldots (1)$. However, 0! is defined to be unity.

Manipulation of Exponential Functions

An exponential function is the Napierian base of the natural logarithms, $e = 2.71828 \ldots$, raised to some power. For example, e^2 is an exponential function and has the value 7.3891. This value can be calculated on most calculators.

Rules that must be followed when manipulating these functions are given here.

Rule 1:

$$e^x \times e^y = e^{x+y}$$

Rule 2:

$$e^{-x} = \frac{1}{e^x}$$

Rule 3:

$$\frac{e^x}{e^y} = e^{x-y}$$

Rounding Data

Reliability calculations are made by using failure rate data. If the failure rate data base is accurate to three places, calculations using these data can be made to three places. Use should be made of the commonly accepted rule (computer's rule) to round the computational results to the proper number of significant figures. The "Mathematics Dictionary" (ref. 2-1) defines rounding off as

> When the first digit dropped is less than 5, the preceding digit is not changed; when the first digit dropped is greater than 5 or 5 and some succeeding digit is not zero, the preceding digit is increased by 1; when the first digit dropped is 5 and all succeeding digits are zero, the commonly accepted rule is to make the preceding digit even, i.e., add 1 to it if it is odd, and leave it alone if it is already even.

For example, if the reliability of a system is 0.8324, 0.8316, or 0.8315, it would take the form 0.832, if rounded off to three places.

Integration Formulas

Only the following integration formulas are used in this manual:

$$\int_a^b x^n \, dx = \left.\frac{x^{n+1}}{n+1}\right|_a^b = \frac{b^{n+1} - a^{n+1}}{n+1} \qquad (1)$$

$$\int_a^b e^{-x} \, dx = -e^{-x}\Big|_a^b = -e^{-b} + e^{-a} = e^{-a} - e^{-b} \qquad (2)$$

$$\int_p^q e^{-ax}\,dx = \left.\frac{-e^{-ax}}{a}\right|_p^q = \frac{e^{-ap} - e^{-aq}}{a} \tag{3}$$

Examples 1:

$$\int x^2\,dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$$

$$\int_2^3 x\,dx = \left.\frac{x^2}{2}\right|_2^3 = \frac{(3)^2 - (2)^2}{2} = \frac{9-4}{2} = \frac{5}{2}$$

Example 2:

$$\int_3^4 e^{-x}\,dx = -e^{-x}\Big|_3^4 = e^{-3} - e^{-4}$$

Example 3:

$$\int_3^4 e^{-2x} = \left.\frac{-e^{-2x}}{2}\right|_3^4 = \frac{e^{-8} - e^{-6}}{2}$$

Differential Formulas

Only the following differential formulas are used in this manual:

$$\frac{d(ax)}{dx} = a \tag{4}$$

$$\frac{d(ax^n)}{dx} = nax^{n-1} \tag{5}$$

Examples 4:

$$\frac{d(x)}{dx} = 1$$

$$\frac{d(4x)}{dx} = 4$$

Examples 5:

$$\frac{d(x^2)}{dx} = 2x^{2-1} = 2x$$

$$\frac{d(4x^3)}{dx} = (3)4x^{3-1} = 12x^2$$

Partial Derivatives

This manual uses the following partial derivative formula:

$$\frac{\partial v}{\partial x_1} = \frac{\partial (xyz)}{\partial x} = yz \tag{6}$$

TABLE 2-1.—BINOMIAL COEFFICIENTS

n	Coefficient of each term of $(a+b)^n$										
	1	2	3	4	5	6	7	8	9	10	11
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1

Example 6:

$$v = 2\text{ ft} \times 3\text{ ft} \times 4\text{ ft} = 24\text{ ft}^3 \begin{cases} x = 2\text{ ft} \\ y = 3\text{ ft} \\ z = 4\text{ ft} \end{cases}$$

$$\frac{\partial v}{\partial x} = yz = 12\text{ ft}^2$$

Expansion of $(a+b)^n$

It will be necessary to know how to transform the expression $(a+b)^n$ into what is called a binomial expansion. This type of problem is easily solved by using table 2-1 and recalling that

$$(a+b)^n = a^n + na^{n-1}b + \frac{(n-1)(n)}{2!}a^{n-2}b^2 + \frac{(n-2)(n-1)(n)}{3!}a^{n-3}b^3 + \ldots + \frac{n(n-1)(n-2)\ldots(n-m+1)}{m!} \times a^{n-m}b^m + \ldots + b^n \tag{7}$$

Example 7:

Expand $(a+b)^4$. From table 2-1 with $n = 4$,

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Failure Physics

When most engineers think of reliability, they think of parts. This is understandable, since parts are the building blocks of products. All agree that a reliable product must have reliable

parts. But would everyone agree on what makes a part reliable? When asked this question, nearly all engineers would say a reliable part is one purchased according to a certain source control document and bought from an approved vendor. Unfortunately, these two qualifications are not always guarantees of reliability, even though we would like to think that they are. To illustrate, consider the following case of the qualified clock.

A clock purchased according to PD 4600008 was procured from an approved vendor for use in the ground support equipment of a missile system and was subjected to qualification tests as part of the reliability program. These tests consisted of high- and low-temperature, mechanical shock, temperature shock, vibration, and humidity tests. The clocks from the then sole-source vendor failed two of the tests: low temperature and humidity. A failure analysis revealed that lubricants in the clock's mechanism froze and that the seals were not adequate to protect the mechanism from humidity. A second approved vendor was selected. His clocks failed the high-temperature test. In the process the dial hands and numerals turned black, making readings impossible from a distance of 2 feet. A third approved vendor's clocks passed all of the tests except mechanical shock, which cracked two of the cases. Ironically, the fourth approved vendor's clocks, though less expensive, passed all the tests.

The point of this illustration is that four clocks, each designed to the same specification and procured from a qualified vendor, all performed differently in the same environments. These various failures are shown in table 2-2. Why did this happen? The answer is simple. The specification did not include the gear lubricant or the type of coating on the hands and numerals or the type of case material.

Many similar examples could be cited, ranging from requirements for glue and paint to complete assemblies and systems, and the key to answering these problems can best be stated as follows: *To know how reliable a product is or how to design a reliable product, you must know how many ways its parts can fail and the types and magnitude of stresses that cause such failures.* Think about this for a while; if you knew every conceivable way a missile could fail, and if you knew the type and level of stress required to produce each of these failures, you could build a missile that would never fail. You could do this because you could

(1) Eliminate as many ways of failure as possible
(2) Eliminate as many stresses as possible
(3) Eliminate the remaining potential failures by controlling the level of the remaining stresses

Sound simple? Well, it would be simple, except for one thing. Despite the thousands of failures observed in industry each day, we still know very little about why things fail and even less about how to control these failures. The situation is not hopeless, however. Through systematic data accumulation and study, we learn more each day. This manual is a small but important part of this systematic development.

As pointed out earlier, this manual introduces some basic concepts of failure physics. These include failure modes (how failures are revealed); failure mechanisms (what produces the failure mode); and failure stresses (what activates the failure mechanisms). It also introduces the theory and the practical tools available for controlling failures.

This chapter presents some basic probability theorems in preparation for a discussion of the various classes of failures that contribute to product unreliability.

Probability Theory

Fundamentals

Because reliability values are probabilities, every student of reliability disciplines should know the fundamentals of probability theory. Probability theory is used in chapter 3 to develop models that represent exactly how failures occur in products.

Probability defined.—Probability can be defined as follows: *If an event can occur in A different ways, all of which are considered equally likely, and if a certain number B of these events are considered successful or favorable, the ratio B/A is called the probability of the event.* Probability by this definition is also called an *a priori* (beforehand) probability because its value is determined without experimentation. It follows that reliability predictions of the success of missile

TABLE 2-2.—RESULTS OF QUALIFICATION TESTS ON SOURCE CONTROL DOCUMENT CLOCK

Vendor	High temperature	Low temperature	Mechanical shock	Temperature shock	Vibration	Humidity
1		Fail				Fail
2	Fail					
3			Fail			
4						

flights which are made before the flights occur are a priori reliabilities. In other words, a priori reliabilities are estimates of what may happen, not observed facts.

After an experiment has been conducted, an *a posteriori* probability or an observed reliability can be defined as follows: *If $f(n)$ is the number of favorable or successful events observed in a total number of n trials or attempts, the relative frequency $f(n)/n$ is called the statistical probability, the a posteriori probability, the empirical probability, or the observed reliability.* Note that the number of favorable events $f(n)$ is a function of the total number of trials or attempts n. Therefore, as the number of trials or attempts changes, $f(n)$ may also change, and consequently the statistical probability (or observed reliability) may change.

Reliability of a coin.--Trying out this theory, consider the physics of a coin. Assume it has two sides, is thin, and is made of homogeneous material. If the coin is tossed, one of two possible events may occur: heads or tails. If landing heads up is considered more favorable than landing tails up, a prediction of success can be made by using the a priori theory. From the a priori definition, the probability of success is calculated as

$$\frac{1 \text{ favorable event}}{2 \text{ possible events}} = 1/2, \text{ or } 50 \text{ percent}$$

This is an estimate of what should be observed if the coin is tossed, but not yet an observed fact. After the coin is tossed, however, the probability of success could be much more specific as shown in table 2-3.

TABLE 2-3.—OBSERVED PROBABILITY OF SUCCESS

Number of tosses, n	1	10	100	1000	10 000
Number of heads observed, $f(n)$	0	7	55	464	5080
Relative frequency of probability of success, $f(n)/n$	0	0.70	0.55	0.464	0.508

The table shows two important phenomena:

(1) As the number of trials changes, the number of favorable events observed also changes. An observed probability of success (or observed reliability) may also change with each additional trial.

(2) If the assumptions made in calculating the a priori probability (reliability prediction) are correct, the a posteriori (observed) probability will approach the predicted probability as the number of trials increases. Mathematically, the relative frequency $f(n)/n$ approaches the a priori probability B/A as the number of trials n increases, or

$$\lim_{n \to \infty} \frac{f(n)}{n} = \frac{B}{A}$$

In the coin toss example, the predicted reliability was 0.50. The observed reliability of 0.508 indicates that the initial assumptions about the physics of the coin were probably correct. If, as a result of 10 000 tosses, heads turned up 90 percent of the time, this could indicate that the coin was incorrectly assumed to be homogeneous and that, in fact, it was "loaded." Inconsistency in the actual act of tossing the coin, a variable that was not considered in the initial assumptions, could also be indicated. Here again, even with a simple coin problem, it is necessary to consider all the ways the coin may "fail" in order to predict confidently how it will perform.

Reliability of missiles.—In the aerospace industry a priori probabilities (reliability predictions) are calculated for missiles in an effort to estimate the probability of flight success. Inherent in the estimate are many assumptions based on the physics of the missile, such as the number of its critical parts, its response to environments, and its trajectory. As in the coin problem the ultimate test of the missile's reliability prediction is whether or not the prediction agrees with later observations.

If during flight tests the observations do not approach the predictions as the number of flights increases, the initial assumptions must be evaluated and corrected. An alternative approach is to modify the missile to match the initial assumptions. This approach is usually pursued when the reliability prediction represents a level of success stated by the customer or when the predicted value is mandatory for the missile to be effective. This subject of reliability predictions is discussed again in chapter 4.

In practice, reliability testing yields the knowledge needed to verify and improve initial assumptions. As experience is gained, the assumptions undergo refinements that make it possible to develop more accurate reliability predictions on new missiles and systems not yet tested or operated. This information also provides design engineers and management with data to guide design decisions toward maximum missile or system reliability. Some reliability problems require the use of Bayes or Markovian probability theorems. Additional information on other topics is available in references 2-2 to 2-5 and in IEEE Reliability Society publications and other documents listed in the reference sections for chapters 3 to 9 and in the bibliography at the end of this manual.

Probability Theorems

The three probability theorems presented here are fundamental and easy to understand. In these theorems and examples the probability of success (reliability) is represented with an R and the probability of failure (unreliability) with a Q. The following section (Concept of Reliability) examines what contributes to the reliability and unreliability of products.

Theorem 1.—If the probability of success is R, the probability of failure Q is equal to $1 - R$. In other words, the probability that all possible events will occur is $Q + R = 1$.

Example 1: If the probability of a missile flight success is 0.81, the probability of flight failure is $1 - 0.81 = 0.19$.

Therefore, the probability that the flight will succeed or fail is 0.19 + 0.81 = 1.0.

Theorem 2.—If R_1 is the probability that a first event will occur and R_2 is the probability that a second independent event will occur, the probability that both events will occur is R_1R_2. A similar statement can be made for more than two independent events.

Example 2: If the probability of completing one countdown without a failure R_1 is 0.9, the probability of completing two countdowns without failure is $R_1R_2 = (0.9)(0.9) = 0.81$. The probability that at least one of the two countdowns will fail is $1 - R_1R_2 = 1 - 0.81 = 0.19$ (from theorem 1). We say that at least one will fail because the unreliability term Q includes all possible failure modes, which in this case is two: one or both countdowns fail.

Example 3: If the probability of failure Q_1 during one countdown is 0.1, the probability of failure during two countdowns is $Q_1Q_2 = (0.1)(0.1) = 0.01$. Therefore, the probability that at least one countdown will succeed is $1 - Q_1Q_2 = 1 - 0.01 = 0.99$. We say that at least one will succeed because the value 0.99 includes the probability of one countdown succeeding and the probability of both countdowns succeeding.

Example 4: If the probability of completing one countdown without failure R_1 is 0.9 and the probability of a second countdown failing is $Q_2 = 0.1$, the probability that the first will succeed and the second fail is $R_1Q_2 = (0.9)(0.1) = 0.09$.

Theorem 3.—If the probability that one event will occur is R_1 and the probability that a second event will occur is R_2 and if not more than one of the events can occur (i.e., the events are mutually exclusive), the probability that either the first or second event, not both, will occur is $R_1 + R_2$. A similar theorem can be stated for more than two events.

Example 5 (true event method): Consider now the probability of completing two countdowns without a failure. Let the probabilities of success for the first and second countdowns be R_1 and R_2 and the probabilities of failure be Q_1 and Q_2. In order to solve the problem using theorem 3, it is best to diagram the possible events as shown in figure 2-1. The mutually exclusive events are

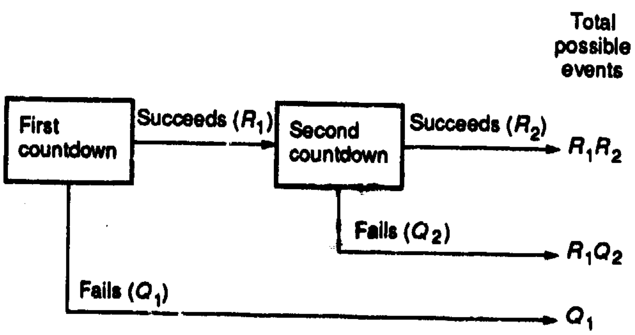

Figure 2-1.—Diagram of possible events—probability of completing two countdowns without a failure.

Q_1 first countdown fails

R_1Q_2 first countdown succeeds and second fails

R_1R_2 both countdowns succeed

From theorem 3 the probability that one of the three events will occur is

$$Q_1 + R_1Q_2 + R_1R_2$$

But because these three events represent all possible events that can occur, their sum equals 1 (from theorem 1). Therefore,

$$Q_1 + R_1Q_2 + R_1R_2 = 1$$

The probability of completing both countdowns without one failure R_1R_2 is the solution to the proposed problem; therefore,

$$R_1R_2 = 1 - (R_1Q_2 + Q_1)$$

If $R_1 = 0.9$, $Q_1 = 0.1$, $R_2 = 0.9$, and $Q_2 = 0.1$ then

$$\begin{aligned} R_1R_2 &= 1 - [(0.9)(0.1) + 0.1] \\ &= 1 - (0.09 + 0.1) = 1 - 0.19 = 0.81 \end{aligned}$$

which agrees with the answer found in example 2 by using theorem 2. The expression for R_1R_2 can also be written

$$\begin{aligned} R_1R_2 = 1 - (R_1Q_2 + Q_1) &= 1 - [(1 - Q_1)Q_2 + Q_1] \\ &= 1 - (Q_1 + Q_2 - Q_1Q_2) \end{aligned}$$

which is the usual form given for the probability of both events succeeding. Note, however, that in this expression, the event indicated by Q_1Q_2 (both countdowns fail) is not a true possible event, because we stipulated in the problem that only one countdown could fail. The term Q_1Q_2 is only a mathematical event with no relation to observable events. In other words, if the first countdown fails, we have lost our game with chance.

Example 6 (mathematical event method): Now consider the same problem as in example 5, ignoring for the time being the restriction on the number of failures allowed. In this case the diagram of the possible events looks like that shown in figure 2-2. In this case the mutually exclusive events are

R_1R_2 both countdowns succeed

R_1Q_2 first countdown succeeds and second fails

Q_1R_2 first countdown fails and second succeeds

Q_1Q_2 both countdowns fail

Keep in mind that in this example both countdowns may fail. From theorem 3 the probability that one of the four events will occur is

$$R_1R_2 + R_1Q_2 + Q_1R_2 + Q_1Q_2$$

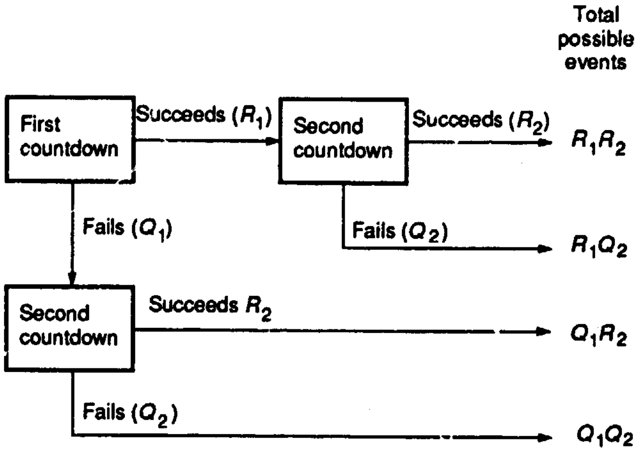

Figure 2-2.—Diagram of possible events—number of failures not restricted.

Again, because the four events represent all possible events that can occur, their sum equals unity (from theorem 1); that is,

$$R_1R_2 + R_1Q_2 + Q_1R_2 + Q_1Q_2 = 1$$

Solving for the probability that both countdowns will succeed is

$$R_1R_2 = 1 - (R_1Q_2 + Q_1R_2 + Q_1Q_2)$$

Substituting $1 - Q_1$ for R_1 and $1 - Q_2$ for R_2 on the right side of the equation gives the answer given in example 5:

$$\begin{aligned} R_1R_2 &= 1 - [(1 - Q_1)Q_2 + Q_1(1 - Q_2) + Q_1Q_2] \\ &= 1 - (Q_2 - Q_1Q_2 + Q_1 - Q_1Q_2 + Q_1Q_2) \\ &= 1 - (Q_1 + Q_2 - Q_1Q_2) \end{aligned}$$

This countdown problem has been solved in two ways to acquaint you with both the true event method and the mathematical event method of determining probability diagrams. The exercises at the end of this chapter may be solved by using whichever method you prefer. Because these exercises will be helpful to you in gaining a working knowledge of the three theorems presented, we suggest that you work the problems before continuing to the next section.

Concept of Reliability

Now that you have an understanding of the concepts of probability and failure physics, you are ready to consider the concept of reliability. First, the most common definition of reliability—in terms of successful operation of a device—is discussed. That definition, to fit the general theme of this manual, is then modified to consider reliability in terms of the absence of failure modes.

Reliability as Probability of Success

The classical definition of reliability is generally expressed as follows: *Reliability is the probability that a device will operate successfully for a specified period of time and under specified conditions when used in the manner and for the purpose intended.* This definition has many implications. The first is that when we say that reliability is a probability, we mean that reliability is a variable, not an absolute value. Therefore, if a device is 90 percent reliable, there is a 10 percent chance that it will fail. And because the failure is a chance, it may or may not occur. As in the coin example, as more and more of the devices are tested or operated, the ratio of total success to total attempts should approach the stated reliability of 90 percent. The next implication concerns the statement ". . . will operate successfully . . ." This means that failures that keep the device from performing its intended mission will not occur. From this comes a more general definition of reliability: that it is the probability of success.

It should be obvious then that a definition of what constitutes the success of a device or a system is necessary before a statement of its reliability is possible. One definition of success for a missile flight might be that the missile leaves the launching pad. Another, that the missile hits the target. Either way, a probability of success, or reliability, can be determined, but it will not be the same for each definition of success. *The importance of defining success cannot be overemphasized.* Without it a contractor and a customer will never reach an agreement on whether or not a device has met its reliability requirements (i.e., the mission).

The latter part of the classical definition indicates that a definition of success must specify the operating time, the operating conditions, and the intended use. Operating time is defined as the time period in which the device is expected to meet its reliability requirements. The time period may be expressed in seconds, minutes, hours, years, or any other unit of time. Operating conditions are defined as the environment in which the device is expected to operate; they specify the electrical, mechanical, and environmental levels of operation and their durations. Intended use is defined as the purpose of the device and the manner in which it will be used. For example, a missile designed to hit targets 1000 miles away should not be considered unreliable if it fails to hit targets 1100 miles away. Similarly, a set of ground checkout equipment designed to be 90 percent reliable for a 1-hour tactical countdown should not be considered unreliable if it fails during 10 consecutive countdowns or training exercises. The probability of success in this case is $(0.9)^{10} = 0.35$ (from probability theorem 2).

In addition to these specified requirements, we must also consider other factors. As explained in the inherent product reliability section of this chapter, these areas have a marked effect on the reliability of any device.

Reliability as Absence of Failure

Although the classical definition of reliability is adequate for most purposes, we are going to modify it somewhat and examine reliability from a slightly different viewpoint. Consider this definition: *Reliability is the probability that the critical failure modes of a device will not occur during a specified period of time and under specified conditions when used in the manner and for the purpose intended.* Essentially, this modification replaces the words "a device will operate successfully" with the words "critical failure modes . . . will not occur." This means that if all the possible failure modes of a device (ways the device can fail) and their probabilities of occurrence are known, the probability of success (or the reliability of a device) can be stated. It can be stated in terms of the probability that those failure modes critical to the performance of the device will not occur. Just as we needed a clear definition of success when using the classical definition, we must also have a clear definition of failure when using the modified definition.

As an example, assume that a resistor has only two failure modes: it can open or it can short. If the probability that the resistor will not short is 0.99 and the probability that it will not open is 0.9, the reliability of the resistor (or the probability that the resistor will not short or open) is given by

$$R_{\text{resistor}} = \text{Probability of no opens} \times \text{Probability of no shorts}$$
$$= 0.9 \times 0.99 = 0.89$$

Note that we have multiplied the probabilities. Probability theorem 2 therefore requires that the open-failure-mode probability and the short-failure-mode probability be independent of each other. This condition is satisfied because an open failure mode cannot occur simultaneously with a short mode.

Product Application

This section relates reliability (or the probability of success) to product failures.

Product failure modes.—In general, critical equipment failures may be classified as catastrophic part failures, tolerance failures, and wearout failures. The expression for reliability then becomes

$$R = P_c P_t P_w$$

where

P_c probability that catastrophic part failures will not occur

P_t probability that tolerance failures will not occur

P_w probability that wearout failures will not occur

As in the resistor example these probabilities are multiplied together. This means they are considered to be independent of each other, but this may not always be true because an out-of-tolerance failure, for example, may evolve into or result from a catastrophic part failure. Nevertheless, in this manual they are considered independent and exceptions are pointed out as required.

Inherent product reliability.—The next step is to consider the inherent reliability of a product. Try to think of the expression $P_c P_t P_w$ as representing the potential reliability of a product as described by the product's documentation. Or to put it another way, let it represent the reliability inherent in the design drawings instead of the reliability of the manufactured hardware. This inherent reliability is predicated upon the decisions and actions of many people. If they should change, the inherent reliability could change.

If the inherent reliability of the design is denoted by R_i, then

$$R_i = P_c P_t P_w$$

Why do we consider inherent reliability? Because the facts of failure are these: When a design comes off the drawing board, the parts and materials have been selected; the tolerance, error, stress, and other performance analyses have been performed; the type of packaging is firm; the manufacturing processes and fabrication techniques have been decided; and usually the test methods and the quality acceptance criteria have been selected. At this point the design documentation represents some potential reliability that can never be increased except by a design change or good maintenance. However, the possibility exists that the actual reliability observed when the documentation is transformed into hardware will be much less than the potential reliability of the design. To understand why this is true, consider the hardware as a black box with a hole in both the top and the bottom. Inside the box are potential failures that limit the inherent reliability of the design. When the hardware is operated, these potential failures fall out the bottom (i.e., operating failures are observed). The rate at which the failures fall out depends on how the box or hardware is operated. Unfortunately, we never have just the inherent failures to worry about because other types of failures are being added to the box through the hole in the top. These other failures are generated by the manufacturing, quality, and logistics functions, by the user or customer, and even by the reliability organization itself. We discuss these added failures and their contributors in the following paragraphs but it is important to understand that, because of the added failures, the observed failures will be greater than the inherent failures of the design.

K Factors

The other contributors to product failure previously mentioned are called *K* factors; they have a value between 0 and 1, and modify the inherent reliability as follows:

$$R_{\text{product}} = R_i(K_q K_m K_r K_t K_u)$$

where

K_q probability that quality test methods and acceptance criteria will not degrade the inherent reliability. An example of K_q is the situation in which the quality control engineer accepts a defective part that later shows up as a field failure and is counted against product reliability.

K_m probability that manufacturing processes and fabrication and assembly techniques will not degrade the inherent reliability. Examples of K_m would be cold-soldered joints, poor lamination of multilayer printed circuit boards, and loose fittings in plumbing installations that can show up as field failures.

K_r probability that activities performed by the reliability engineer will not degrade the inherent reliability. An example of K_r would be an inaccurate test analysis that forces a design change which degrades rather than improves the hardware performance.

K_ℓ probability that logistics activities will not degrade the inherent reliability. An example of K_ℓ would be an inaccurate procedure in a repair manual that, if followed, would create more failures than it fixes.

K_u probability that the user or customer will not degrade the inherent reliability. Examples of K_u are operator errors that cause a field failure because correct operating procedures are not followed. This factor has been observed to be quite large for many systems. In one missile system, 11 out of every 100 countdowns were aborted because of operator errors (i.e., $K_u = 0.89$).

There are many other K factors, but these are the main ones. Even if each K factor could be made equal to unity (which, of course, is the goal), we would still be left with R_i, the inherent reliability of the design. It is also clear that any one of the factors can cause the product reliability to go to zero. The achievement of inherent reliability during production of a product and the achievement of reliability growth during the build, use, and test phases are of major concern to many reliability engineers.

Concluding Remarks

Chapter 2 has explained two principal concepts:

(1) To design a reliable product or to improve a product, you must understand first how the product can fail and then how to control the occurrence of those failures.

(2) There is an upper limit on how reliable a product can be when a certain traditional way of design and fabrication is used. That limit is the inherent reliability. Therefore, the most effective reliability engineer is the designer because all the designer's decisions directly affect the product's reliability. The three probability theorems were also illustrated in this chapter.

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Reliability Training[1]

1a. What notation means to take the sum of the x_i's from $i = 1$ to $i = n$?

A. $\Sigma\, x$'s B. $\sum_{i=1}^{\infty} x_k$ C. $\sum_{i=1}^{n} x_i$

1b. If $\bar{x} = 100$, $x_1 = 90$, $x_2 = 70$, and $x_3 = 50$, what is $\sum_{i=1}^{n} (\bar{x} - x_i)^2$?

A. 350 B. 35×10^2 C. 35 000

2a. What notation means to take the n^{th} root of x?

A. x^n B. ϵ^n C. $\sqrt[n]{x}$

2b. If $\bar{x} = 100$, $x_1 = 90$, $x_2 = 70$, and $x_3 = 50$, what is $\sqrt{\sum_{i=1}^{n} (\bar{x} - x_i)^2}$?

A. 3.6 B. 59.2 C. 640

3a. What notation means to take the product of the x_i's from $i = 1$ to n?

A. $\Pi\, x$'s B. $\prod_{i=0}^{\infty} x_k$ C. $\prod_{i=1}^{n} x_i$

3b. If $x_1 = 0.9$, $x_2 = 0.99$, and $x_3 = 0.999$, what is $\prod_{i=1}^{3} x_i$?

A. 0.890 B. 0.800 C. 0.991

4a. The notation $x!$ refers to what shorthand method of writing?

A. Poles B. Factorial C. Polynomials

4b. What does 10!/8! equal?

A. 800 B. 900 C. 90

5a. Describe the three rules for manipulation of exponential functions.

i. Products

A. Substract exponents B. Add exponents C. Multiply exponents

ii. Negative exponent

A. Cancel exponents B. Balance exponents C. 1/Exponent

iii. Division

A. Add exponents B. Subtract exponents C. Multiply exponents

5b. Simplify, $\epsilon^6\epsilon^3/\epsilon^4$.

A. ϵ^2 B. ϵ^4 C. ϵ^5

6. What is the integral of the following functions?

a. $\int_{x_1}^{x_2} x^3\, dx$

A. $x^4/4$ B. $x^4/4\Big|_0^{x_2}$ C. $[(x_2)^4 - (x_1)^4]/4$

[1]Answers are given at the end of this manual.

b. $\int_{x_1}^{x_2} \epsilon^{-ax}\, dx$

A. $-\epsilon^{-ax}/a$ B. $\left[\epsilon^{-ax_1} - \epsilon^{-ax_2}\right]/a$ C. $-\epsilon^{-ax}/a\Big|_0^{x}$

7. What is the derivative of the following functions?

a. $10x^4$

A. $40x^2$ B. $40x^3$ C. $10x^3$

b. ϵ^{2x}

A. ϵ^{2x} B. $\epsilon^{2x}/2$ C. $2\epsilon^{2x}$

8a. Write the first two terms of the binomial expansion $(a + b)^n$.

A. $a^n + (n - 1)a^{n-1}b + \ldots$ B. $a^n - na^{n-1}b + \ldots$ C. $a^n + na^{n-1}b + \ldots$

8b. Expand $(a + b)^3$ by using table 2-1.

A. $a^3 + 2a^2b + b^3$ B. $a^3 - 3a^2b - 3ab^2 + b^3$ C. $a^3 + 3a^2b + 3ab^2 + b^3$

9. What needs to be done to design a reliable product?

A. Test and fix it

B. Know how its parts fail

C. Know the type and magnitude of stresses that cause such failures

D. Both B and C

10. What are a priori reliabilities estimates of?

A. What may happen B. What will happen C. What has happened

11. What are a posteriori reliabilities observing?

A. What may happen B. What has happened C. What will happen

12. If the probability of success is R, what is the probability of failure Q?

A. $1 + R$ B. $1 - R^2$ C. $1 - R$

13. If R_1, R_2, and R_3 are the probabilities that three independent events will occur, what is the probability that all three will occur?

A. $R_1 + R_2 + R_3$ B. $R_1(R_2 + R_3)$ C. $\prod_{i=1}^{3} R_i$

14. If R_1, R_2, and R_3 are the probabilities that three independent events will occur and not more than one of the events can occur, what is the probability that one of these events will occur?

A. $R_1R_2R_3$ B. $R_3(R_1 + R_2)$ C. $\sum_{i=1}^{3} R_i$

15. What do we need to know if a device is to perform with classical reliability?

A. Operating time and conditions

B. How it will be used

C. The intended purpose

D. All of the above

16. What do we need to know if a device is to perform with reliability defined as the absence of failure?

A. Critical failure modes

B. Operating time and conditions

C. How it will be used

D. The intended purpose

E. All of the above

17. What is the inherent reliability R_i of the product you are working on?

A. P_c (the probability that catastrophic part failures will not occur)

B. P_t (the probability that tolerance failures will not occur)

C. P_w (the probability that wearout failures will not occur)

D. The product of all of the above

18. What is the reliability of your product?

A. K_q (the probability that quality test methods will not degrade R_i)

B. K_m (the probability that manufacturing processes will not degrade R_i)

C. K_r (the probability that reliability activities will not degrade R_i)

D. K_ℓ (the probability that logistic activities will not degrade R_i)

E. K_u (the probability that the user will not degrade R_i)

F. The product of all of the above and R_i

Chapter 3

Exponential Distribution and Reliability Models

An expression for the inherent reliability of a product was given in chapter 2 as (ref. 3-1)

$$R_i = P_c P_t P_w$$

where

P_c probability that catastrophic part failures will not occur

P_t probability that tolerance failures will not occur

P_w probability that wearout failures will not occur

In chapter 3, we discuss the term P_c and develop and explain its mathematical representation in detail. We then use the probability theorems to establish methods of writing and solving equations for product reliability in terms of series and redundant elements.

Exponential Distribution

To understand what is meant by exponential distribution, first examine a statistical function called the Poisson distribution. This distribution is expressed as (ref. 3-2)

$$P(x,t) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

where

λ average failure rate

t operating time

x observed number of failures

This distribution states that if an observed average failure rate λ is known for a device, it is possible to calculate the probability $P(x,t)$ of observing $x = 0,1,2,3, \ldots,$ number of failures when the device is operated for any period of time t.

To illustrate, consider a computer that has been observed to make 10 arithmetic errors (or catastrophic failures) for every hour of operation. Suppose we want to know the probability of observing 0, 1, and 2 failures during a 0.01-hour program. From the data given, then

λ (failure rate) = 10 failures/hour

t (operating time) = 0.01 hour

x (observed failures) = 0, 1, and 2

The probability of observing no failures $P(0, 0.01)$ is then

$$P(0, 0.01) = \frac{(10 \times 0.01)^0 e^{-(10\times 0.01)}}{0!}$$

$$= \frac{1 \times e^{-0.1}}{1} = e^{-0.1} = 0.905$$

The probability of observing one failure $P(1, 0.01)$ is

$$P(1, 0.01) = \frac{(10 \times 0.01)^1 e^{-(10\times 0.01)}}{1!}$$

$$= \frac{(0.1)^1 e^{-0.1}}{1} = 0.1 \times 0.905 = 0.091$$

The probability of observing two failures $P(2, 0.01)$ is

$$P(2, 0.01) = \frac{(10 \times 0.01)^2 e^{-(10\times 0.01)}}{2!}$$

$$= \frac{(0.1)^2 e^{-0.1}}{2 \times 1} = \frac{0.01 \times 0.905}{2}$$

$$= \frac{0.00905}{2} = 0.0045$$

Remember that the definition of P_c is the probability that no catastrophic failures will occur. So for the computer $P_c = P(0, 0.01) = 0.905$. In other words, there is a 90.5-percent chance that no arithmetic errors will occur during the 0.01-hour program. This is the reliability of the computer for that particular program.

Again the Poisson distribution for $x = 0$ (i.e., no observed failures) is

$$P(0,t) = \frac{(\lambda t)^0 e^{-\lambda t}}{0!} = e^{-\lambda t}$$

The term $e^{-\lambda t}$ is called the exponential distribution and is the simplest form of P_c. Consequently, for a device that has an average failure rate λ the probability of observing no failures for a period of time t is (ref. 3-3)

$$P_c = e^{-\lambda t}$$

The expression for inherent reliability now takes the form

$$R_i = e^{-\lambda t} P_t P_w$$

or in the more general expression for total product reliability

$$R = e^{-\lambda t} P_t P_W (K_q K_m K_r K_l K_u)$$

At this point it is probably a good idea to digress for a moment to explain why these expressions for reliability may differ from those used elsewhere. During the conceptual and early research and development phases of a program, it is common practice (and sometimes necessary because of a lack of information) to assume that $P_t = 1$ (the design is perfect), that $P_w = 1$ (no wearout failures will occur), and that the K factors all equal 1 (there will be no degradation of inherent reliability). These assumptions reduce the inherent reliability and product reliability expressions to

$$R_i = R = e^{-\lambda t}$$

Frequently, these assumptions are not realistic and the resultant reliability predictions are usually high. They may bear little resemblance to the reliability finally observed when the product is tested. Later in this manual we will let

$$P_c = R = e^{-\lambda t}$$

to keep the notation simple.

On the other hand, it is also common to use $e^{-\lambda t}$ to represent the observed product reliability. In this case the observed average failure rate λ represents the combination of all types of failures including catastrophic, tolerance, and wearout. If the total product failure rate is λ', then

$$R = e^{-\lambda' t} = e^{-\lambda t} P_t P_w (K_q K_m K_r K_l K_u)$$

Failure Rate Definition

The failure rate λ as used in the exponential distribution $e^{-\lambda t}$ represents random catastrophic part failures that occur in so short a time that they cannot be prevented by scheduled maintenance (ref. 3-4). Random means that the failures occur randomly in time (not necessarily from random causes as many people interpret random failure) and randomly from part to part. For example, suppose a contractor uses 1 million integrated circuits in a computer. Over a period of time she may observe an average of one circuit failure every 100 operating hours. Even though she knows the failure rate, she cannot say which one of the million circuits will fail. All she knows is that, on the average, one will fail every 100 hours. In fact, if a failed circuit is replaced with a new one, the new one, theoretically, has the same probability of failure as any other circuit in the computer. In addition, if the contractor performs a failure analysis on each of the failed circuits, she may find that every failure is caused by the same mechanism, such as poorly welded joints. Unless she takes some appropriate corrective action, she will continue to observe the same random failures even though she knows the failure cause.

A catastrophic failure is an electrical open or short, a mechanical or structural defect, or an extreme deviation from an initial setting or tolerance (a 5-percent-tolerance resistor that deviated beyond its end-of-life tolerance, say to 20 percent, would be considered to have failed catastrophically).

The latter portion of the failure rate definition refers to the circumstance under which a failure is revealed. If a potential operating failure is corrected by a maintenance function, such as scheduled preventive maintenance, where an out-of-tolerance part could be replaced, that replacement cannot be represented by λ because it did not cause an operating or unscheduled failure. Here we see one of the many variables that affect the operating failure rate of a product: the maintenance philosophy.

Failure Rate Dimensions

Failure rate has the dimension of failure per unit of time, where the time is usually expressed in 10^x hours or cycles. Some Government documents express λ in percent failures per 10^3 hours. Table 3-1 shows the most common usage. Generally, the form that allows calculations using whole numbers, rather than decimal fractions, is chosen.

"Bathtub Curve"

In the Poisson distribution, λ was referred to as an average failure rate, indicating that λ may be a function of time $\lambda(t)$.

TABLE 3-1.—COMMON FAILURE RATE DIMENSIONS

Failures/hour, percent	Failures/ 10^6 hours	Failures/ 10^9 hours
10.0	100.0	100 000.0
1.0	10.0	10 000.0
.1	1.0	1 000.0
.01	.1	100.0
.001	.01	10.0
.0001	.001	1.0
.00001	.0001	.1
.000001	.00001	.01
.0000001	.000001	.001

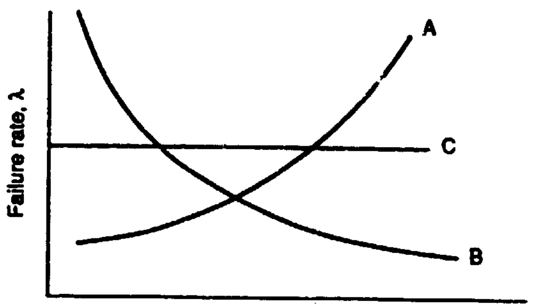

Figure 3-1.—Failure rate curves.

Figure 3-1 shows three general curves representing $\lambda(t)$ possibilities. Curve A shows that as operating time increases, failure rate also increases. This type of failure rate is found where wearout or age is a dominant failure mode stress (e.g., slipped clutches or tires). Curve B shows that as operating time increases, the failure rate decreases. This type of failure rate has been observed in some electronic parts, especially semiconductors. Curve C shows that as operating time increases, the failure rate remains constant. This type of failure rate has been observed in many complex systems and subsystems. In a complex system (i.e., a system with a large number of parts) parts having decreasing failure rates reduce the effect of those having increasing failure rates. The net result is an observed near-constant failure rate for the system. Therefore, part failure rates are usually given as a constant, although in reality they may not be. This manual deals only with constant part failure rates because they are related to system operation. Even if the failure rates might be changing over a period of time, the constant-failure-rate approximation is used.

If the failure rate for a typical system or complex subsystem is plotted against operating life, a curve such as that shown in figure 3-2 results. The curve is commonly referred to as a "bathtub" curve. The time t_0 represents the time at which the system is first put together. The interval from t_0 to t_1 represents a period during which assembly errors, defective parts, and compatibility problems are found and corrected. As shown, the system failure rate decreases during this debugging, or burn-in, interval as these gross errors are eliminated. The interval from t_1 to t_2 represents the useful operating life of the equipment and is generally considered to have a constant failure rate. It is during this time that the expression $P_c = e^{-\lambda t}$ is used. Therefore, when using $e^{-\lambda t}$, we assume that the system has been properly debugged. In practice this assumption may not be true, but we may still obtain an adequate picture of the expected operating reliability by accepting the assumption. The interval from t_2 to t_3 represents the wearout period, during which age and deterioration cause the failure rate to increase and render the system inoperative or extremely inefficient and costly to maintain.

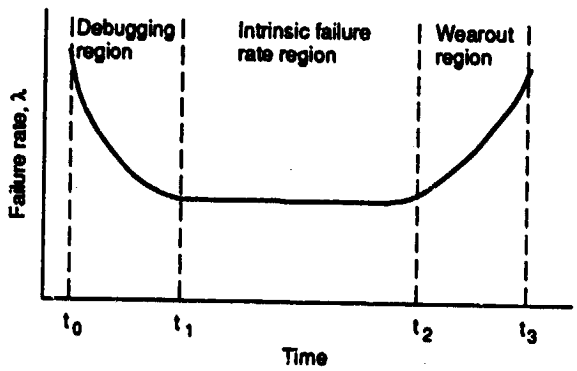

Figure 3-2.—Failure rate versus operating time.

The following analogy should help summarize the concepts of failure and failure rate: A company picnic is planned to be held on the edge of a high cliff. Because families will be invited, there will be various types of people involved: large, small, young, and old, each type with its own personality and problems. Picnic officials are worried about the possibility of someone falling over the cliff. The question is, What can be done about it? Four possible solutions are presented:

(1) Move the picnic farther back from the cliff. The farther back the picnic, the less the chance that someone will walk as far as the cliff and fall over.

(2) Keep the picnic short. The shorter the picnic, the less time anyone has to walk to the cliff.

(3) Look over the cliff to see if anyone has fallen. This is a good idea because they would know when to call the ambulance—but it hardly helps to keep others from falling. It is possible, however, that if they go to the bottom of the cliff to see who has fallen over, they might observe that every 15 minutes one person over the age of 99 falls over the cliff. Knowing this, all persons over 99 could be sent home and the picnic could be saved from further tragedy.

(4) Finally, they could build a high fence to separate the cliff from the picnic. Obviously, this is the best solution, because it is doubtful that anyone would climb the fence just to get to the cliff.

Now, let us look at the analogy of this picnic-to-failure rate. Say that we are building a system (picnic) made of many parts (people) and that there are many types of parts; some are large, some small, and some new and untried, such as integrated circuits. Some of these parts, the composition resistors for instance, are old and mature. Each part has its own personality (the way it was fabricated). Our problem is how to keep these parts from failing (falling over the cliff). And again we have four possible solutions:

(1) Reduce the stresses on the parts (move the picnic back from the cliff); the lower the stresses, the fewer the failures.

(2) Keep the operating time (the picnic) short; the shorter the operating time, the less chance a part has to fail.

(3) Establish part failure rates (look over the cliff to see if anyone has fallen), but this only helps if we know what parts (people) are failing. Once we know this, we can eliminate those parts from our system.

(4) Eliminate the failure mechanisms of the part (build a fence to separate the cliff from the picnic). This is the best answer, of course, because if we eliminate the cause of part failures, we cannot have any system failures.

Mean Time Between Failures

For the exponential distribution the reciprocal of failure rate is called the mean time between failures (MTBF) and is the integral of the exponential distribution:

$$\text{MTBF} = \frac{1}{\lambda}\int_0^\infty e^{-\lambda t}dt = -\frac{1}{\lambda}\,(e^{-\lambda t})_0^\infty$$

$$= -\frac{1}{\lambda}\left(\frac{1}{e^\infty} - \frac{1}{e^0}\right) = -\frac{1}{\lambda}\,(0-1) = \frac{1}{\lambda}$$

Therefore, if a device has a failure rate of one failure per 100 hours its MTBF is 100 hours.

If the time dimension is given in cycles, the MTBF becomes mean cycles between failures (MCBF), a term also in common use. For a nonrepairable device, mean time to failure (MTTF) is used instead of MTBF. For a repairable device MTBF is usually equal to MTTF.

If a device has an MTBF of, for example, 200 hours, this does not mean that the device will not fail until 200 operating hours have accumulated, nor does it mean that the device will fail automatically at 200 hours. MTBF is exactly what is says: a mean or average value. This can be seen from

$$e^{-\lambda t} = e^{-t/\text{MTBF}}$$

When the operating time t equals the MTBF, the probability of no failure is

$$e^{-\text{MTBF/MTBF}} = e^{-1} = 0.368$$

(using exponential tables or a slide rule), which means that there is a $1 - 0.368 = 0.632$ chance that the device will fail before its MTBF is reached. In other words, if a device has an MTBF of 1000 hours, replacing the device after 999 hours of operation will not improve reliability. To show the concept of a mean value in another way, consider the following empirical definition of MTBF:

$$\text{MTBF} = \frac{\text{Total test hours}}{\text{Total observed failures}}$$

For example, if 100 transistors are tested for 1000 hours each and five failures are observed, the observed MTBF is

$$\text{MTBF} = \frac{100 \times 1000}{5} = \frac{100\,000}{5} = 20\,000 \text{ hours}$$

Note that when the failures were observed is not indicated. The assumption of a constant failure rate leads to a constant time between failures, or MTBF.

Calculations of P_c for Single Devices

If a failure rate for a device is known, the probability of observing no failures for any operating period t can be calculated.

Example 1: A control computer in a missile has a failure rate of 1 per 10^2 hours. Find P_c for a flight time of 0.1 hour.

Solution 1:

$$P_c = e^{-\lambda t} = e^{-(1/10^2)(0.1)} = e^{-1\times 10^{-3}} = e^{-0.001} = 0.999$$

Therefore, there is one chance in a thousand that the control computer will fail. (Note: if λt or t/MTBF is less than 0.01, $P_c \cong 1 - \lambda t$, or $1 - t/\text{MTBF}$.) For example.

$$P_c = e^{-0.001} \cong 1 - 0.001 = 0.999$$

If λt, or t/MTBF, is greater than 0.01, use exponential tables to find P_c, as shown here.

$$P_c = e^{-0.08} = 0.923$$

Example 2: The same type of problem can be solved if the MTBF is known. The MTBF of a tape reader used in ground support equipment is 100 hours. Find P_c for a 2-hour operation.

Solution 2:

$$P_c = e^{-t/\text{MTBF}} = e^{-2/100} = e^{-0.02} = 0.980$$

If a specific P_c is required for a specified operating time, the required failure rate, or MTBF, can be calculated.

Example 3: A relay is required to have a 0.999 probability of not failing for 10 000 cycles. Find the required failure rate and MCBF.

Solution 3:

$$R = e^{-\lambda t}$$

$$0.999 = e^{-0.001} = e^{-\lambda(10^4 \text{ cycles})}$$

Equating exponents gives

$$\lambda(10^4 \text{ cycles}) = 0.001$$

$$\lambda = \frac{0.001}{10^4} = \frac{1 \text{ failure}}{10^7 \text{ cycles}}$$

The required MCBF is therefore

$$\text{MCBF} = \frac{1}{\lambda} = 10^7 \text{ cycles}$$

Reliability Models

In the following sections we replace $P_c = e^{-\lambda t}$, the reliability of a part, with a plain R to keep the notation simple.

Calculation of Reliability for Series-Connected Devices

In reliability, devices are considered to be in series if each device is required to operate without failure to obtain system success (ref. 3-5). A system composed of two parts is represented in a reliability diagram, or model, as shown in figure 3-3. If the reliability R for each part is known, from probability theorem 2, chapter 2, the probability that the system will not fail is

$$R_s = R_1 R_2$$

(We assume that the part reliabilities are independent; i.e., the success or failure of one part will not affect the success or failure of another part.) If there are n parts in the system, each one required for system success, the total system reliability is given by

$$R_s = R_1 R_2 R_3 \ldots R_n = \prod_{i=1}^{n} R_i$$

where

R_s probability that system will not fail

R_j reliability of j^{th} part

n total number of parts

The expression

$$R_s = \prod_{j=1}^{n} R_j$$

is often called the product rule.

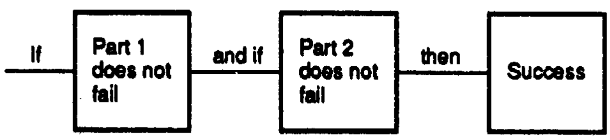

Figure 3-3.—Series model.

Example 4: A system has 100 parts, each one required for system success. Find the system reliability R_s if each part has $R = 0.99$.

Solution 4:

$$R_s = \prod_{j=1}^{n} R_j = \prod_{j=1}^{100} R_j = R_1 R_2 R_3 \ldots R_{100}$$

$$= (0.99)(0.99)(0.99) \ldots (0.99) = (0.99)^{100}$$

$$= \left(e^{-0.01}\right)^{100} = e^{-1} = 0.368$$

Therefore, the probability that the system will succeed is about 37 percent.

Example 5: For a typical missile that has 7000 active parts and a reliability requirement of 0.90, each part would have to have a reliability R_p of 0.999985. This is calculated from

$$(R_p)^{7000} = 0.90 = e^{-0.105}$$

Solution 5: Therefore,

$$R_p = \left(e^{-0.105}\right)^{1/7000} = e^{-1.5\times 10^{-5}} = e^{-0.000015}$$

$$= 1 - 0.000015 = 0.999985$$

The product rule can also be expressed as

$$R_s = \prod_{j=1}^{n} R_j = R_1 R_2 R_3 \ldots R_n$$

$$= e^{-\lambda_1 t_1} e^{-\lambda_2 t_2} e^{-\lambda_3 t_3} \ldots e^{-\lambda_n t_n}$$

$$= e^{-(\lambda_1 t_1 + \lambda_2 t_2 + \lambda_3 t_3 + \ldots \lambda_n t_n)}$$

$$= \exp\left(-\sum_{j=1}^{n} \lambda_j t_j\right)$$

where

λ_j failure rate of j^{th} part

t_j operating time of j^{th} part

Therefore, if for each series-connected part in a system the failure rate and operating time are known, the system reliability can be calculated by finding $\sum_{j=1}^{n} \lambda_j t_j$ and raising e to the $-\left(\sum_{j=1}^{n} \lambda_j t_j\right)$ power.

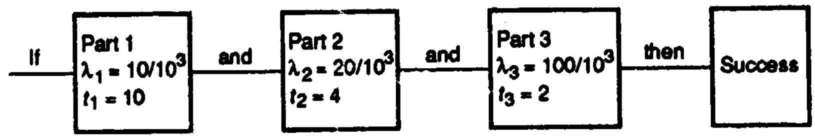

Figure 3-4.—Series model using failure rates and operating times.

Example 6: Find the system reliability from the model shown in figure 3-4.

Solution 6: Step 1

$$\sum_{j=1}^{3} \lambda_j t_j = \lambda_1 t_1 + \lambda_2 t_2 + \lambda_3 t_3$$

$$= 10/10^3(10) + 20/10^3(4) + 100/10^3(2)$$

$$= 100/10^3 + 80/10^3 + 200/10^3 = 380/10^3$$

Step 2

$$R_s = \exp\left(-\sum_{j=1}^{3} \lambda_j t_j\right) = e^{-380/10^3} = e^{-0.38} = 0.684$$

If the t_j's are equal (i.e., each part of the device operates for the same length of time), the product rule can further be reduced to

$$R_s = \exp\left(-\sum_{j=1}^{n} \lambda_j\right) t_c$$

where t_c is the common operating time.

Example 7: Find the reliability of the system shown in figure 3-5.

Solution 7: Step 1

$$\sum_{j=1}^{3} \lambda_j = \lambda_1 + \lambda_2 + \lambda_3 = 7/10^3 + 5/10^3 + 6/10^3 = 18/10^3$$

Step 2

$$R_s = \exp\left(-\sum_{j=1}^{3} \lambda_j\right) t_c = e^{-18/10^3(10)} = e^{-180/10^3}$$

$$= e^{-0.18} = 0.835$$

Calculation of Reliability for Devices Connected in Parallel (Redundancy)

In reliability, devices are considered to be in parallel if one or more of the devices can fail without causing system failure but at least one of the devices must succeed for the system to succeed. First we consider simple redundancy.

Simple redundancy.—If n devices are in parallel so that only one of the devices must succeed for the system to succeed, the devices are said to be in simple redundancy. The diagram, or model, of a two-part redundancy system presented in figure 3-6 illustrates this concept. In other words, if part 1 fails, the system can still succeed if part 2 does not fail, and vice versa. However, if both parts fail, the system fails.

From probability theorem 3, chapter 2, we know that the possible combinations of success R and failure Q of two devices is given by

$$R_1R_2 + R_1Q_2 + Q_1R_2 + Q_1Q_2$$

where

R_1R_2 both parts succeed

R_1Q_2 part 1 succeeds and part 2 fails

Q_1R_2 part 1 fails and part 2 succeeds

Q_1Q_2 both parts fail

We also know that the sum of these events equals unity, since they are mutually exclusive (i.e., if one event occurs the others cannot occur). Therefore,

$$R_1R_2 + R_1Q_2 + Q_1R_2 + Q_1Q_2 = 1$$

Because at least one of the parts or devices must succeed in simple redundancy, the probability of this happening is given by

$$R_1R_2 + R_1Q_2 + Q_1R_2 = 1 - Q_1Q_2$$

In simple terms, if the only way the redundant system can fail is by all redundant parts failing, the probability of success must be equal to 1 minus the probability that all redundant parts

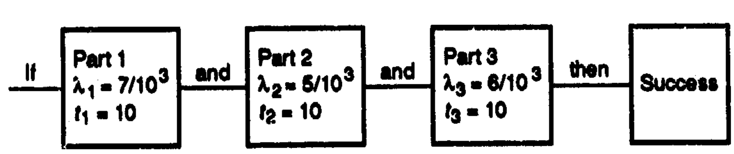

Figure 3-5.—Series model with operating times equal.

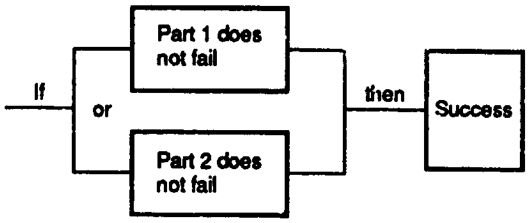

Figure 3-6.—Simple redundancy model.

will fail (i.e., $R = 1 - Q$), from probability theorem 1, chapter 2. This reasoning can be extended to n redundant parts if at least one of the n parts must succeed for the system to succeed.

Example 8: Suppose there are three ways that a space capsule can be guided: (1) automatically with $R_1 = 0.9$, (2) semiautomatically with $R_2 = 0.8$, (3) manually with $R_3 = 0.7$. The model or diagram of successful guiding, assuming that the three ways are independent of each other, is shown in figure 3-7. From probability theorem 3, chapter 2, the possible events are given by

$$R_1R_2R_3 + R_1R_2Q_3 + R_1Q_2R_3 + Q_1R_2R_3 + R_1Q_2Q_3 + Q_1Q_2R_3 + Q_1R_2Q_3 + Q_1Q_2Q_3$$

Because the sum of these probabilities is equal to unity and at least one of the control systems must operate successfully, the probability that guidance will be successful R_{guidance} is

$$\begin{aligned} R_{\text{guidance}} &= R_1R_2R_3 + R_1R_2Q_3 + R_1Q_2R_3 + Q_1R_2R_3 \\ &\quad + R_1Q_2Q_3 + Q_1Q_2R_3 + Q_1R_2Q_3 \\ &= 1 - Q_1Q_2Q_3 = 1 - [(1 - R_1)(1 - R_2)(1 - R_3)] \\ &= 1 - [(1 - 0.9)(1 - 0.8)(1 - 0.7)] \\ &= 1 - [(0.1)(0.2)(0.3)] \\ &= 1 - (0.006) = 0.994 \end{aligned}$$

In general, then, for simple redundancy

$$R_{\text{simple redundant}} = 1 - \prod_{j=1}^{n} Q_j = 1 - \left(Q_1Q_2Q_3 \ldots Q_n\right)$$

where

$\prod_{j=1}^{n} Q_j$ total probability of failure

Q_j total probability of failure of j^{th} redundant part

n total number of redundant parts

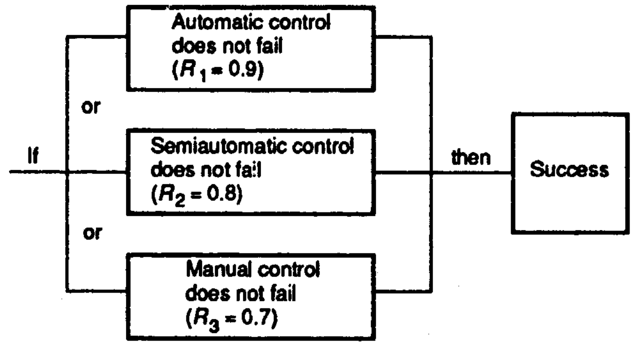

Figure 3-7.—Space capsule guidance model.

Example 9: Find the reliability of the redundant system shown in figure 3-8.

Solution 9: Step 1—Solve for the reliability of parts 1 and 2.

$$R_1 = e^{-\lambda_1 t_1} = e^{-[(120/10^6)\times 10^3]} = e^{-0.120} = 0.887$$

$$R_2 = e^{-\lambda_2 t_2} = e^{-[(340/10^6)\times 10^3]} = e^{-0.340} = 0.712$$

Step 2—Solve for the unreliability of each part.

$$Q_1 = 1 - R_1 = 0.113$$

$$Q_2 = 1 - R_2 = 0.288$$

Solve for the reliability of the redundant system.

$$\begin{aligned} R_{\text{simple redundant}} &= 1 - Q_1Q_2 = 1 - (0.113)(0.288) \\ &= 1 - 0.033 = 0.967 \end{aligned}$$

There is a 96.7 percent chance, therefore, that both parts will not fail during the 1000-hour operating time.

Compound redundancy.—Compound redundancy exists when more than one of n redundant parts must succeed for the system to succeed. This can be shown in a model of a three-element redundant system in which at least two of the elements must succeed, as shown in figure 3-9.

From probability theorem 3, chapter 2, the possible events are

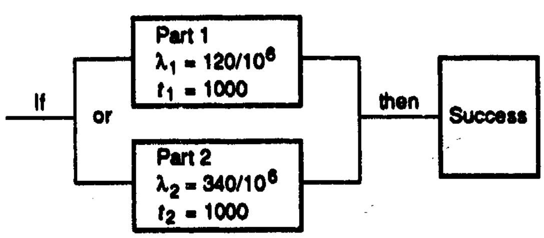

Figure 3-8.—Simple redundancy model using failure rates and operating times.

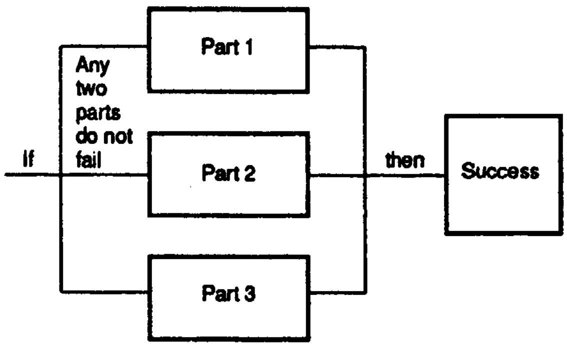

Figure 3-9.—Compound redundancy model.

$$R_1R_2R_3 + R_1R_2Q_3 + R_1Q_2R_3 + Q_1R_2R_3 + R_1Q_2Q_3 + Q_1Q_2R_3 + Q_1R_2Q_3 + Q_1Q_2Q_3$$

To simplify the notation, let $R_1 = R_2 = R_3$ and $Q_1 = Q_2 = Q_3$. This reduces the expression to

$$R^3 + R^2Q + R^2Q + R^2Q + RQ^2 + RQ^2 + RQ^2 + Q^3$$

or

$$R^3 + 3R^2Q + 3RQ^2 + Q^3$$

Because the sum of these probabilities equals unity and at least two of the three parts must succeed, the probability for success is given by

$$R_s = R^3 + 3R^2Q = 1 - (3RQ^2 + Q^3)$$

where $3RQ^2$ represents one part succeeding and two parts failing and Q^3 represents all three parts failing.

Example 10: Assume that there are four identical power supplies in a fire control center and that at least two of them must continue operating for the system to be successful. Let each supply have the same reliability, $R = 0.9$ (which could represent $e^{-\lambda t}$ or R_i or R). Find the probability of system success $R_{\text{simple redundant}}$.

Solution 10: The number of possible events is given by

$$(R + Q)^4 = R^4 + 4R^3Q + 6R^2Q^2 + 4RQ^3 + Q^4$$

The sum of the probabilities of these events equals unity; therefore, the expression for two out of four succeeding is

$$R_s = R^4 + 4R^3Q + 6R^2Q^2 = 1 - (4RQ^3 + Q^4)$$

Substituting $R = 0.9$ and $Q = 1 - 0.9$ gives

$$R_s = 1 - (4RQ^3 + Q^4) = 1 - [4(0.9)(0.1)^3 + (0.1)^4]$$

$$= 1 - [(3.6)(0.001) + 0.0001] = 1 - (0.0036 + 0.0001)$$

$$= 1 - 0.0037 = 0.996$$

Calculation of Reliability for Complete System

To find the reliability for a complete system, begin by developing a model for the system, write the equation for the probability of success from the model, and then use the failure rates and operating times of the system elements to calculate the reliability of the system (refs. 3-6 to 3-8).

Example 11: Consider the system model with series and redundant elements shown in figure 3-10.

Solution 11: The equation can be written directly as

$$R_s = R_1R_2R_3(1 - Q_4Q_5Q_6)$$

where $R_1R_2R_3$ represents the probability of success of the series parts and $(1 - Q_4Q_5Q_6)$ represents the probability of success of the three parts in simple redundancy. If we know that

$$R_1 = 0.99 = e^{-0.01} \qquad R_4 = 0.85$$

$$R_2 = 0.999 = e^{-0.001} \qquad R_5 = 0.89$$

$$R_3 = 0.95 = e^{-0.05} \qquad R_6 = 0.78$$

where R may represent $e^{-\lambda t}$, inherent reliability R_i, or observed product reliability depending on the stage of product

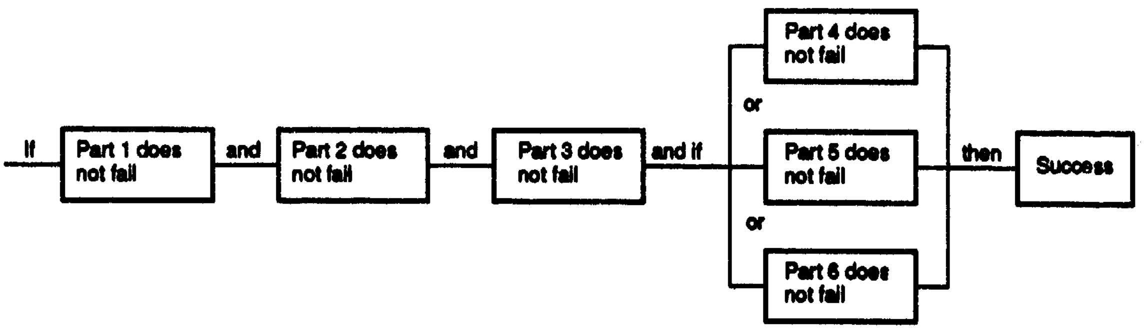

Figure 3-10.—Model of system with series and redundant elements.

development, then the reliability of the system is

$$R_s = e^{-0.01}\, e^{-0.001}\, e^{-0.05}[1 - (1 - 0.85)(1 - 0.89)(1 - 0.78)]$$

$$= e^{-0.061}[1 - (0.15)(0.11)(0.22)] = e^{-0.061}(1 - 0.00363)$$

$$= e^{-0.061}\, e^{-0.0036} = e^{-0.065} = 0.935$$

However, this does not mean that there will be no equipment failures. The system will still succeed even though one or two of the redundant paths have failed.

Example 12: Write the equation for the system shown in figure 3-11.

Solution 12: The equation can be written directly as

$$R_s = R_1R_2[1 - (R_3Q_4Q_5 + Q_3R_4Q_5 + Q_3Q_4R_5 + Q_3Q_4Q_5)](1 - Q_6Q_7)$$

where R_1R_2 is the probability that the two parts in series will not fail, $1 - (R_3Q_4Q_5 + \ldots + Q_3Q_4Q_5)$ is the probability that two out of three of the compound redundant parts will not fail, and $(1 - Q_6Q_7)$ is the probability that both of the simple redundant parts will not fail. If data giving the reliabilities of each part are available, insert this information into the system success equation to find the system reliability.

Example 13: Write the equation for the system shown in figure 3-12.

Solution 13: The equation can be written directly as

$$R_s = R_1R_6R_7\{1 - [Q_2Q_3(1 - R_4R_5)]\}$$

where $R_1R_6R_7$ is the reliability of the series parts, $(1 - R_4R_5)$ is the probability that R_4 or R_5 will fail in the bottom redundant path, and $\{1 - [Q_2Q_3(1 - R_4R_5)]\}$ is the reliability of the three paths in simple redundancy.

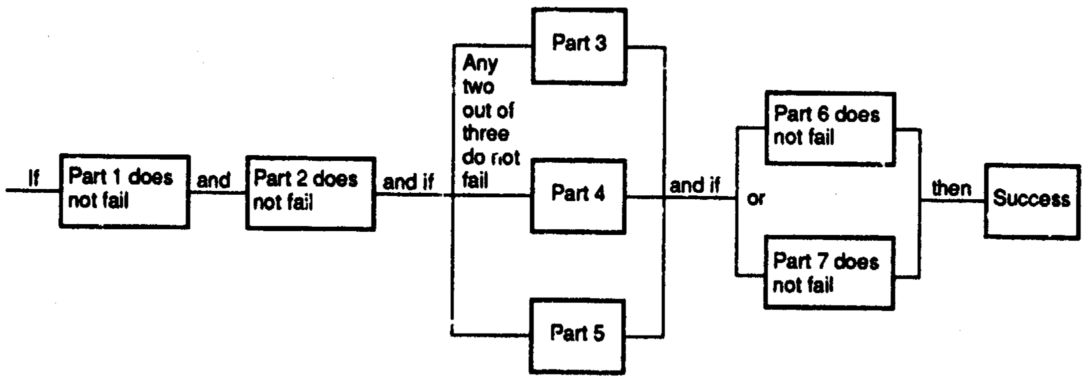

Figure 3-11.—System reliability model using series, simple redundancy, and compound redundancy elements.

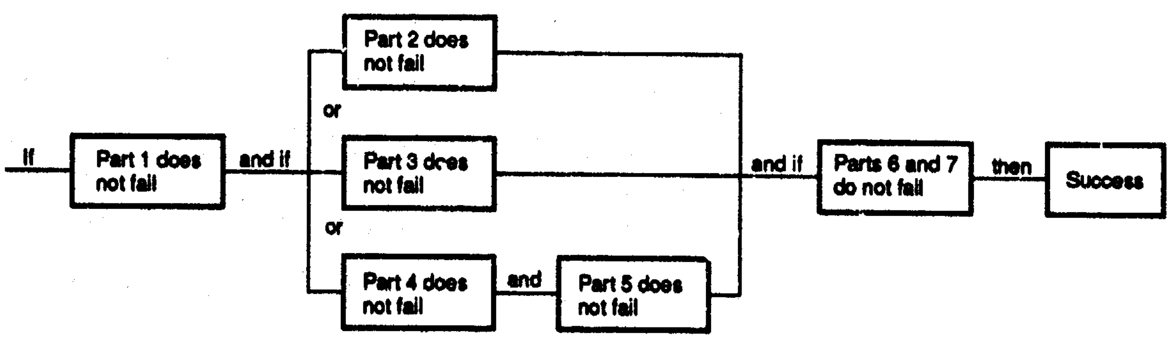

Figure 3-12.—Model with series elements in redundant paths.

Concluding Remarks

Chapter 3 has presented several inportant concepts that you should have clearly in mind:

(1) The exponential distribution $e^{-\lambda t}$ represents the probability that no catastrophic part failures will occur in a product.

(2) The failure rate λ as used in $e^{-\lambda t}$ is a constant and represents the rate at which random catastrophic failures occur.

(3) Although the cause of failure is known, random failures may still occur.

(4) The mean time between failures (MTBF) is the reciprocal of the fail re rate.

(5) In reliability, devices are in series if each one is required to operate successfully for the system to be successful. Devices are parallel or redundant if one or more can fail without causing system failure but at least one of the devices must succeed for the system to succeed.

In addition, you should be able to calculate the following:

(1) The reliability of a device, given failure rate and operating time.

(2) The reliability of devices connected in series from the product rule:

$$R_s = \prod_{j=1}^{n} R_j$$

(3) The reliability of devices connected in simple redundancy from

$$R_{\text{simple redundant}} = 1 - \prod_{j=1}^{n} Q_j$$

(4) The reliability of n devices connected in compound redundancy by expanding $(R + Q)^n$ and collecting the appropriate terms.

And finally, you should be able to combine the four methods described above to calculate the reliability of a total system.

In 1985, alternative methodologies were introduced in the form of computer reliability analysis programs. One such underlying model uses a Weibull failure rate during the burn-in, or "infant mortality," period and a constant failure rate during the steady-state period for electronic devices. Initial results indicate that given a 15- to 40-year system life the infant mortality period is assumed to last for the first year. Of course, the higher the stress of the environment, the shorter the infant mortality period. The point is that there are many ways of performing reliability studies, and different methodologies could be equally appropriate or inappropriate. Appendix C describes five distribution functions that can be used for reliability analysis. Table C-1 shows the time to failure fit for various systems. The basic criteria relate to the distribution of failures with time.

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Reliability Training[1]

1a. Of 45 launch vehicle flights, 9 were determined to be failures. What is the observed reliability?

A. 0.7 B. 0.8 C. 0.9

1b. What is the observed reliability if the next five flights are successful?

A. 0.72 B. 0.82 C. 0.87

1c. After the five successes of part 1b, how many more successes (without additional failures) are required for a reliability of $R = 0.90$?

A. 20 B. 30 C. 40

2. A three-stage launch vehicle has a reliability for each stage of $R_1 = 0.95$, $R_2 = 0.94$, $R_3 = 0.93$.

   a. What is the probability of one successful flight?

   A. 0.83 B. 0.85 C. 0.87

   b. What is the probability of flight failure for part a?

   A. 0.00021 B. 0.15 C. 0.17

   c. What is the probability of two successful flights?

   A. 0.689 B. 0.723 C. 0.757

3. You are taking a trip in your car and have four good tires and a good spare. By expanding $(R + Q)^5$,

   a. How many events (good tires or flats) are available?

   A. 16 B. 32 C. 64

   b. How many combinations provide four or more good tires?

   A. 6 B. 7 C. 16

   c. If $R = 0.99$ for each tire, and a successful trip means you may have only one flat, what is the probability that you will have a successful trip?

   A. 0.980 B. 0.995 C. 0.9990

4. A launch vehicle system is divided into five major subsystems, three of which have already been built and tested. The reliability of each is as follows: $R_1 = 0.95$, $R_2 = 0.95$, $R_3 = 0.98$. The reliability of the overall system must be equal to, or greater than, 0.85. What will be the minimum acceptable reliability of subsystems 4 and 5 to ensure 85-percent reliability?

   A. 0.92 B. 0.95 C. 0.98

5a. A launch vehicle test program consists of 20 test firings requiring 90-percent reliability. Five tests have already been completed with one failure. How many additional successes must be recorded to successfully complete the test program?

A. 13 B. 14 C. 15

5b. Based on the probability (four successes in five flights) what is the probability of achieving successful completion of the test program?

A. 0.04 B. 0.167 C. 0.576

---

[1]Answers are given at the end of this manual.

6. During individual tests of major launch vehicle subsystems, the reliability of each subsystem was found to be

Subsystem 1 = 0.95
Subsystem 2 = 0.99
Subsystem 3 = 0.89
Subsystem 4 = 0.75

Since all subsystems are required to function properly to achieve success, what increase in reliability of subsystem 4 would be necessary to bring the overall system reliability to 0.80?

A. 15 percent B. 20 percent C. 25 percent

7. Solve for the following unknown values:

a. $\lambda = 750 \times 10^{-6}$ failures/hour; $t = 10$ hours; $R = ?$

A. 0.9925 B. 0.9250 C. 0.9992

b. $\lambda = 8.5$ percent failures/10^3 hours; $t = 3000$ hours; $R = ?$

A. 0.9748 B. 0.7986 C. 0.0781

c. MTBF = 250 failures/hour; $t = 0.5$ hour; $R = ?$

A. 0.9802 B. 0.9980 C. 0.9998

d. $R = 0.999$; $t = 10$ hours; $\lambda = ?$

A. 1000×10^{-9} failures/hour B. 10×10^{-6} failures/hour C. 10 percent failures/10^3 hours

e. MTBF = ?

A. 10^4 failures/hour B. 10^5 failures/hour C. 10^6 failures/hour

8. The a priori MTBF prediction of a printed circuit board was 12.5×10^6 hours. Find the number of expected failures during a 10^8-hour (accelerated) life test of 10 circuit board samples.

A. 12.5 B. 80 C. 125

9a. Write the reliability equation for the battery activation success diagram shown below:

If		And	And	And	Then
Battery activate command (part 1)	Passes umbilical path (part 2)	Initiates EBW 1 (part 3) or EBW 2 (part 4)	Ignites initiator 1 (part 5) or initiator 2 (part 6)	Battery activates (part 7)	Success

A. $R_s = R_1R_2(1 - R_3R_4)(1 - R_5R_6)R_7$ B. $R_s = R_1R_2(1 - Q_3Q_4)(1 - Q_5Q_6)R_7$

9b. If $R = 0.9$ for all series and $R = 0.8$ for all parallel parts, solve for R_s.

A. 0.73 B. 0.26 C. 0.67

10. A launch vehicle subsystem is required to be stored for 10 years (use 9000 hours = 1 year). If the subsystem reliability goal is 0.975.

a. What λ is required with no periodic checkout and repair?

A. 2800×10^{-9} B. 28×10^{-9} C. 280×10^{-9}

b. What λ is required with checkout and repair every 5 years? (Assume 100-percent checkout.)

A. 5600×10^{-9} B. 56×10^{-9} C. 560×10^{-9}

c. What λ is required with checkout and repair every year? (Assume 100-percent checkout.)

A. 2800×10^{-9} B. 28×10^{-9} C. 280×10^{-9}

Chapter 4

Using Failure Rate Data

Now that you have a working knowledge of the exponential distribution $e^{-\lambda t}$ and have the fundamentals of series and redundant models firmly in mind, the next task is to relate these concepts to your everyday world. To do this, we explore further the meaning of failure rates, examine variables that affect part failure modes and mechanisms, and then use part failure rate data to predict equipment reliability. We introduce a simple technique for allocating failure rates to elements of a system. The concepts discussed in this chapter are tools the designer can use for trading off reliability with other factors such as weight, complexity, and cost. These concepts also provide guidelines for designing reliability into equipment during the concept stage of a program.

Variables Affecting Failure Rates

In chapter 3 failure rate λ was defined as a constant in time representing the rate of occurrence of random catastrophic failures in the equipment. An actual observation of a constant failure rate is shown in figure 4-1. The results of two tests are shown in this figure. One is an operating life test lasting 4500 hours; the other, a storage test lasting 7000 hours. Each test is discussed separately.

Operating Life Test

The tests involved 7575 parts—3930 resistors, 1545 capacitors, 915 diodes, 1080 transistors, and 105 transformers. One-third of the parts were operated at −25 °F, one-third at 77 °F, and one-third at 125 °F. The parts, tested in circuits (printed circuit boards), were derated no more than 40 percent. The ordinate of the curve shows cumulative failures as a function of operating time. For example, at about 240 hours the first failure was observed, at about 385 hours the second, etc. Several important observations can be made concerning failure rates and failure modes.

Constant failure rate.—Figure 4-1 shows that the failure rate for the first 1600 hours is constant at one failure every 145 hours. This agrees with the constant-λ theory. Bear in mind that constant failure rate is an observation and not a physical law. Depending on the equipment, failure rates may decrease or increase for a period of time.

Random nature.—Notice that the failures in this constant-failure-rate region are random (in occurrence). For example, two diodes fail, then three transistors, then a silicon switch, then a diode, then a trimpot and a resistor, etc.

Repetitive failures.—Figure 4-1 also shows that during the first 1600 hours only two of these failures involved the same type of device. This is important because in most systems the problems that get the most attention are the repetitive ones. It should be apparent in this case that the repetitive failures are not the ones that contribute the most to unreliability (failure rate). And taking corrective action on the repetitive type of failure would only improve the observed failure rate by 18 percent.

Failure modes.—Table 4-1 shows the observed failure modes (the way the failures were revealed) for the transistor, diode, and resistor failures given in figure 4-1. Note in table 4-1(a) that the short failure mode for transistors had an occurrence rate five times that of any other mode. Note also that the eight transistor failures were distributed about evenly in the three environments but that some different failure modes were observed in each environment.

Observe again in table 4-1(b) that the short failure mode for diodes occurred most frequently. The failures were not distributed evenly in each environment, but a different failure mode occurred in each environment.

Resistors failed in two modes (table 4-1(c)): one intermittent resistor at low temperatures and one tolerance failure at high temperatures.

Burn-in.—As shown in figure 4-1 after 1600 hours the failure rate of the 7575 parts dropped by a factor of 7 for the remaining 2900 test hours (3 failures per 2900 hours, failures 12, 13, and 14, as compared with 11 failures per 1600 hours). This is an example of what are commonly called burn-in failures. The first 11 failures represent parts that had some defect not detected by the normal part screening or acceptance tests. Such defects do not reveal themselves until the part has been subjected to operation for some time. As mentioned earlier, eliminating the repetitive failure would only decrease the failure rate in the first 1600 hours by about 18 percent, but if screening tests were sensitive enough to detect all defects, the failure rate would approach the intrinsic failure rate shown in figure 4-1 right from the start.

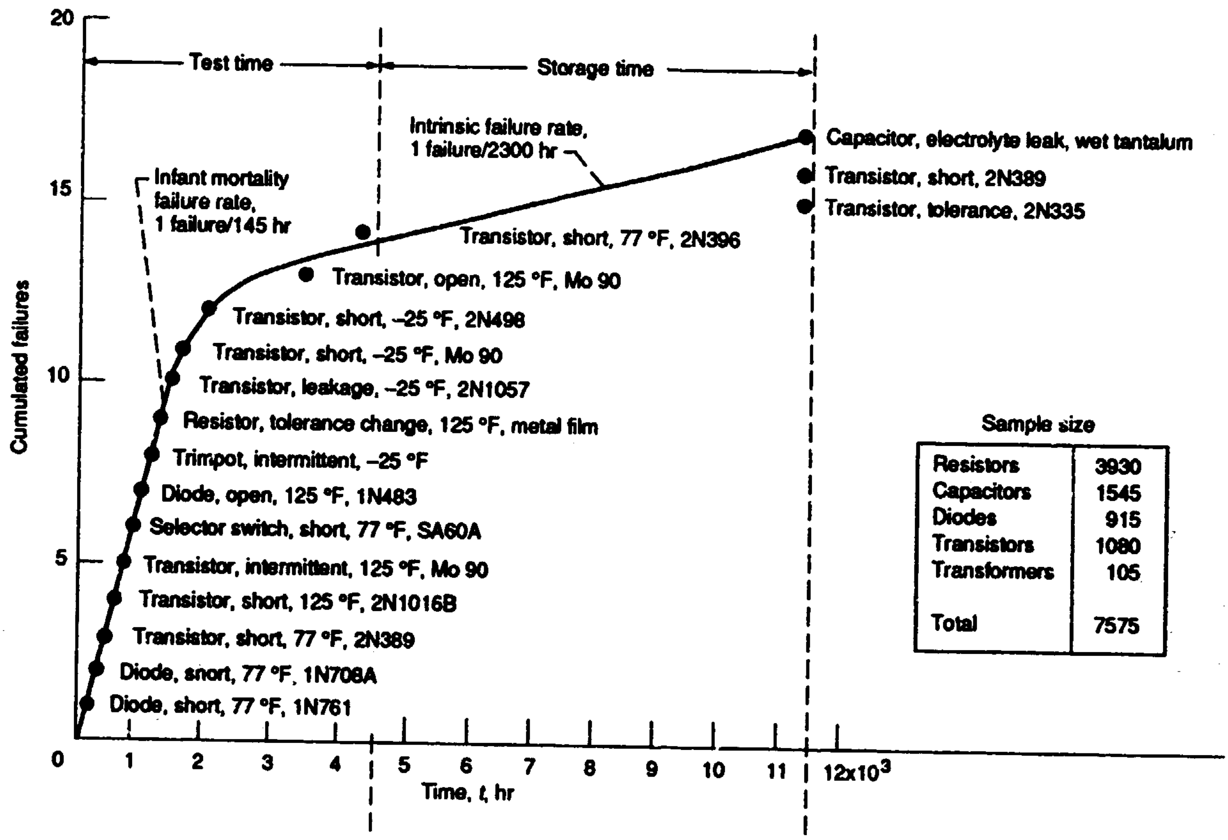

Figure 4-1.—Observed part failures versus test and storage time.

In summary, some of the observed properties of operating failure rates are as follows:

(1) For complex equipment the intrinsic failure rate of electronic parts is usually constant in time.

(2) Failures are random, with repetitive failures representing only a small portion of the problems.

(3) Failure modes of parts and equipment vary, depending on the operating environment.

(4) Most parts have a dominant failure mode. For example, the dominant failure mode for semiconductors is shorting.

(5) Rigid part screening and acceptance criteria can substantially reduce operating failure rates by eliminating early failures.

Storage Test

After the operating test the parts were put in storage for approximately 7000 hours (10 months) and then retested to determine the effect of storage on parts. As shown in figure 4-1, three failures (14, 15, and 16) were observed at the end of the storage period. Note that the average failure rate observed in storage (one failure per 2300 hours) is close to the same rate observed in the previous 2900 hours of operation. Thus, it can be concluded that storage does produce part failures and that the storage failure rate may be as high as the operating rate. Industry is conducting a great deal of research on this problem because storage failure rates become a significant factor in the reliability of unmanned systems and affect considerably the maintenance policy of manned systems.

Summary of Variables Affecting Failure Rates

Part failure rates are thus affected by

(1) Acceptance criteria
(2) All environments
(3) Application
(4) Age or storage

To find ways of reducing the occurrence of part failures, we observe failure modes, learn what caused the failure (the failure stress), determine why it failed (the failure mechanism), and then take action to eliminate the failure. For example, one of the failure modes observed during the storage test was an "open" in a wet tantalum capacitor. The failure mechanism was deterioration of the end seals, which allowed the electrolyte to leak. One obvious way to avoid this failure mode in a system that must be stored for long periods without maintenance is not to use wet tantalum capacitors. If this is impossible, the next best thing would be to redesign the end seals. This would no doubt require further testing to isolate the exact failure stress that produces the failure mechanism. Once isolated, the failure mechanism can often be eliminated through redesign or additional process controls.

TABLE 4-1.—FAILURE MODES

(a) Transistors

Observed part failure mode	Temperature, °F			Total failures	Observed failure rate, failures/hr
	−25	77	125		
Open	------	-----	MD-90	1	$0.206/10^6$
Short	MD-90 2N498	2N389 2N396	2N1016B -------	5	$1.03/10^6$
Intermittent	------	-----	MD-90	1	$.206/10^6$
Leakage	2N1057	-----	-------	1	$.206/10^6$
Totals	3	2	3	8	$1.65/10^6$

(b) Diodes

Open	----	------	1N483	1	$0.24/10^6$
Short	----	1N761 1N708A SA60A	-----	3	$.73/10^6$
Totals	0	3	1	4	$0.97/10^6$

(c) Resistors

Intermittent	Trimpot	-----	-------	1	$0.06/10^6$
Tolerance	------	-----	Metal film	1	$.06/10^6$
Totals	1	0	1	2	$0.12/10^6$

TABLE 4-3.—STRESS RATIOS THAT MEET ALLOCATION REQUIREMENT

Part temperature, °C	Stress ratio, W					
	0.1	0.2	0.3	0.4	0.5	0.6
	Failure rate of derated part per 10^6 hr, λ_D					
30					0.23	0.22
40				0.24		
50			0.24			
60		0.25				
70	0.25					

TABLE 4-2.—FAILURE RATE CALCULATION

(a) Tactical fire control station logic gate

Component	Stress ratio	Number used, N	Failure rate of derated part at 40 °C λ_D, failures/10^6 hr	Application factor for vehicle, ground mounted, K_A	Total failure rate, $\lambda_T = N\lambda_D K_A$, failures/$10^6$ hr
Resistor, composition (2000 Ω)	0.5	1	0.0035	10	0.035
Resistor, composition (180 000 Ω)	.5	1	.0035	↓	.035
Resistor, composition (22 000 Ω)	.6	1	.0038		.038
Resistor, composition (6500 Ω)	.5	2	.0035		.070
Transistor, germanium (PNP type)	< 1 W; 0.4 normalized junction temperature	1	1.3	8	10.400
Diode, 1N31A	.3	1	3.5	5	17.500
					Total, $\lambda_c = \Sigma\ \lambda_T = 29.68$

(b) Proposed logic gate

Component	Stress ratio	Number used, N	Failure rate of derated part at 40 °C λ_D, failures/10^6 hr	Application factor for vehicle, ground mounted, K_A	Total failure rate, $\lambda_T = N\lambda_D K_A$, failures/$10^6$ hr
Resistor, film (1300 Ω)	0.8	1	0.19	0.3	0.057
Resistor, film (3320 Ω)	.2	↓	.14	.3	.042
Resistor, film (46 600 Ω)	.2		.14	.3	.042
Transistor, silicon (NPN type)	< 1 W; 0.15 normalized junction temperature		.165	8	1.320
Diode, 1N31A	.2	5	3.0	5	75.000
					Total, $\lambda_c = \Sigma\ \lambda_T = 76.46$.

One of the best known methods of representing part failures is the use of failure rate data. Figure 4-2 (from ref. 4-1) shows a typical time-versus-failure-rate curve for flight hardware. This is the well-known "bathtub curve," which over the years has become widely accepted by the reliability community. It has proven to be particularly appropriate for electronic equipment and systems. It displays the sum of three failure rate quantities: quality (QFR), stress (SFR), and wearout (WFR).

Zone I, the infant mortality period, is characterized by an initially high failure rate (QFR). This is normally the result of poor design, use of substandard components, or lack of adequate controls in the manufacturing process. When these mistakes are not caught by quality control operations, an early failure is likely to result. Early failures can be eliminated by a "burn-in" period during which time the equipment is operated at stress levels closely approximating the intended actual operating conditions. The equipment is then released for actual use only when it has successfully passed through the burn-in period. For most well-described complex equipment, a 100-hour failure-free burn-in is usually adequate to cull out a large proportion of the infant mortality failures caused by stresses on the parts.

Zone II, the useful life period, is characterized by an essentially constant failure rate (SFR). This is the period dominated by chance failures. Chance failures are those failures that result from strictly random or chance causes. They cannot be eliminated by either lengthy burn-in periods or good preventive maintenance practices.

Equipment is designed to operate under certain conditions and to have certain strength levels. When these strength levels are exceeded because of random unforeseen or unknown events, a chance failure will occur. Although reliability theory and practice are concerned with all three types of failure, the primary concern is with chance failures, since they occur during the useful life of the equipment. Figure 4-2 is somewhat deceiving because zone II is usually much longer than zone I or III. The time when a chance failure will occur cannot be predicted, but the likelihood or probability that one will occur during a given period of time within the useful life can be determined by analyzing the equipment design. If the probability of a chance failure is too great, either design changes must be introduced or the operating environment made less severe.

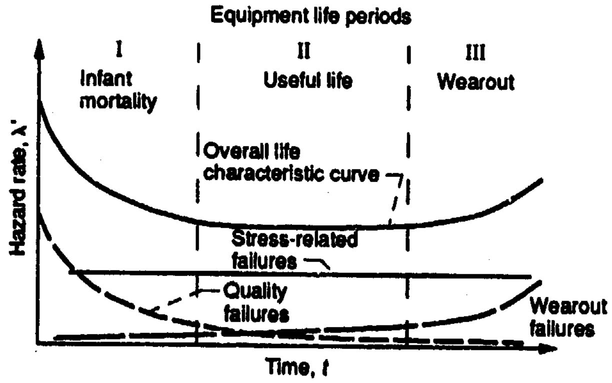

Figure 4-2.—Hazard rate versus equipment life periods.

The SFR period is the basis for the application of most reliability engineering design methods. Because it is constant, the exponential distribution of time to failure is applicable and is the basis for the design and prediction procedures spelled out in documents such as MIL-HDBK-217E (ref. 4-2).

The simplicity of the approach (utilizing the exponential distribution, as previously indicated) makes it extremely attractive. Fortunately, it is widely applicable for complex equipment and systems. If complex equipment consists of many components, each having a different mean life and variance that are randomly distributed, then the system malfunction rate becomes essentially constant as failed parts are replaced. Thus, even though the failures might be wearout failures, the mixed population causes them to occur at random intervals with a constant failure rate and exponential behavior. This has been verified for much equipment from electronic systems to rocket motors.

Zone III, the wearout period is characterized by an increasing failure rate (WFR) as a result of equipment deterioration due to age or use. For example, mechanical components, such as transmission bearings, will eventually wear out and fail regardless of how well they are made. Early failures can be postponed and the useful life extended by good design and maintenance practices. The only way to prevent failure due to wearout is to replace or repair the deteriorating component before it fails.

Because modern electronic equipment is almost completely composed of semiconductor devices that really have no short-term wearout mechanism, except for perhaps electromigration, one might question whether predominantly electronic equipment will even reach zone III of the bathtub curve.

Different statistical distributions might be used to characterize each zone. Hazard rate has been defined for five different failure distribution functions, see figure C-1 in the appendix. Depending on which distribution fits the hazard rate data best, a failure distribution function can be selected. The infant mortality period for the typical hazard rate in figure 4-2 might be represented by the Weibull distribution, the useful life period by the exponential distribution, and the wearout period by the log normal distribution.

Part Failure Rate Data

It is common in the field of reliability to represent part integrity or reliability in terms of failure rate or mean time between failures (MTBF). In general, part failure rates are presented as a function of temperature and electrical stress as shown in figure 4-3. The family of curves on the graph represents different applied electrical stresses in terms of a stress ratio or derating factor. For example, if a part is to operate at temperature A and is derated 20 percent (stress ratio, 0.8), that part will have a failure rate of $\lambda = 0.8$ as shown.

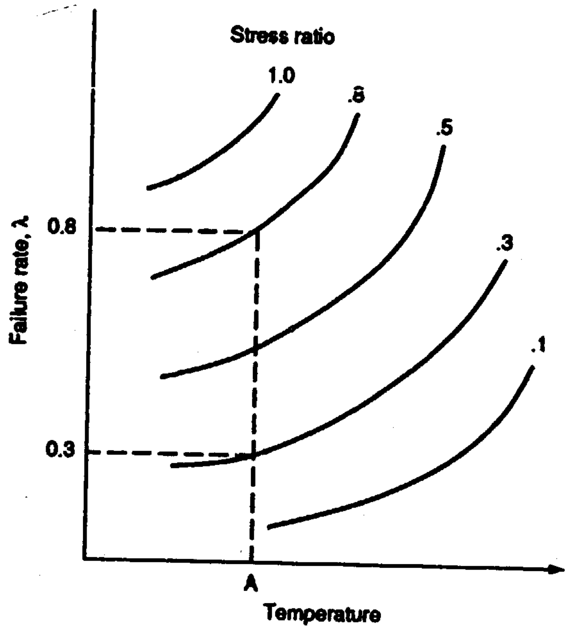

Figure 4-3.—Failure rate versus electrical stress ratio and temperature.

If the part is derated 70 percent (stress ratio, 0.3), the part will have a failure rate of $\lambda = 0.3$, etc. Failure rate is usually given in failures per 10^6 hours, although as indicated in chapter 3 other dimensions are used depending on who publishes the data.

The current authoritative failure rate data published by the Department of Defense are in MIL-HDBK-217E (ref. 4-2). The MIL-HDBK-217 series is a direct result of the 1952 AGREE effort mentioned in chapter 1. The publications listed in table 1-1 as well as references 4-3 to 4-5 are also offshoots of this effort to meet the need for authoritative, statistically based part failure rates. Because new data on both existing and new state-of-the-art parts are constantly being generated and analyzed, failure rate handbooks do change. Therefore, be sure to use the latest version available. Even the latest version of the data used for compiling the handbook may not represent the parts you are using. The best procedure is to use your own failure rate data.

As emphasized in chapter 3 failure rates are statistical, and there is no such thing as an absolute failure rate. Consider the simple definition of failure rate:

$$\lambda = \frac{\text{Number of observed failures}}{\text{Total operating time}}$$

Obviously, if today we observe two failures in 100 hours and tomorrow we accumulate no more failures, the new failure rate is two failures in 124 hours. Then, if a failure occurs in the next 1-hour period, the failure rate is three failures in 125 hours. Therefore, we can never know what the true failure rate is, but we can determine representative failure rates or best estimates from many hours of observed operating time. This type of failure rate data is presented in the MIL-HDBK-217 series.

Improving System Reliability Through Part Derating

The best way to explain how to derate a component is to give an example. Consider two 20-V wet slug tantalum capacitors, both to be operated at a component temperature of 60 °C. One is to be operated at 20 V and the other at 12 V. First, find the stress ratio or operating-to-rated ratio for both applications:

$$\text{Stress ratio} = \frac{\text{Operating voltage}}{\text{Rated voltage}}$$

Hence, one capacitor has a stress ratio of 1.0,

$$\text{Stress ratio} = \frac{20\ \text{V}}{20\ \text{V}} = 1.0$$

and the other, a stress ratio of 0.6,

$$\text{Stress ratio} = \frac{12\ \text{V}}{20\ \text{V}} = 0.6$$

(A stress ratio of 0.6 means the same as "derating" the component 40 percent.) To find the failure rate λ for each capacitor, go to the MIL-HDBK-217E (ref. 4-2) table for MIL-C-3965 glass-sealed wet slug capacitors. Move horizontally across the 60 °C line to the vertical 0.6 and 1.0 stress ratio columns and read directly:

$$\lambda_{0.6} = 0.12 \text{ failure per } 10^6 \text{ hours}$$

$$\lambda_{1.0} = 0.57 \text{ failure per } 10^6 \text{ hours}$$

As mentioned earlier, failure rates are not absolute; therefore, the failure rates just calculated for the two capacitors are not absolute. In other words, we cannot state definitely that one will fail at the rate of 0.12 per 10^6 hours and the other at 0.57 per 10^6 hours when used in the system. We can say, at least, that the nonderated capacitor is expected to have a failure rate 4.75 times that of the derated one. If we derated still further, say 90 percent, $\lambda_{0.1} = 0.0013/10^6$, we could expect the capacitor to be 438 times more reliable than the nonderated capacitor. This is, of course, the reason for derating in the first place.

The same failure rate information is presented on the opposite page of MIL-HDBK-217E in figure 4-3 format. Although the λ values must be approximated from the curves, this form of presentation shows graphically the effects of temperature and stress on failure rate and also the effect of not derating.

Use of Application Factor

Thus far only the stress ratio and the ambient temperature of the component have been considered in the derated failure rate λ_D. However, other stresses, such as vibration, shock, and humidity, also affect failure rate. These environmental factors are taken into account by assigning a weighting application factor K_A. Thus, the total failure rate λ_T becomes

$$\lambda_T = \lambda_D K_A$$

The K_A varies from component to component and by application. MIL-HDBK-217E (ref. 4-2) lists five applications: ground, vehicle-mounted ground, shipboard, airborne, and missile. Thus, if our two capacitors are used in a missile system, their failure rates become

$$\lambda_{0.6} = 0.12/10^6 \times 25 = 3.0/10^6$$

$$\lambda_{1.0} = 0.57/10^6 \times 25 = 14.25/10^6$$

The K_A factor includes the failure rate for the connection technique normally associated with that part class, except for wires and cables.

Predicting Reliability From Part Failure Rate Data

We have shown so far that the failure rate of a part is given by $\lambda_D K_A$ and, as shown in chapter 3, the reliability of a part used in a circuit or system can be estimated from $R = e^{-\lambda t}$. Further, we can estimate the reliability of a system from

$$R_s = \exp\left(-\sum_{i=1}^{n} \lambda_i t_i\right)$$

where

λ_i failure rate of i^{th} part

t_i operating time of i^{th} part

This is also discussed in chapter 3.

For example, table 4-2(a) shows a reliability estimate for a tactical fire control station logic gate. The total failure rate of each part type in the circuit is shown as $\lambda_T = N\lambda_D K_A$. The expected failure rate of the circuit λ_e is then found from

$$\lambda_e = \sum_{i=1}^{n} \lambda_i = 29.68 \text{ failures}/10^6 \text{ hours}$$

The reliability estimate for a logic gate proposed for another system is shown in table 4-2(b). Note that the complexity (number of parts) is higher for the proposed circuit than for the tactical circuit by a ratio of 9:7 and the estimated failure rate is higher by a factor of 2.6. This is possible because, in spite of greater derating, the failure rates of most of the proposed components are higher and the failure contribution of the five diodes alone is double the total failure rate of the tactical circuit.

These calculations are for an operating circuit. Now consider the effects of a nonoperating circuit on the mission model. From figure A-3 in appendix A, the operating application factor for ground electronics equipment is given as 5×10^3. The nonoperating application factor is 8×10^2. The scale factor for a nonoperating circuit using operating failure rates is given by

$$K_s = \frac{K_{A_{NO}}}{K_{A_O}} = \frac{0.8\times10^3}{5.0\times10^3} = 0.16$$

The expected failure rate for a nonoperating circuit is given by

$$\lambda_e = K_s \sum \lambda_t = 0.16 \times 29.68/10^6$$

$$= 4.75 \text{ failures}/10^6 \text{ hours}$$

The operating and nonoperating times during a mission are used in the model to calculate reliability.

The reliability of either circuit, operating or not, as discussed in chapter 3, would be given by

$$R_c = e^{-\lambda_c t_c}$$

where

λ_c circuit failure rate

t_c operating time of circuit

Predicting Reliability by Rapid Techniques

The preceding logic gate illustration is an example of a reliability prediction based on detailed knowledge of parts population and stress. In many situations, however, this type of detailed prediction is not possible. Some situations that come to mind are concept and tradeoff studies where detailed parts counts are not available, where operating stress levels have not been determined, and where time or personpower is limited. Fortunately, a number of rapid reliability prediction techniques are available. One good technique is presented in detail in MIL-HDBK-217E (ref. 4-2). Usually, one or more of these techniques can be used. Although the results lack the detail of the logic gate example, these methods aid in quickly screening candidate designs and help managers make sound decisions.

Use of Failure Rates in Tradeoffs

The failure rate tables and derating curves are useful from the designer's point of view because they provide knowledge for making reliability tradeoffs and permit a more practical method of establishing derating requirements. For example,

suppose we have two design concepts for performing some function. If concept A is found to have a failure rate that is 10 times higher than that of concept B, it can be expected that concept B will fail one-tenth as often as concept A. If it is desirable to use concept A for other reasons, such as cost, size, performance, or weight, the derating failure rate curves can be used to improve concept A's failure rate (e.g., select components with a lower failure rate, derate the components more, or both). An even better approach is to find ways to reduce the complexity and thus the failure rate of concept A.

As another example of the use of failure rate data in tradeoffs, consider figure 4-4. This figure gives a failure-rate-versus-temperature curve for the electronics of a complex (over 35 000 parts) piece of ground support equipment. The curve was developed as follows:

(1) A failure rate prediction was performed by using component failure rates and their application factors K_A for an operating temperature of 25 °C. The resulting failure rate was chosen as a reference point, as indicated on the curve.

(2) Predictions were then made by using the same method for temperatures of 50, 75, and 100 °C. The ratios of these predictions to the reference point, 25 °C, were plotted versus component operating temperature, with the resulting curve for the equipment. This curve was then used to provide tradeoff criteria for using air-conditioning versus blowers to cool the equipment. To illustrate, suppose the maximum operating temperatures expected are 50 °C with air-conditioning and 75 °C with blowers. Suppose further that the required failure rate for the equipment, if the equipment is to meet its reliability goal, is one failure per 50 hours. A failure rate prediction at 25 °C might indicate a failure rate of one per 100 hours. Referring to figure 4-4, we see that the maximum allowable

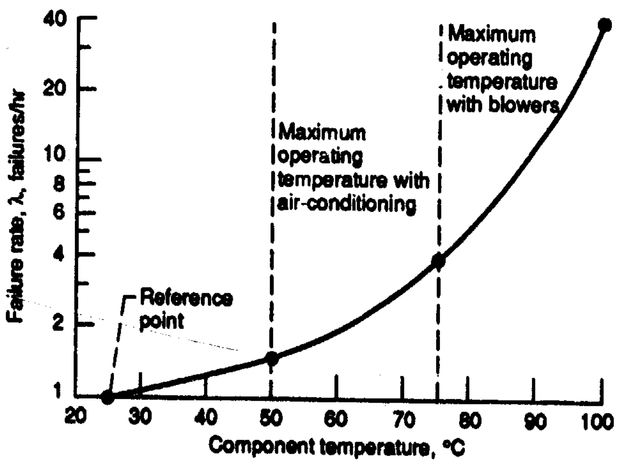

Figure 4-4.—Predicted failure rate ratios versus temperature for ground support equipment (electronics).

operating temperature is therefore 60 °C, since the maximum allowable failure rate ratio is $\lambda = 2$. In other words, at 60 °C the equipment failure rate will be $(1/100) \times 2 = 1/50$, which is the required failure rate. If blowers are used for cooling, the equipment must operate at temperatures as high as 75 °C; if air-conditioning is used, the temperature need not exceed 50 °C. Therefore, it would appear that we must use air-conditioning if we are to meet the reliability requirement.

But other factors must be examined before we arrive at a final decision. Whatever type of cooling equipment is selected, total system reliability now becomes

$$R_T = R_s R_c$$

Therefore, the effect on the system of the cooling equipment's reliability must be calculated. An even more important consideration is the effect on system reliability should the cooling equipment fail. Because temperature control appears to be critical, loss of temperature control may have serious system consequences. Therefore, it is too soon to rule out blowers entirely. A failure mode, effects, and criticality analysis (FMECA) must be made on both cooling methods to examine all possible failure modes and their effects on the system. Only then will we have sufficient information with which to reach a sound decision.

Nonoperating Failures

As pointed out in discussing figure 4-1, parts continue to fail when not in use. These nonoperating failures are converted to nonoperating failure rates. In general, electronic parts fail less frequently in the nonoperating mode than in the operating mode. Certain hydraulic and mechanical parts, however, fail more frequently in the nonoperating mode. For many military weapon systems the nonoperating role is the norm. Missiles may remain in storage depots or in a dormant standby condition for months or years before being fired. Likewise, many subsystems in orbiting satellites are passive most of the time. In these cases, system reliability becomes

$$R_s = R_{\text{operating}} R_{\text{nonoperating}} = e^{-\Sigma\lambda_o t_o} e^{-\Sigma\lambda_n t_n}$$

$$= e^{-\Sigma(\lambda_o t_o + \lambda_n t_n)}$$

Because nonoperating time t_n can be many orders of magnitude greater than operating time t_o, nonoperating failures often represent a major portion of total system failures. There is, hence, increased interest in how, why, and at what rate nonoperating parts fail. Some recent studies gave indications that nonoperating failure rates may not be as high as some handbooks might indicate. Turn-on and test stress failures affect the count of true nonoperating failures.

Applications of Reliability Predictions to Control of Equipment Reliability

Even though we have indicated that reliability predictions do not give absolute answers, several things can be done to make these predictions more meaningful. Consider the concept stage—the most important stage to reliability because the potential reliability of the system is fairly well defined by the time the concept is selected. To predict the potential reliability, we usually must

(1) Predict the number and types of parts to be used in the system
(2) Choose a part derating factor
(3) Choose a maximum operating environment

Now, to make the prediction meaningful, we must

(1) Place a complexity limit (the limit predicted in (1) above) on the system
(2) Direct the minimum amount of derating allowed
(3) Approve part applications to ensure that parts are used in the correct manner and will be operating within the assumed environmental limits

Standardization as a Means of Reducing Failure Rates

Another means of establishing control over the failure rate (reliability) of a product is to employ standardization principles. As an extreme illustration, suppose we need 1000 transistors for a system and allow each transistor to be a different type, bought from a different vendor. If each vendor part has five failure mechanisms peculiar to that vendor, the system will have $5 \times 1000 = 5000$ failure mechanisms. If, through testing, we find one failure mechanism and eliminate it, we have reduced the failure mechanisms of the system by a factor of only 1/5000. If, on the other hand, we could require that the 1000 transistors be of the same type and bought from the same vendor and if this vendor's part has five failure mechanisms, the system also has only five ways to fail. If we then eliminate one of these failure mechanisms by testing, we have reduced the failure mechanisms of the system by one-fifth, or 20 percent. You can readily see, though, that an initial reliability prediction would be the same in both cases because each system uses 1000 transistors. Also the chance of observing the failure mode will increase by five times. Quick failure mode detection and correction is important in reliability work.

Allocation of Failure Rates and Reliability

In most Government contracts reliability goals or requirements are specified at the system level only. The apportioning of these goals to elements of the system is left to the contractor. This apportioning process is called allocation in reliability engineering.

In a similar fashion the reliability organization usually allocates the system reliability or failure rate requirements only to the assembly or subassembly level. The designers, therefore, must allocate goals to the part level for the component for which they are responsible. All allocations at any level are performed in such a manner that, when the failure rates or reliabilities of the system elements are combined (by using the prediction methods discussed in chapter 3), the goal or requirement for the system is obtained. The allocation process, together with part failure rate data, provides the designer with a method for determining how good the parts must be if the design is to meet the specified reliability.

The first method of allocating failure rates is called the assembly method. If the reliability requirement of a system, subsystem, or assembly, as well as the operating time interval, is known, the required failure rate may be calculated from

$$R = e^{-\lambda t}$$

The resulting failure rate can then be divided by the anticipated number of parts to be used to allocate the average failure rate requirement down to the part level.

Example 1: Consider a missile that has a reliability requirement of 0.99 for a flight period of 0.5 hour. The estimated complexity of the missile is 10 000 active parts. Find the average failure allocation for each part.

Step 1—Write the reliability equation for the missile.

$$R_m = e^{-\lambda_m t_m}$$

Step 2—Solve for the failure rate of the missile.

$$R_m = 0.99 = e^{-0.01} = e^{-\lambda_m (0.5)}$$

Equating exponents gives

$$\lambda_m(0.5) = 0.01 = \frac{0.01}{0.5} = 0.02 \text{ failure per hour}$$

Step 3—Solve for the average part failure rate λ_p

$$\lambda_p = \frac{\lambda_m}{\text{Number of parts}} = \frac{0.02}{10\ 000} = 2/10^{-6}$$

The assumptions made in this example and the method of allocating are as follows:

(1) All parts are required for system success.
(2) All parts fly the entire mission.

Thus, if the system reliability requirement is to be met, high failure contributors must be offset by low ones so that their average failure rate $\lambda_p \leq 2/10^{-6}$.

Let us continue this example by further examining one specific part class. For MIL-HDBK-217E (ref. 4-2) values this flight failure rate λ_p includes the K_A value associated with each part class (i.e., 1.5 to 100 for resistors). Thus, the λ_p for the fixed-film resistors (MIL-R-22684) in the system becomes

$$\lambda_p = \lambda_D K_A$$

$$\lambda_D = \frac{\lambda_p}{K_A} = \frac{2/10^{-6}}{8} = 0.25/10^{-6}$$

Now that we have determined the λ_D requirement, we are ready for the next step.

A quick scan of the λ_D values, extracted from MIL-HDBK-217E (ref. 4-2) and shown in table 4-3, for this type of fixed-film resistor reveals the part temperature and stress ratio combinations that provide $\lambda_D \leq 0.25/10^{-6}$. The anticipated operating temperature, say 40 °C maximum, would further reduce the acceptable combinations, leading to the conclusion that this type of part must be derated 60 percent or more to meet the reliability apportionment.

The second method of allocating failure rates is called the equal-risk method and can also be applied when allocating reliability goals to several elements within a system (see fig. 4-5).

The reliability assigned to each element is given by

$$R_e = \sqrt[n]{R_s}$$

where n is the number of elements. The same reliability goal is assigned to each element, hence, the name "equal risk."

Example 2: A fire control system computer is made up of 10 logic racks and has a reliability requirement of 0.999. Allocate a reliability goal to each of the logic racks.

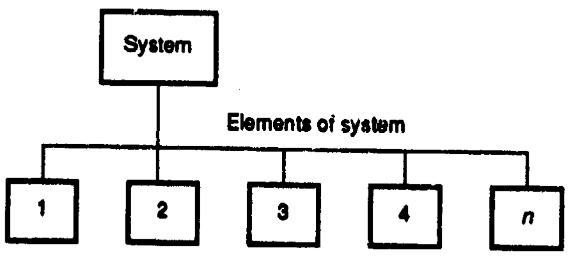

Figure 4-5.—System elements model.

$$R_{\text{rack}} = \sqrt[10]{0.999} = \left(e^{-0.001}\right)^{1/10} = e^{-0.001} = 0.9999$$

The part failure rates λ_p of each rack can then be allocated as shown in example 1:

(1) All subelements operate for approximately the sa[me] period.

(2) There is no significant difference in the failure rate complexity of the subelements.

Many other methods of allocating reliability goals take into account operating time, complexity, cost, maintainability, the state of the art, and other factors. See references 4-2 to 4-12.

Importance of Learning From Each Failure

When a product fails, a valuable piece of information about this product has been generated. We now have the opportunity to learn how to improve the product if we take the right actions.

Failures can be classified into categories:

(1) Catastrophic failures—for example, a shorted transistor or an open wire-wound resistor

(2) Degradation failures—for example, change in the gain of a transistor or the value of a resistor

(3) Wearout failures—for example, the wear of brushes in an electric motor

These three principal failure categories can be broken down further:

(1) Independent failures—For example, a shorted capacitor in a radiofrequency amplifier has nothing to do with a low-emission cathode in a picture tube.

(2) Cascade failures—For example, the shorted capacitor in the radiofrequency amplifier causes excessive current to flow in its transistor and burns the collector beam lead open.

(3) Common mode failures—For example, uncured resin is present in motors.

By using these categories and a good failure reporting, analysis, corrective action, and concurrence system, much can be learned from each failure. Failure analysis is required to determine what caused the part to fail. Corrective action ensures that something was done about the cause. Concurrence keeps management informed on what is being done to avoid another failure. These data enable all personnel involved to compare the part ratings with the use stresses and thus verify that the part is being used with a known margin.

Failure Reporting, Analysis, Corrective Action, and Concurrence

A number of different methods can be used to record reliability data for any given project. The Department of Defense has standardized a method on DD form 787-1. A

simple form that tells the whole story on one sheet of paper is NASA-C-8192 (fig. 4-6). The method that you use to record reliability data will have to fit your needs. Keep your form simple and easy to fill out, and get approval from management.

Case Study—Achieving Launch Vehicle Reliability

Design Challenge

The launch vehicle studied requires the highest acceleration and velocity and the shortest reaction time of any developed. As such, the design challenges were formidable; typical in-flight environments include random vibration of 61 g's rms up to 3 kHz, mechanical shock at 25 000 g's peak (between 5 and 10 kHz), linear acceleration well in excess of 100 g's, acoustics of 150 dB, and aerodynamic heating up to 6200 °F. The development philosophy was for a vehicle to be launched from a tactical silo with the initial design. Although many changes occurred during the 13-year development, the first flight test vehicle was not greatly different from the 70 now deployed.

Subsystem Description

The vehicle is launched from an underground silo, which also serves as a "storage container" during the multiyear design life. Adjacent to the silo and integral to it is a small compartment housing the ground support equipment. This equipment is used to conduct periodic tests of the vehicle electronics, to prepare the vehicle for launch, and to launch the vehicle. It also maintains the silo environment at 80 ± 10 °F and 50 percent or less relative humidity.

The vehicle is predominantly in a power-off storage mode when deployed in its silo. A periodic test of flight electronics is conducted automatically every 4 weeks. In a multiyear design life the flight electronics accumulate about 11 min of operating time and 43 830 hours of storage time. The ratio of storage time to operating time is nearly 240 000:1.

Approach to Achieving Reliability Goals

Reliability mathematical models were developed early in the research and development program. From these models it was apparent that the following parameters were the most important in achieving the reliability goals:

(1) Electronic storage failure rate during a multiyear design life (i.e., storage failures)

(2) Percent testability of missile electronics (i.e., MIL-STD-471A, ref. 4-6)

(3) Periodic test interval for missile electronics

(4) Severity of in-flight environments (acceleration, shock, vibration, and aerodynamic heating)

Launch and Flight Reliability

The flight test program demonstrated the launch and flight reliability of the vehicle. The ultimate flight program success ratio of 91 percent exceeded the overall availability-reliability goal by a comfortable margin.

Field Failure Problem

Twenty-six guidance sections failed the platform caging test portion of the launch station periodic tests (LSPT's). These failures resulted in a major alarm powerdown. An investigation was conducted.

Description of launch station periodic tests.—The system test requirements at the site include a requirement for station periodic tests upon completion of cell or vehicle installation and every 28 days thereafter. LSPT's check the overall system performance to evaluate the readiness of a cell. During an LSPT the software initiates a test of the vehicle and ground equipment, data processing system, and radar interfaces. Any nonconformance during an LSPT is logged by the data processor and printed out, and the time from initiation of LSPT to failure is recorded. During an LSPT the platform spin motor is spun up and held at speed for approximately 10 sec. After this the system is returned to normal.

An LSPT consists of two phases:

(1) Spinup—a powerup phase to spin the gyros, align the platform, verify platform null, and check airborne power supply operation

(2) A detailed test of airborne electronics in the radio-frequency test phase

Initial failure occurrence.—Cell 3 on remote farm 1 (R1C3) experienced an LSPT failure (a major alarm powerdown) 5.936 sec after "prep order," the command to get the vehicle ready to launch. The failure did not repeat during four subsequent LSPT's. R1C3 had previously passed three scheduled LSPT's before failure. A total of four cells on remote farms 1 and 2 had experienced similar failures. Two of the failures occurred at 5.360 sec (an inverter test to determine if ac power is available). Two occurred at 5.936 sec (caging test to determine if the platform is nulled to the reference position; see fig. 4-7).

Replacement of failed guidance and control sections (G&C) 28, 102, and 86 led to successful LSPT's. G&C 99, which failed only once during in-cell testing, was left on line. G&C's 28, 102, and 86 were returned to Martin Marietta, Orlando, for analysis of the presumed failed condition.

Failure verification and troubleshooting.—A test plan was generated that permitted testing of the failed G&C's in a horizontal marriage test and a G&C test to maximize the probability of duplicating the field failures. Test results confirmed site failures for both the caging null and the inverter null during a horizontal marriage test on G&C 102, a G&C level test on G&C's 28 and 86, and an autopilot level test on G&C 102. G&C 102 failed caging null four times and inverter

NASA
Lewis Research Center

PROBLEM REPORT

REPORT #: PR
Page 1 of ___

SECTION A

1. ASSEMBLY NAME:
2. ASSEMBLY NO.:
3. PROJECT NAME:
4. S/N:
5. HARDWARE TYPE: Eng/Qual ☐ Flight ☐ GSE ☐
6. PROCEDURE NO. & PARAGRAPH:
7. TEST TYPE:
8. LOCATION (building & room):
9. TEST HOURS/CYCLES COMPLETED:
10. CONDITIONS (temp., humidity, freq., rate, etc.):
11. REFERENCE DOCUMENT:
12. REQUIREMENT:
13. DATE OF OCCURRENCE
14. PROBLEM DESCRIPTION:
15. INITIATOR (sign & date):
16. LOCATION (mail stop):
17. PHONE EXT.:

SECTION B

18. PROBLEM ANALYSIS:
19. ROOT CAUSE OF PROBLEM:
20. SOFTWARE TROUBLE REPORT #:
21. DEFECTIVE SUB-ASSY INFORMATION:
Name: ___ Number: ___ S/N: ___
22. DEFECTIVE COMPONENT INFORMATION:
Name: ___ Number: ___
S/N: ___ Date/Lot Code: ___ Model No./Value: ___
Supplier (name & city): ___
23. ANALYST (sign & date):
24. LOCATION (mail stop):
25. PHONE EXT.:

SECTION C

26. DISPOSITION OF ASSEMBLY:
REWORK ( ) REPAIR ( ) SCRAP ( )
USE AS IS ( ) RETURN TO SUPPLIER ( )
27. DISPOSITION OF COMPONENT:
REWORK ( ) REPAIR ( ) SCRAP ( )
USE AS IS ( ) RETURN TO SUPPLIER ( )
28. CORRECTIVE ACTION (ECO, retest, penalty cycle, etc.):
29. IMPLEMENTATION DATE:
30. CRITICALITY (code):
31. PROBLEM RISK RATING (code):
32. FOLLOW-UP DATE:
33. FOLLOW-UP COMPLETE (REPORT CLOSED):
34. PROJECT ENGINEER (sign & date):
35. QA REPRESENTATIVE (sign & date):

SECTION D

36. APPROVAL:

Comments: ___

Project Manager (sign & date)

Product Assurance Mgr. (sign & date)

NASA-C-8192 (Rev. 2/91) Copy Distribution: WHITE—OMS&A YELLOW—Project Manager GREEN—Hardware (PAI# 140)

Figure 4-6.—Failure report and analysis form.

PROBLEM REPORT INSTRUCTIONS

USE BLACK INK, NO ERASURES/WHITEOUT. CORRECT BY LINING OUT ERROR, WRITE IN CORRECT DATA, THEN INITIAL & DATE.
IF A BLOCK IS NOT APPLICABLE, WRITE " NA " IN THE SPACE PROVIDED.
IF MORE SPACE IS REQUIRED, THEN USE CONTINUATION SHEET (NASA-C-10032).
FOR INFORMATION CONCERNING PROBLEM REPORT PROCESSING SEE PAI# 140.
BLOCKS 1-34 MUST BE COMPLETE (as applicable) PRIOR TO ACTUAL CORRECTION OF THE PROBLEM.
RECORD ALL REWORK AND/OR REPAIR ON FORM (NASA-C-10031).

SECTION A - Person Reporting Problem (Initiator)

Completes all blocks (1-17) in Section A of Problem Report.

SECTION B - Problem Troubleshooter (Analyst)

Completes applicable blocks of Section B.

Software related problems require information in blocks 18-20, and 23-25. Software Trouble Report is form NASA-C-10033.

All other problems require information in blocks 18-25.

NOTE: ALL REWORK, REPAIR OR TROUBLESHOOTING IS TO BE DOCUMENTED ON THE REWORK/REPAIR HISTORY LOG (NASA-C-10031)

SECTION C - Problem Corrective Action

Project Engineer - Complete Blocks 26-30, and 34 of Section C.

Block 30 (criticality code) codes and definitions (code must be two digits: numeric-alpha):

1. Negligible - Negligible or no effect on mission or spacecraft/science instrument performance or lifetime.
   - A. No appreciable change in subsystem functional capability.
   - B. Minor degradation of engineering/science data.
2. Significant - Significantly degrading to mission or spacecraft/science instrument performance.
   - A. Appreciable change in subsystem functional capability.
   - B. Appreciable degradation of engineering or science data.
   - C. Causes significant operational difficulties or constraints.
   - D. Reduction in lifetime.
   - E. Significant impact on system safety.
3. Catastrophic - Catastrophic or major degradation of mission or spacecraft/science instrument performance.
   - A. Complete or nearly complete loss of major science objective.
   - B. Serious degradation to spacecraft systems causing loss of major science output.

QA Representative - Complete Blocks 31-33, and 35 of Section C.

Block 31 (Problem Risk Rating) codes and definitions:

A. Known Cause/Certain Corrective Action - Cause of the problem has been determined with confidence by analysis and/or test. Corrective action has been implemented, and verified. Residual risk is considered low.

B. Unknown Cause/Certain Corrective Action - Cause of the problem has not been determined with confidence. Corrective action has been determined, implemented, and verified. Residual risk is considered low.

C. Known Cause/Uncertain Corrective Action - The cause is considered to be "known" and "understood" with confidence. Corrective action has not been determined or implemented. Residual risk is considered unknown.

D. Unknown Cause/Uncertain Corrective Action - The cause has not been determined with confidence. Corrective action has not been determined or implemented. Residual risk is considered unknown.

SECTION D - Problem Closure

Project Manager - Reviews report and signs-off as indicated in Section D, Block 36.

Product Assurance Manager - Reviews report and signs-off approval in Section D, Block 36.
(required before Problem Report is considered complete.)

NOTE: PROBLEM REPORTS ARE NOT CLOSED UNTIL THE FOLLOW-UP HAS BEEN PERFORMED, IS ACCEPTABLE, AND BLOCK 33 IS INITIALED AND DATED.

NASA-C-8192 (Rev. 2/91) (Reverse)

Figure 4-6.—Concluded.

null once at horizontal marriage. Evaluation of the inverter null failure revealed that a high caging amplifier output caused the launch sequencer level detector to become offset during inverter monitoring, resulting in the major alarm even though the autopilot inverter voltage was normal. Launch sequencer offset may or may not occur with an uncaged platform depending on the amplitude of the caging amplifier output when the inverter voltage is monitored. Therefore, both the inverter null and the caging null LSPT failures at site were due to failure of the platform to cage.

An autopilot acceptance test tool was modified to permit monitoring of the platform spin motor voltage (800 Hz, 8 V, 3 ϕ) and the spin motor rotation detector (SMRD). During a spinup test on autopilot 69 (G&C 102), recordings indicated sustained caging oscillation. The SMRD showed no evidence of spin motor operation even though all autopilot voltages were correct, including the spin motor excitation voltage at the platform terminals. Further verification was obtained by listening for characteristic motor noises with a stethoscope.

G&C 86 failed the G&C level test due to caging null and inverter null alarms. Then, 3.5 sec into the third run the caging loop stopped oscillating, but the platform did not cage in time to pass the test. The next run met all G&C test requirements. It appeared obvious that the spin motor started spinning in the middle of the run.

G&C 28 failed one run of the G&C level test; however, it met all requirements in the autopilot level test. This means

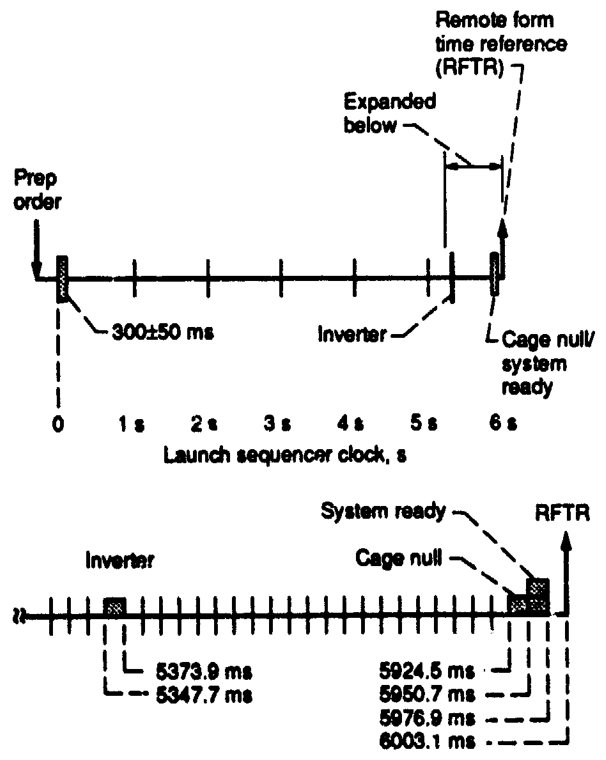

Figure 4-7.—System spinup tests. (Gate times are within ±50 ms of that shown because of data processor tolerances.)

that the spin successfully met its acceptance test procedure requirements. A hesitation was noted during two of the seven spinup tests conducted. Platform 127 was heated to normal on the gyro test set. Its resistances were checked and found to meet specification requirements. No attempt was made to start platform 127's spin motor at platform level. Both units were hand-carried to the subcontractor for failure analysis. The subcontractor was familiar with the construction of the platform and had the facilities to disassemble the platform without disturbing the apparently intermittent failure condition.

Verification test conclusions.—Verification tests isolated the site LSPT failures to a failure of the platform spin motor to spin up, thereby causing major alarms at the inverter null or caging null gate. During testing, three of the first four failed platforms caged upon repeated application of voltage. Once the platform caged, the platform, autopilot, and G&C met all system test requirements. On the basis of these results, it was decided to repeat LSPT's up to 10 times after a site failure before removing the G&C. If the LSPT's were successful, the G&C would be left on line.

Measurements at platform level indicated the problem was internal to the platform and that all resistances and the platform temperature were correct. Subcontractor representatives reviewed the test results and concurred that the problem was internal to the platform.

Mechanical Tests

The spin motor breakaway torque was measured with a gram gage on platform 127 and was found to be normal (750 dyne cm). Dynamometer tests were performed on both platforms. The dynamometer is an instrument that measures rotation torque by slowly rotating the rotor of the spin motor while recording the stator rotational torque. The dynamometer is used during initial builds to establish the spin motor bearing preload (torque). The spin motor generates approximately 4000 dyne cm of starting torque with normal excitation voltage; 800 dyne cm of this torque is used to overcome the inertia and frictional torque of the motor.

Platform 140 was tested on the dynamometer and produced the torque peaks of 3400 and 3100 dyne cm shown in figure 4-8. The torque peaks were three revolutions apart. This is four times the normal running torque level for a new spin motor and about four times the torque level for this spin motor for the rest of its run. The torque increase lasted for about one-half of a revolution and repeated within three revolutions. The spin motor bearings were cleaned and reassembled. Two large torque spikes of approximately 3000 dyne cm were observed on the first revolution. A 2200-dyne cm torque hump, one revolution in duration, was centered at the beginning of the second revolution. From these results it was concluded that something in the spin motor bearing was causing an abnormal frictional load in the bearing. This result isolated the problem to the spin motor bearing area and eliminated the motor electrical characteristics as being a contributor.

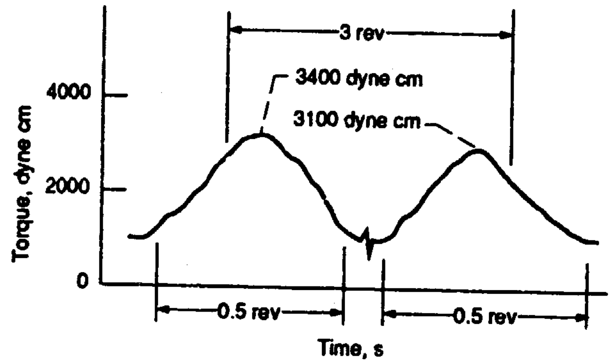

Figure 4-8.—Platform dynamometer torque test.

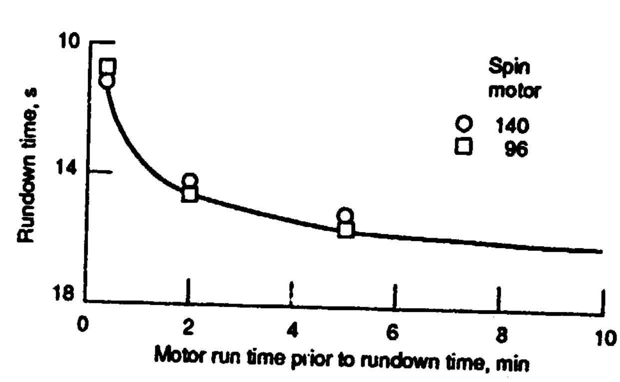

Figure 4-9.—Rundown time versus motor run time.

Runup and Rundown Tests

A series of tests were performed on spin motors 96 and 140 to determine the effect of motor running time on spin motor start and running torque. Figure 4-9 shows the change in rundown time with change in motor run time.

Summary of Case Study

Field problem cause.—The 26 LSPT failures at the site were caused by the failure of the G&C platform spin motors to spin up within 6 sec after the command to get the vehicle ready for launch. It was determined that the spin motors did not start with normal application of voltage. A polymer film had formed on the bearing surfaces during testing at 175 °F and caused the balls to stick to the outer race. This film was identified as from the alkyl phenol and alkyl benzene families. Its source was determined to be uncured resins from the bearing retainer.

Polymer film.—A film approximately 900 Å thick had formed on the metal surfaces of the bearings of failed spin motors. The amount of material generated was $\sim 10^{-7}$ g/ball. To put this number in proper perspective, 2×10^{-4} g of oil is put on the bearing race during initial build, and 2×10^{-3} g of oil is impregnated in the bearing retainer.

Alkyl phenol/alkyl benzene is a generic identification of a family of organic compounds. Further analysis identifies the major compounds in the family as phenol and methyl phenol (alkyl phenols) and toluene, xylene, and benzene (alkyl benzenes). A phenolic polymeric film would have the gummy, adhesive, and insolubility properties detected in the analysis. There is little doubt the gummy film detected was a phenol-based material.

Source of phenol.—Phenols are used in three areas of the spin motor. A phenolic adhesive bonds the stator laminations together and bonds the hysteresis ring to the rotor. The bonding processes adequately cure the phenol to the point where uncured phenols would not be present. Also, the stator laminations are coated with epoxy after bonding. The remaining source is the paper phenolic retainer, which serves as a spacer and a lubrication source for the spin motor bearings. Mass spectral analysis of the retainers yielded spectra essentially identical to the spectrum of the coating on the failed bearings. The conclusion of this analysis is that the source of the phenolic is uncured phenolic resins or resin compounds in the retainer.

Retainer processing.—The retainer material is manufactured by a vendor to military specifications and screened to tighter vehicle requirements for specific gravity. There is no specific requirement concerning uncured resins in the retainer material. The vendor estimated an upper limit of 1 percent of uncured resin in the retainer raw material. One percent would provide 3×10^{-5} g of uncured resins, more than sufficient to cause the spin motor problem.

The finished retainer material is cleaned by an extraction process with benzene or hexane. This process does not remove a significant amount of uncured resins. Therefore, if uncured resins survive the vendor processing, they will remain in the uncured state in the installed retainers.

Mechanism of film formation.—It is theorized that the uncured resins are transferred from the retainer to the bearing surfaces through the natural lubricating process of the retainer. Running the spin motors generates centrifugal forces that sling the excess oil off the rotating surfaces, leaving a thin film of oil. The force of gravity during subsequent storage of the motor causes the already thin film to become thinner on the top surfaces and thicker on the lower surfaces. This redistribution process involves only the oil and leaves more viscous contaminants in place. Subsequent running of the motor will cause replacement of oil on the oil-free surfaces. The source of the replacement oil is the retainer capillaries. This replacement process will cause the oil to bring any uncured phenolics to the surface of the retainer. The metal surfaces will then become lubricated with oil containing a small percentage of uncured resins. Subsequent storage cycles and running will continue this redistribution process, steadily increasing the phenolic concentration. Exposure to a temperature of 175 °F and extended operational maintenance gradually cures these phenolics in two stages. Initially, a highly viscous gummy residue is formed; finally, a hard insoluble polymer film is formed on the metal surfaces. The film forms a bond between the balls and the races. The coating builds up to the point where the spin motor torque cannot overcome the bond at initial power application.

Extent of problem.—Analysis of failed and unfailed field units proved that not all platforms are susceptible to this failure. Obviously, a high percentage are susceptible, since 26 failures have been experienced. It is likely that many unfailed platforms contain some small percentage of uncured resins.

The significantly higher failure rate in the units with higher serial numbers points to a process (or common) failure mode. All evidence points to lot-to-lot variations in the amount of uncured resins present in the retainer raw material. Traceability from retainer lot to individual platform spin motor was not possible in this case, but such records should be available. The 26 units that have failed and the failure rate at the 14-day interval bound the total platform failure rate. The number of spares available is adequate to meet system life and reliability requirements.

Site reliability.—The site system reliability goal allows approximately two G&C failures per month for any cause. Analysis of test data indicates the goal can be achieved at either a 7-day test interval (0.8 failure/month) or a 14-day test interval (1.5 failures/month). It cannot be achieved at a 21-day interval (7.7 failures/month) or a 28-day interval (8.6 failures/month). Even though at least 74 percent of the site failures were restarted, a limited number of spare G&C's are available.

Tests at the site revealed that most failed spin motors can be restarted within 10 power applications and, once started, will perform properly. The site procedure was revised to leave any failed G&C's that restart within 10 attempts on line. Platforms that did not start within 10 attempts were returned to the contractor and were restarted by repetitive application of overvoltage or reverse voltage up to the motor saturation limit. These data support the conclusion that the failure mode was the formation of a film bond on the race and that increasing the inverter output voltage to the motor saturation limit would not eliminate the problem.

Current site operating procedures provide a 14-day LSPT interval with a 10-min run time. This enables the G&C failure rate to meet system reliability goals. The vehicle site is currently being deactivated. If reactivation should be required, the repair of all defective or support platforms should be included as part of that effort.

Concluding Remarks

Now that you have completed chapter 4, several concepts should be clear.

(1) The failure rate of complex equipment is usually considered to be a constant.

(2) Most failures are random, with repetitive failures representing a small portion of unreliability.

(3) The rate at which failures occur depends upon
 (a) The acceptance criteria, which determine how effectively potential failures are detected
 (b) All applied stresses, including electrical, mechanical, and environmental. (As these stresses increase, the failure rate usually increases.)

(4) Published failure rate data represent the potential failures expected of a part. The rate at which these failures are observed depends on the applied electrical stresses (the stress ratio) and the mechanical stresses (the K_A factor).

(5) In general, failure rate predictions are best applied on a relative basis.

(6) Failure rate data can be used to provide reliability criteria to be traded off with other performance parameters or physical configurations.

(7) The reliability of a device can be increased only if the device's failure mechanisms and their activation causes are understood.

In addition, you should be able to use failure rate data to predict the failure rate expected of a design, and consequently, to calculate the first term, P_c, of inherent reliability. Finally, you should be able to allocate failure rate requirements to parts after having been given a reliability goal for a system or the elements of a system.

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Reliability Training[1]

1a. Using the failure rate data in table 4–4 (on p. 51), calculate the flight failure rate for a launch vehicle electronic subsystem consisting of the following parts (assume $K_A = 1000$):

Component	Number of parts, N
Resistor, G657109/10	5
Resistor, variable, 11176416	1
Capacitor, G657113	3
Diode, G657092	3
Transistor, 11176056	4
Integrated circuit, analog, 11177686	1

A. 195 failures per 10^9 hours
B. 195 000 failures per 10^9 hours
C. 195 000 failures per 10^6 hours

1b. Assume the flight failure rate for this circuit is 500 000 failures per 10^9 hours. Calculate the reliability of the circuit for a 0.01-hour flight.

A. 0.9995 B. 0.99995 C. 0.999995

2. The a posteriori flight failure rate of a launch vehicle is 440 000 failures per 10^9 hours.

a. If the storage failure rate is 0.3 of the operating rate, how long can the vehicle be stored with a 90.4 percent probability of no failures?

A. 30 days B. 40 days C. 50 days

b. After 1450 hours (2 months) in storage the vehicle is removed and checked out electronically. If the vehicle passes its electronic checkout and the checkout equipment can detect only 80 percent of the possible failures, what is the probability that the vehicle is good? (Ignore test time.)

A. 0.962 B. 0.858 C. 0.946

3. A subassembly in a piece of ground support equipment has a reliability requirement of 0.995. Preliminary estimates suggest that the subassembly will contain 300 parts and operate for 200 hours. What is the average part failure rate required to meet the reliability goal?

A. 25×10^{-6} B. $16\,667 \times 10^{-9}$ C. 83×10^{-9}

4. A piece of ground support equipment has a reliability goal of 0.9936. It contains four subassemblies of approximately equal risk.

a. What is the allocated reliability goal of each of the four subassemblies?

A. 0.99984 B. 0.9984 C. 0.9884

b. Allocating further into subassembly 1. Assume the goal is 0.998. Solve for the average part failure rate given the following:

Estimated parts count: 100
Estimated operating time: 10 hours

A. $20\,000 \times 10^{-9}$ B. 2000×10^{-9} C. 200×10^{-9}

[1] Answers are given at the end of this manual.

TABLE 4-4.—SELECTED LISTING—APPROVED ELECTRONIC FAILURE RATES FOR LAUNCH VEHICLE APPLICATION[a]

Part number	Part	Operating mode[b]	Nonoperating mode[b]
		Failure rate, failures/10^9 hr	
Integrated circuits			
11177680/81/82/83/84/85	Digital	10	3
11177686	Analog	30	10
Transistors			
6557155	Double switch	10	3
6557318/19	Medium-power switch	20	
6557046	PNP type of transistor		
11176911	Medium-power switch		
11176056	High-speed switch		
11177685	Field-effect transistor		
6310038	2N5201	10	
6557072	2N918 (unmatched)	50	5
Diodes			
6557061	Rectifier and logic (5 V)	20	3
6557092	Rectifier and logic (30 V)	5	
6557123	Rectifier and logic (50 V)		
6557125	Rectifier and logic (600 V)		
11176912	Rectifier and logic (400 V)		
Resistors			
6557018	2.5-W wirewound	2	1
6557015	1/8-W wirewound	3	2
6557016/17	1- and 2-W wirewound	2	.5
6557030	1/10-W fixed film	1	.5
6557031	6-W wirewound	5	.5
6557109/10	1/4-W fixed composition	1	.2
6557329	1/8-W fixed film	1	.3
11176416	1-W variable metal film	50	10.3
Capacitors			
G657020/21/22	Fixed glass	0.1	0.1
G657113/173	Fixed ceramic	5	1
G657114	Fixed ceramic	10	1
G657119/120	Solid tantalum	2	1
G657202	Precision, fixed ceramic	50	3
Relays			
11176326/453	DPDT armature	100	20
Transformers (RF)			
11301034/35/43/49		10	5
11301064		1	5
RF coil			
G657140/41		3	2
G657178/81		10	2
RF filter			
G657189		50	5

[a]Current failure rate data are available from two sources (refs. 4-1 and 4-4).
[b]Applies to all slash numbers of parts shown. (Worst case shown.)

Chapter 5

Applying Probability Density Functions

The inherent reliability of equipment is defined in chapter 3 as

$$R_i = e^{-\lambda t} P_t P_w$$

where

- R_i probability of no failures
- $e^{-\lambda t}$ probability of no catastrophic part failures
- P_t probability of no tolerance failures
- P_w probability of no wearout failures

Before discussing the P_t and P_w terms in the next chapter, it is necessary to develop an understanding of probability density functions and cumulative probability functions. These concepts form another part of probability theory not discussed in chapter 2. First, in this chapter the theory of density and cumulative functions is discussed in general; then the normal, or Gaussian, distribution is discussed in detail. This normal distribution is used extensively later in the manual.

Probability Density Functions

If a chance variable x can take on values only within some interval, say between a and b, the probability density function $p(x)$ of that variable has the property that (ref. 5-1)

$$\int_a^b p(x)\,dx = 1$$

In other words the area under the curve $p(x)$ is equal to unity. This is shown in figure 5-1.

In the language of probability, the probability of x being within the interval (a,b) is given by

$$P(a \le x \le b) = \int_a^b p(x)\,dx = 1$$

In other words the probability that x lies between a and b is 1. This should be clear, since x can take only values between a and b.

In a similar fashion we can find the probability of x being within any other interval, say between c and d, from

$$P(c \le x \le d) = \int_c^d p(x)\,dx$$

This is shown in figure 5-2.

Example 1: Suppose we were to perform an experiment in which we measured the height of oak trees in a 1-acre woods. The result, if our measuring accuracy is ±5 feet, might look like the histogram shown in figure 5-3.

The value at the top of each histogram cell (or bar) indicates the number of trees observed to have a height within the boundaries of that cell. For example, 19 trees had a height between 0 and 10 feet, 17 trees had a height between 10 and 20 feet, etc. The figure shows that 100 trees were observed.

Now let us calculate values for the ordinate of the histogram so that the area under the histogram equals unity. Then, we will establish a probability density function for the tree heights. Since we observed 100 trees, it should be apparent that if the calculated ordinate of a cell times the width of the cell (the cell area) yields the percentage of 100 trees in that cell, the sum of the percentage in all cells will have to equal 100 percent. Of, if the percentages are expressed as decimal fractions, their sum will equal 1, which will be the total area under the histogram. Therefore,

$$\text{Ordinate of cell} = \frac{\text{Percent of trees in cell}}{\text{Width of cell}}$$

For the cell 0 to 10 feet, which has 19 percent of the trees in it,

$$\text{Cell ordinate} = \frac{19}{100} \times \frac{1}{10} = 0.019$$

As a check, we can see that

Cell ordinate = 0.019 × Cell width (10) = 0.19, or 19 percent

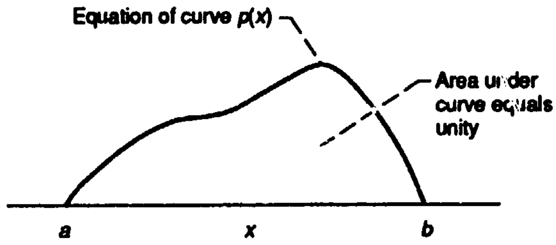


Figure 5-1.—Probability density function curve.

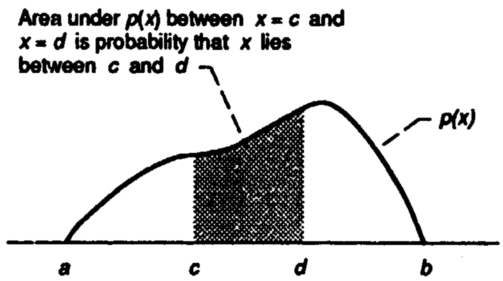


Figure 5-2.—Application of probability density function.

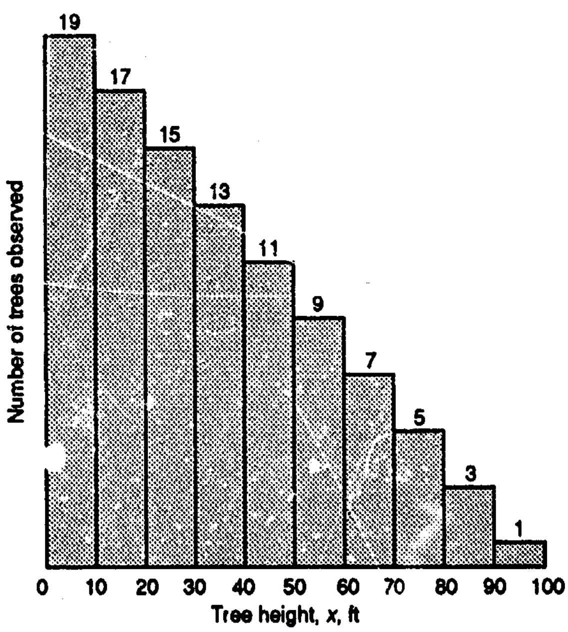


Figure 5-3.—Height of trees observed in 1-acre woods.

In a similar fashion the ordinates for the other cells can be calculated and are shown in table 5-1 and figure 5-4.

The next step (fig. 5-4) is to draw a line through the midpoint of the cells. The equation of this line is called the probability density function $p(x)$ and has the form

$$p(x) = -0.0002x + 0.02$$

TABLE 5-1.—CALCULATION OF CELL ORDINATES FOR TREE DATA

Cell	Ordinate	Area, cell width times cell ordinate
0–10	$\frac{19}{100 \times 10} = 0.019$	0.19
10–20	$\frac{17}{10^3} = 0.017$	.17
20–30	$\frac{15}{10^3} = 0.015$	.15
30–40	$\frac{13}{10^3} = 0.013$	.13
40–50	$\frac{11}{10^3} = 0.011$	.11
50–60	$\frac{9}{10^3} = 0.009$	.09
60–70	$\frac{7}{10^3} = 0.007$	.07
70–80	$\frac{5}{10^3} = 0.005$	.05
80–90	$\frac{3}{10^3} = 0.003$	.03
90–100	$\frac{1}{10^3} = 0.001$	.01
	Total area	1.00

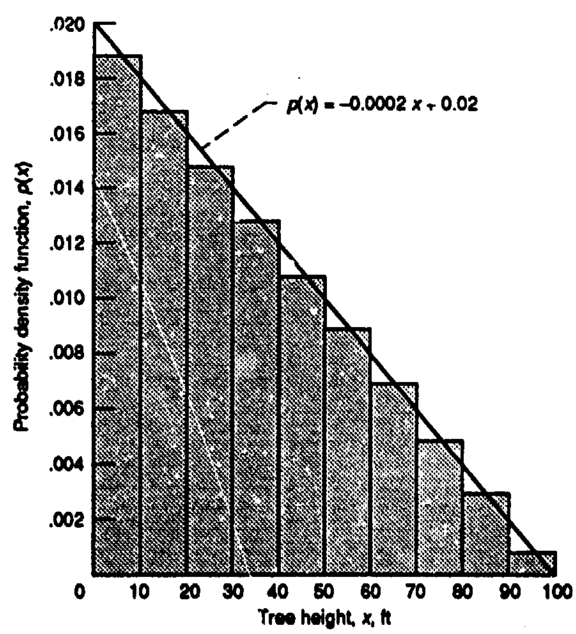


Figure 5-4.—Probability density function for tree heights.

The area under the curve is (ref. 5-2)

$$\text{Area} = \int_0^{100} p(x)\,dx = \int_0^{100} (-0.0002x + 0.02)\,dx$$

$$= -\frac{x^2}{10^4} + 0.02x\Big|_0^{100} = -\frac{(100)^2}{10^4} + 0.02(100)$$

$$= -\frac{10^4}{10^4} + 2 = -1 + 2 = 1$$

This agrees with our requirement that the area under a probability density function equal unity.

Application of Density Functions

Now let us see how we can apply the density function to the tree data. To find the percentage of trees between 60 and 80 feet high, solve for

$$P(60 \le x \le 80) = \int_{60}^{80} p(x)\,dx = \int_{60}^{80} (-0.0002x + 0.02)\,dx$$

$$= -\frac{x^2}{10^4} + 0.02x\Big|_{60}^{80} = -\frac{1}{10^4}(80^2 - 60^2) + 0.02(80 - 60)$$

$$= -\frac{1}{10^4}(2800) + 0.4 = -0.28 + 0.4$$

$$= 0.12, \text{ or } 12 \text{ percent}$$

Figure 5-3 shows that this answer is correct, since 12/100 trees were observed to have a height between 60 and 80 feet.

Another way to look at this example is that there is only a 12-percent chance that a tree picked at random from the 1-acre area would have a height between 60 and 80 feet. In a similar fashion we can calculate the probability that a tree would have any range of heights within the boundary of 0 to 100 feet.

In the tree example, we were able to measure the trees in a particular part of the woods and to obtain a height density function for those trees. But what do we do if we are interested in a different area of woods and for some reason we are not able to go out and measure the trees? We would probably assume that the acre we measured was representative of all other acres in the same woods. If we accept this assumption, we could then use our experience (the established density function) to predict the distribution of tree heights in an unmeasured acre. And this is exactly what is done in industry.

As you can see, if we know what the density functions are for such things as failure rates, operating temperatures, and missile accuracy, it is easy to determine the probability of meeting a failure rate requirement for equipment (such as a missile) specified to operate in some temperature range with a required accuracy.

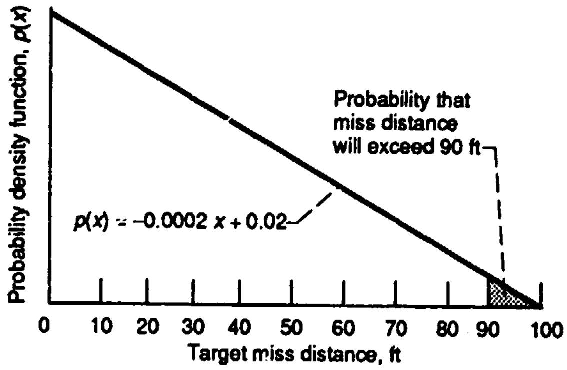


Figure 5-5.—Probability density function for missile target miss distance.

Example 2: Suppose a missile has a maximum target miss distance requirement of 90 feet and that after several hundred firings the probability density function for miss distance is

$$p(x) = -0.0002x + 0.02 \qquad \text{where } 0 \le x \le 100$$

which is the same as the $p(x)$ for the tree example. This is shown in figure 5-5.

To predict the probability that the next missile fired will miss the target by more than 90 feet, solve for

$$P(90 \le x \le 100) = \int_{90}^{100} (-0.0002x + 0.02)\,dx$$

$$= -\frac{x^2}{10^4} + 0.02x\Big|_{90}^{100}$$

$$= -\frac{1}{10^4}(100^2 - 90^2) + 0.02(100 - 90)$$

$$= -\frac{1900}{10^4} + 0.02(10)$$

$$= -0.19 + 0.2 = 0.01, \text{ or } 1 \text{ percent}$$

In other words there is a 99-percent chance that the missile will hit within 90 feet of the target and a 1-percent chance that it will not. This is shown as the shaded area under the density function in figure 5-5.

Cumulative Probability Distribution

Another practical tool in probability calculation is the cumulative probability distribution, denoted by $F(x)$ (ref. 5-3). An $F(x)$ curve for the tree example in the preceding section is shown in figure 5-6. The curve represents the cumulative

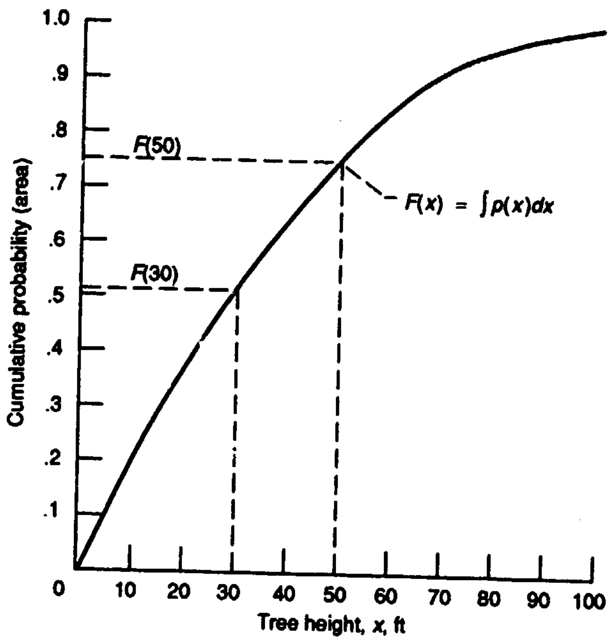


Figure 5-6.—Cumulative probability function for tree heights.

area under the probability density function $p(x)$. The ordinates of the curve were calculated as shown in table 5-2.

The cumulative curve can be used to solve the same problems as the density curve.

Example 3: Referring again to example 1, suppose we want to know the probability that a particular tree selected at random from the woods will have a height between 30 and 50 feet.

Solution 3A: Using the density function for tree height,

$$P(30 \le x \le 50) = \int_{30}^{50} (-0.0002x + 0.02)\,dx$$

$$= -\frac{x^2}{10^4} + 0.02x \Big|_{30}^{50}$$

$$= -\frac{1600}{10^4} + 0.40$$

$$= -0.16 + 0.40 = 0.24, \text{ or } 24 \text{ percent}$$

Solution 3B: Using the cumulative curve shown in figure 5-5,

$$P(30 \le x \le 50) = F(50) - F(30) = 0.75 - 0.51$$

$$= 0.24, \text{ or } 24 \text{ percent}$$

which agrees with solution 3A.

Note that in working out solution 3A the next-to-last step (0.75 − 0.51) is the same as the next-to-last step of solution 3B. The reason for this is that the equation of the cumulative

TABLE 5-2.—ORDINATES FOR CUMULATIVE DISTRIBUTION OF TREE DATA

Tree height, ft	Area under $p(x)$ curve	Ordinate of $p(x)$ curve (cumulative area)
0-10	0.19	0.19
10-20	.17	.36
20-30	.15	.51
30-40	.13	.64
40-50	.11	.75
50-60	.09	.84
60-70	.07	.91
70-80	.05	.96
80-90	.03	.99
90-100	.01	1.00

probability function $F(x)$ is found from

$$F(x) = \int p(x)\,dx$$

and

$$\int_a^b p(x)\,dx = F(b) - F(a)$$

For the tree example

$$F(x) = \int (-0.0002x + 0.02)\,dx = -\frac{x^2}{10^4} + 0.02x$$

Consequently, we can find the probability of a variable x being within some interval by using the cumulative function $F(x)$ even though the cumulative graph is not available.

Example 4: What is the probability that a tree selected at random will have a height less than 20 feet?

Solution 4:

$$P(0 \le x \le 20) = \int_0^{20} p(x)\,dx = F(20) - F(0)$$

$$= -\frac{x^2}{10^4} + 0.02x \Big|_0^{20}$$

$$= \left[-\frac{20^2}{10^4} + 0.02(20)\right] - 0$$

$$= -0.04 + 0.4 = 0.36, \text{ or } 36 \text{ percent}$$

which agrees with a graphical solution.

Some general rules for the use of the cumulative function $F(x)$ are

(1) $P(x \le a) = F(a)$

(2) $P(x \ge a) = 1 - F(a)$

(3) $P(a \le x \le b) = F(b) - F(a)$

Example 5: Suppose we would like to know the probability of equipment seeing tropic zone temperatures above 120 °F

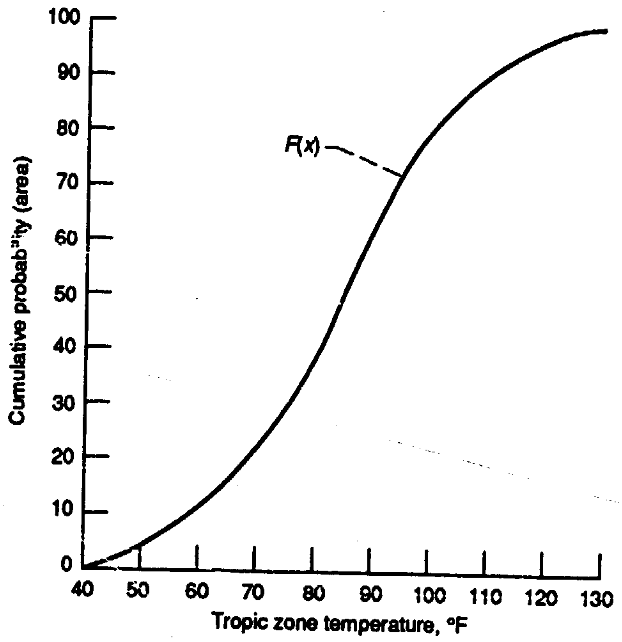


Figure 5-7.—Cumulative distribution of tropic zone temperatures.

during operation because it is above 120 °F we have to add a costly air-conditioning system to the equipment. If we could obtain the temperature data, we might find that the cumulative distribution for tropic zone temperatures would be that shown in figure 5-7.

Solution 5: From the curve the probability of observing a temperature at or above 120 °F is given by

$$P(\text{temp} \geq 120\ ^\circ\text{F}) = 1 - F(120\ ^\circ\text{F}) = 1 - 0.97$$

$$= 0.3, \text{ or } 3 \text{ percent}$$

With only a 3-percent chance of temperatures above 120 °F, we probably would decide against air-conditioning (all other parameters, such as failure rate, being equal).

Normal Distribution

One of the most frequently used density functions in reliability engineering is the normal, or Gaussian, distribution. A more descriptive name, however, is the normal curve of error because it represents the distribution of errors observed from repeated measurements of an object or some physical phenomenon (ref. 5-4).

Example 6: Assume that we need to measure the heights of eighth grade children. A histogram of the children's heights would resemble the curve in figure 5-8. If, as in our tree example, we calculate an ordinate for the histogram so that the area under the histogram equals unity and then connect the midpoints of each cell, we obtain a smooth curve as shown in figure 5-8. This curve represents the density function

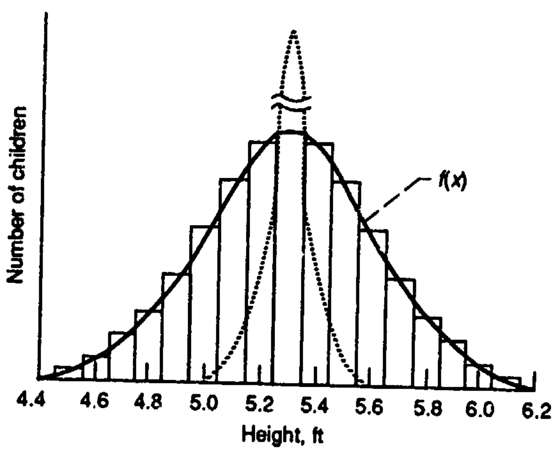


Figure 5-8.—Histogram and density function for heights of children.

for the heights of the children. Such a curve (sometimes called a bell curve) is the shape of the normal distribution. We say that the children's heights are distributed normally.

Normal Density Function

The equation for the density function $p(x)$ of the normal distribution is

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\bar{x})^2/2\sigma^2}$$

This curve is shown in figure 5-9. The function $p(x)$ has two parameters. The first is the mean $\bar{x}$ calculated from

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

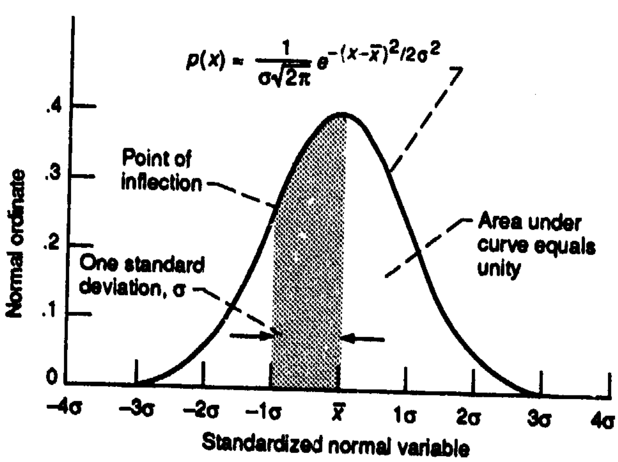


Figure 5-9.—Normal density function.

where

n total number of measurements or observations

x_i value of i^{th} measurement

The mean, therefore, is the arithmetic average of the measurements. From example 6 we would add up all the heights observed and then divide by the number of children measured to obtain a mean or average height. The mean of all the children's heights from the data in figure 5-8 is 5.3 feet.

The second parameter of $p(x)$ is the standard deviation σ calculated from

$$\sigma = \left[\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} \right]^{1/2}$$

where

$\bar{x}$ mean of measurements

x_i value of i^{th} measurement

n total number of measurements

Note that $n - 1$ is used in the equation in order to give an unbiased sampling distribution. In the general definition of σ, n instead of $n - 1$ would be used.

The standard deviation is the square root of the variance, which is denoted by σ^2. The magnitude of the variance, as well as the standard deviation, indicates how far all the measurements deviate from the mean. The standard deviation of the children's height data, for example, is approximately 0.3 foot. If the range of heights observed had been from 5 to 5.6 feet, the standard deviation would have been approximately 0.1 foot. And with this standard deviation the distribution would look squeezed together as shown by the dashed curve in figure 5-8. However, the area under the dashed curve would still equal the area under the solid curve.

Properties of Normal Distribution

The normal density function is a continuous distribution from $-\infty$ to ∞. It is symmetrical about the mean and has an area equal to unity as required for probability density functions. For the normal distribution the standard deviation is the distance on the abscissa from the mean $\bar{x}$ to the intercept on the abscissa of a line drawn perpendicular to the abscissa through the point of inflection on the curve. This is shown in figure 5-9. It is also shown that equal increments of the standard deviation can be laid out to the left ($-$) and the right ($+$) of the mean $\bar{x}$.

As you will recall, in determining probabilities from a density function, we need to calculate the area under the curve $p(x)$. When using the normal density function, it is common practice to relate areas to the standard deviation. In general, for the area under the curve between the values of z and $-z$,

TABLE 5-3.—AREAS BETWEEN $-z$ AND z

z	Area under curve	Probability, $-z \le x \le z$
1	0.683	$P(-1\sigma \le x \le 1\sigma)$
2	.9545	$P(-2\sigma \le x \le 2\sigma)$
3	.9973	$P(-3\sigma \le x \le 3\sigma)$
4	.999937	$P(-4\sigma \le x \le 4\sigma)$
5	.999999426	$P(-5\sigma \le x \le 5\sigma)$
6	.99999999803	$P(-6\sigma \le x \le 6\sigma)$
7	.999999999992	$P(-7\sigma \le x \le 7\sigma)$

standard deviations can be found from

$$p[-z \le x \le z] = \text{Area} = \int_{-z}^{z} \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2(z)^2} dz$$

The areas for various values of z are shown in table 5-3. This table says that the area under the normal curve between 1σ and -1σ is 0.683, or 68.3 percent; the area under the normal curve between 2σ and -2σ is 0.9545, or 95.45 percent, etc.

Example 7: The term "3σ limit" refers to the area under the normal curve between 3σ and -3σ, which is 0.9973, or 99.73 percent, as shown in table 5-3. Therefore, if a power supply output is defined as 28 ± 3 V and the ± 3 V represents a 3σ limit, 99.73 percent of all such power supplies will have an output between 25 and 31 V. The percentage of supplies having an output greater than 31 V and less than 25 V will be $1 - 0.9973 = 0.0027$, or 0.27 percent. This is shown in figure 5-10.

Up to now we have been working with areas under the normal density function between integers of σ, that is, 1, 2, 3, etc. In practice, however, we are usually interested in the area between decimal fractions of σ, those being 1.1, 2.3, etc. We have also been using z to represent the number of standard deviations that a particular limit value is from the mean. For instance, in the power supply example 25 V was given as being three standard deviations from the mean of 28 V. It is better when working in decimal fractions of σ to let $z = (x - \bar{x})/\sigma$, where $x - \bar{x}$ is the distance from the mean $\bar{x}$ to the limit value and σ is the standard deviation. Going back to the supply example, our lower limit was 25 V. This was 3 V from the mean of 28 V, and the standard deviation was 1 V; therefore, $z = (25 - 28)/1 = -3$.

Symmetrical Two-Limit Problems

In this discussion the term "symmetrical two-limit problems" refers to the area under the density function at equal values of z from both sides of the mean. The power supply example was of this type, since we were concerned with the area between -3σ and 3σ from the mean $\bar{x}$. To work these problems when z is a decimal fraction, we use tables of areas in the two tails of the normal curve.

TABLE 5-4.—AREAS IN TWO TAILS OF NORMAL CURVE AT SELECTED VALUES OF z
[From reference 5-1.]

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	1.0000	0.9920	0.9840	0.9761	0.9681	0.9601	0.9522	0.9442	0.9362	0.9283
.1	.9203	.9124	.9045	.8966	.8887	.8808	.8729	.8650	.8572	.8493
.2	.8415	.8337	.8259	.8181	.8103	.8026	.7949	.7872	.7795	.7718
.3	.7642	.7566	.7490	.7414	.7339	.7263	.7188	.7114	.7039	.6965
.4	.6892	.6818	.6745	.6672	.6599	.6527	.6455	.6384	.6312	.6241
.5	.6171	.6101	.6031	.5961	.5892	.5823	.5755	.5687	.5619	.5552
.6	.5485	.5419	.5353	.5287	.5222	.5157	.5093	.5029	.4965	.4902
.7	.4839	.4777	.4715	.4654	.4593	.4533	.4473	.4413	.4354	.4295
.8	.4237	.4179	.4122	.4065	.4009	.3953	.3898	.3843	.3789	.3735
.9	.3681	.3628	.3576	.3524	.3472	.3421	.3371	.3320	.3271	.3222
1.0	.3173	.3125	.3077	.3030	.2983	.2937	.2891	.2846	.2801	.2757
1.1	.2713	.2670	.2627	.2585	.2543	.2501	.2460	.2420	.2380	.2340
1.2	.2301	.2263	.2225	.2187	.2150	.2113	.2077	.2041	.2005	.1971
1.3	.1936	.1902	.1868	.1835	.1802	.1770	.1738	.1707	.1676	.1645
1.4	.1615	.1585	.1556	.1527	.1499	.1471	.1443	.1416	.1389	.1362
1.5	.1336	.1310	.1285	.1260	.1236	.1211	.1188	.1164	.1141	.1118
1.6	.1096	.1074	.1052	.1031	.1010	.0989	.0969	.0949	.0930	.0910
1.7	.0891	.0873	.0854	.0836	.0819	.0801	.0784	.0767	.0751	.0735
1.8	.0719	.0703	.0688	.0672	.0658	.0643	.0629	.0615	.0601	.0588
1.9	.0574	.0561	.0549	.0536	.0524	.0512	.0500	.0488	.0477	.0466
2.0	.0455	.0444	.0434	.0424	.0414	.0404	.0394	.0385	.0375	.0366
2.1	.0357	.0349	.0340	.0332	.0324	.0316	.0308	.0300	.0293	.0285
2.2	.0278	.0271	.0264	.0257	.0251	.0244	.0233	.0232	.0226	.0220
2.3	.0214	.0209	.0203	.0198	.0193	.0188	.0183	.0178	.0173	.0168
2.4	.0164	.0160	.0155	.0151	.0147	.0143	.0139	.0135	.0131	.0128
2.5	.0124	.0121	.0117	.0114	.0111	.0108	.0105	.0102	.00988	.00960
2.6	.00932	.00905	.00879	.00854	.00829	.00805	.00781	.00759	.00736	.00715
2.7	.00693	.00673	.00653	.00633	.00614	.00596	.00578	.00561	.00544	.00527
2.8	.00511	.00495	.00480	.00465	.00451	.00437	.00424	.00410	.00398	.00385
2.9	.00373	.00361	.00350	.00339	.00328	.00318	.00308	.00298	.00288	.00279

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
3	0.00270	0.00194	0.00137	$0.0^{3}967$	$0.{}^{3}674$	$0.0^{3}465$	$0.0^{3}318$	$0.0^{3}216$	$0.0^{3}145$	$0.0^{4}962$
4	$.0^{4}633$	$.0^{4}413$	$.0^{4}267$	$.0^{4}171$	$.0^{4}108$	$.0^{5}680$	$.0^{5}422$	$.0^{5}260$	$.0^{6}159$	$.0^{6}958$
5	$.0^{6}573$	$.0^{6}340$	$.0^{6}199$	$.0^{6}116$	$.0^{7}666$	$.0^{7}380$	$.0^{7}214$	$.0^{7}120$	$.0^{8}663$	$.0^{8}364$
6	$.0^{8}197$	$.0^{8}106$	$.0^{9}565$	$.0^{9}298$	$.0^{9}155$	$.0^{10}803$	$.0^{10}411$	$.0^{10}208$	$.0^{10}105$	$.0^{11}520$

Table 5-4 shows tabulated areas in two tails of the normal curve for selected values of z from the mean $\bar{x}$. For example, when $z = 3.0$, the table shows that 0.00270 of the total area lies in the two tails of the curve below -3σ and above 3σ. Because the curve is symmetrical, 0.00135 of the area will lie to the left of -3σ and 0.00135 to the right of 3σ. Note that this agrees with figure 5-10 for the power supply example.

Example 8 (using table 5-4): Suppose that a circuit design requires that the gain β of a transistor be no less than 30 and no greater than 180. The mean $\bar{x}$ of the β density function of a particular transistor is 105 with a standard deviation of 32. What percentage of the transistors will have a β within the required limits?

Solution 8: Step 1—Solve for z.

$$x - \bar{x} = 105 - 30 = 180 - 105 = 75$$

Since σ is given as 32,

$$z = \frac{75}{32} = 2.34$$

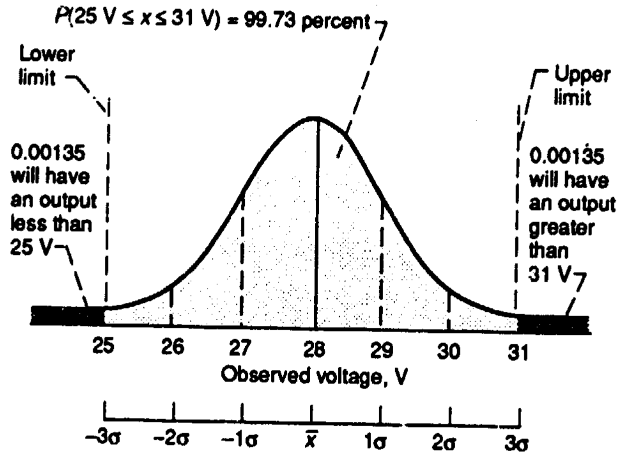


Figure 5-10.—Probability density functions for power supply outputs.

Step 2—From table 5-4 the area in the two tails when $z = 2.34$ is 0.0193. Therefore, because of symmetry, 0.00965 of the transistors will have a β below 30 and 0.00965 will have a β above 180.

Step 3—Now find $P(30 \leq \beta \leq 180)$. Since 0.0193 of the transistors will have a β below 30 or above 180, then $1 - 0.0193$ must give the percentage that will lie between 30 and 180. This is $1 - 0.0193 = 0.9807$, or 98.07 percent, as shown in figure 5-11. If we were to buy 100 000 of these transistors, we would expect 98 070 of them to have a β between 30 and 180. The remaining 1930 would not meet our β requirements.

One-Limit Problems

In many applications engineers are interested only in one-sided limits, an upper or lower limit, rather than a two-sided upper and lower limit. In these cases they are interested in

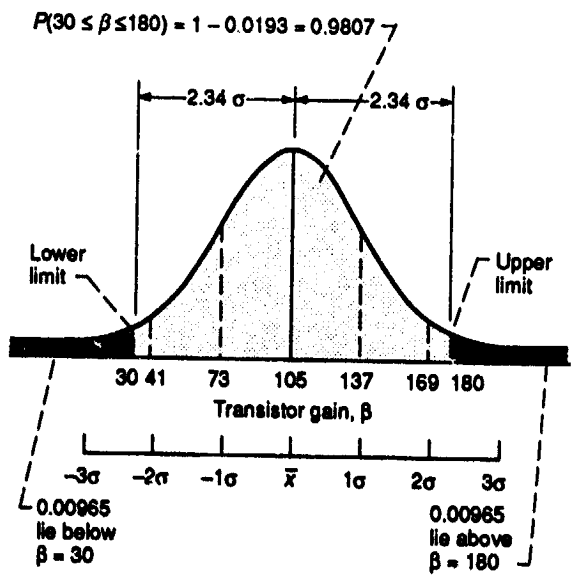


Figure 5-11.—Transistor gain.

the area under one tail of the density function as shown in figure 5-12. Tabulated values of the area in one tail of the normal density function at selected values of z are given in table 5-5.

Example 9: Suppose an exploding bridgewire (EBW) power supply is required to produce an output voltage of at least 1500 V. At this output voltage or greater, all of the bridgewire detonators will explode. If the mean output of all such supplies is known to be 1575 V and the standard deviation is 46 V, what is the probability that an output of 1500 V or greater will be observed?

Solution 9: Step 1—Calculate z.

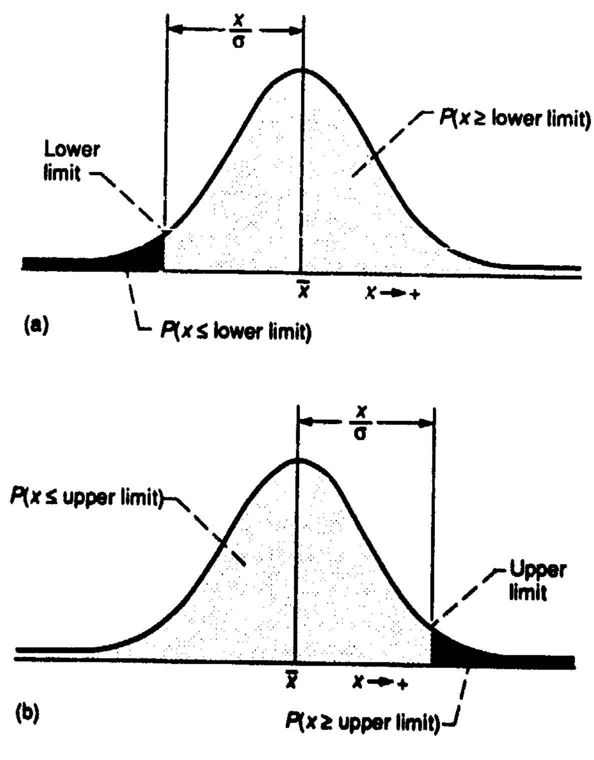
$$z = \frac{\text{Mean} - \text{limit}}{\sigma} = \frac{1575 - 1500}{46} = \frac{75}{46} = 1.63$$

Step 2—Find the area in one tail of the normal curve at z from the mean. From table 5-5 the tail area at $z = 1.63$ from the mean is given as 0.0516. Therefore, there is a 0.0516 probability that an observed output will be below 1500 V.

Step 3—Find the probability that the output will be 1500 V or greater. Since from step 2, $P(x \leq 1500) = 0.0516$,

$$P(x > 1500) = 1 - P(x \leq 1500) = 1 - 0.0516$$

$$= 0.9484, \text{ or } 94.84 \text{ percent}$$

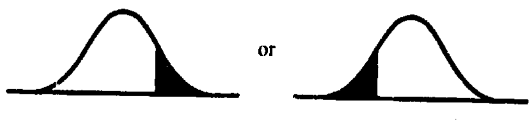


(a) Lower limit.
(b) Upper limit.

Figure 5-12.—Example of one-limit problems.

TABLE 5-5.—AREAS IN ONE TAIL OF NORMAL CURVE AT SELECTED VALUES OF z

[From reference 5-1.]



z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0016	.0113	.0110
2.3	.0107	.0104	.0102	.00990	.00964	.00939	00914	.00889	.00866	.00842
2.4	.00820	.00798	.0076	.00755	.00734	.00714	.00695	.00676	.00657	.00639
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
2.6	00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.007272	.00264
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139

z	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
3	0.00135	$0.0^{3}968$	$0.0^{3}687$	$0.0^{3}483$	$0.0^{3}337$	$0.0^{3}233$	$0.0^{3}159$	$0.0^{3}108$	$0.0^{4}723$	$0.0^{4}481$
4	$.0^{4}317$	$.0^{4}207$	$.0^{4}133$	$.0^{5}854$	$.0^{5}541$	$.0^{5}6340$	$.0^{5}211$	$.0^{5}130$	$.0^{6}793$	$.0^{6}479$
5	$.0^{6}287$	$.0^{6}170$	$.0^{7}996$	$.0^{7}579$	$.0^{7}333$	$.0^{7}190$	$.0^{7}107$	$.0^{8}599$	$.0^{8}332$	$.0^{8}182$
6	$.0^{9}987$	$.0^{9}530$	$.0^{9}282$	$.0^{9}149$	$.0^{10}777$	$.0^{10}402$	$.0^{10}206$	$.0^{10}104$	$.0^{11}523$	$.0^{11}260$

We can therefore expect to obtain a 1500-V output voltage level 94.84 percent of the time. Or to express it another way, 94.84 percent of the supplies will produce an output above the minimum requirement of 1500 V. This result is shown in figure 5-13. Associated with the probability density function $p(x)$ of the normal distribution is a cumulative probability distribution denoted by $F(x)$. As shown in the integral formulas of chapter 2 the relation between the two is given by

$$F(x) = \int p(x)\, dx$$

So, for the normal distribution

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int e^{-1/2[(x-\bar{x})/\sigma]^2}\, dx$$

or in z notation

$$F(z) = \frac{1}{\sqrt{2\pi}} \int e^{-(1/2)z^2}\, dz$$

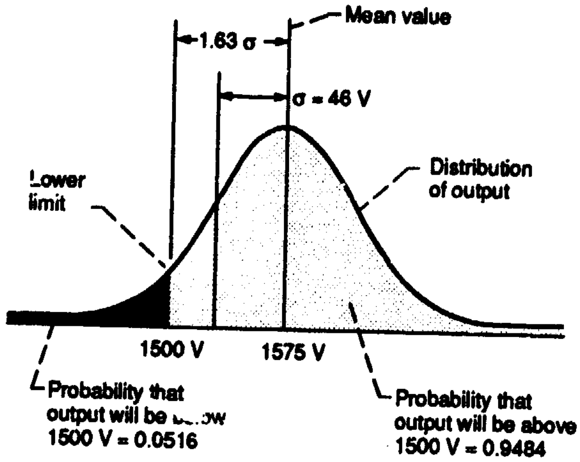


Figure 5-13.—Exploding bridgewire power supply output.

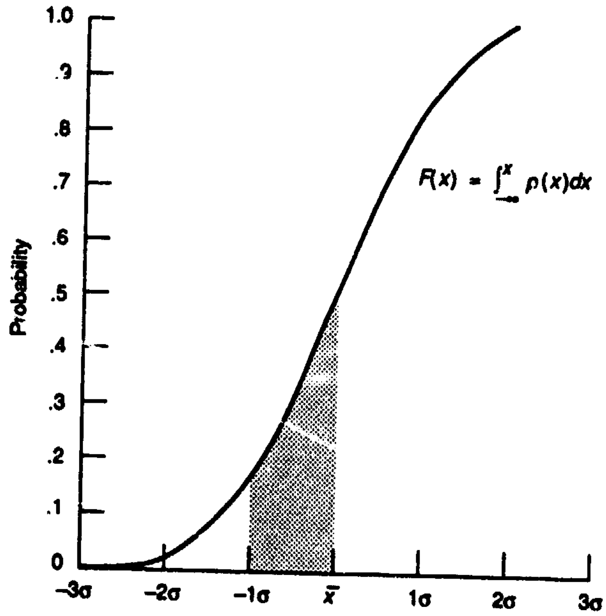


Figure 5-14.—Cumulative normal curve.

A graph of $F(x)$ is shown in figure 5-14. Recall that in discussing cumulative functions earlier, $F(x)$ was called the cumulative area under the density curve. Looking at figure 5-14, then, you can see

(1) That $F(\bar{x}) = 0.5$, or that 50 percent of the area under the normal distribution is between $-\infty$ and the mean $\bar{x}$; or that there is a 50-percent probability that a variable x lies in the interval $(-\infty, \bar{x})$
(2) That $1 - F(\bar{x}) = 0.5$, or that 50 percent of the area under the normal distribution is between the mean $\bar{x}$ and ∞; or that there is a 50-percent probability that a variable x lies in the interval $(\bar{x}, \infty)$

(3) That the area between -1σ and $\bar{x}$ is

$$P(-1\sigma \le x \le \bar{x}) = F(\bar{x}) - F(-1\sigma)$$

$$= 0.5 - 0.16 = 0.34$$

or that there is a 0.34 probability that a variable x will lie between the mean $\bar{x}$ and -1σ

For more accurate work the cumulative areas for selected values of z have been tabulated and are shown in tables 5-6 and 5-7. Table 5-6 shows the cumulative areas for values of z from $-\infty$ to 0, which is illustrated in figure 5-15. Table 5-6 shows

(1) That at $z = 0$ (i.e., when the distance from the limit to $\bar{x}$ is 0) the cumulative area from $-\infty$ to $\bar{x}$ is 0.5000, or 50 percent
(2) That at $z = -1.0$ the cumulative area from $-\infty$ to -1σ is 0.1587, or 15.87 percent
(3) That at $z = -2.0$ the cumulative area from $-\infty$ to -2σ is 0.02275, or 2.275 percent

Table 5-7 shows the cumulative areas for values of z from 0 to ∞. This is illustrated in figure 5-16.

In both tables the value of z is the same as $F(x)$. It therefore follows

(1) That the probability of the variable x lying between $-\infty$ and $\bar{x}$ is

$$P(-\infty \le x \le \bar{x}) = F(\bar{x}) - F(-\infty)$$

$$= F(z=0) - F(z=-\infty)$$

$$= 0.5 - 0 = 0.5, \text{ or 50 percent}$$

(2) That the probability of the variable x lying between -2.1σ and 3.2σ is

$$P(-2.1\sigma \le x \le 3.2\sigma) = F(3.2) - F(-2.1)$$

$$= F(z=3.2) - F(z=-2.1)$$

$$= 0.9993129 - 0.01786$$

$$= 0.9814529, \text{ or 98 percent}$$

Nonsymmetrical Two-Limit Problems

The cumulative function is useful for solving nonsymmetrical two-limit problems, which are, in practice, the most frequently encountered.

Example 10: Suppose that a time-delay relay is required to delay the transmission of a signal at least 90 sec but no more than 98 sec. If the mean "time out" of the specific type of relay is 95 sec and the standard deviation is 2.2 sec, what is the probability that the signal will be delayed within the specified times?

TABLE 5-6.—CUMULATIVE NORMAL DISTRIBUTION FROM $z = -\infty$ to 0
[From reference 5-2.]

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
− .1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
− .2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
− .3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
− .4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
− .5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
− .6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
− .7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
− .8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
− .9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.09853
−1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
−1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
−1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
−1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
−1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03864	.03754	.03673
−1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
−1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
−2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
−2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
−2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
−2.3	.01072	.01044	.01017	$.0^{2}9903$	$.0^{2}9642$	$.0^{2}9387$	$.0^{2}9137$	$.0^{2}8894$	$.0^{2}8656$	$.0^{2}8424$
−2.4	$.0^{2}8198$	$.0^{2}7976$	$.0^{2}7760$	$.0^{2}7549$	$.0^{2}7344$	$.0^{2}7143$	$.0^{2}6947$	$.0^{2}6756$	$.0^{2}6569$	$.0^{2}6387$
−2.5	$.0^{2}6210$	$.0^{2}6037$	$.0^{2}5868$	$.0^{2}5703$	$.0^{2}5543$	$.0^{2}5386$	$.0^{2}5234$	$.0^{2}5085$	$.0^{2}4940$	$.0^{2}4799$
−2.6	$.0^{2}4661$	$.0^{2}4527$	$.0^{2}4396$	$.0^{2}4269$	$.0^{2}4145$	$.0^{2}4025$	$.0^{2}3907$	$.0^{2}3793$	$.0^{2}3681$	$.0^{2}3573$
−2.7	$.0^{2}3467$	$.0^{2}3364$	$.0^{2}3264$	$.0^{2}3167$	$.0^{2}3072$	$.0^{2}2980$	$.0^{2}2690$	$.0^{2}2803$	$.0^{2}2718$	$.0^{2}2635$
−2.8	$.0^{2}2555$	$.0^{2}2477$	$.0^{2}2401$	$.0^{2}2327$	$.0^{2}2256$	$.0^{2}2186$	$.0^{2}2118$	$.0^{2}2052$	$.0^{2}1988$	$.0^{2}1926$
−2.9	$.0^{2}1866$	$.0^{2}1807$	$.0^{2}1750$	$.0^{2}1695$	$.0^{2}1641$	$.0^{2}1589$	$.0^{2}1538$	$.0^{2}1489$	$.0^{2}1441$	$.0^{2}1395$
−3.0	$.0^{2}1350$	$.0^{2}1306$	$.0^{2}1264$	$.0^{2}1223$	$.0^{2}1183$	$.0^{2}1144$	$.0^{2}1107$	$.0^{2}1070$	$.0^{2}1035$	$.0^{2}1001$
−3.1	$.0^{3}9676$	$.0^{3}9354$	$.0^{3}9043$	$.0^{3}8740$	$.0^{3}8447$	$.0^{3}8164$	$.0^{3}7888$	$.0^{3}7622$	$.0^{3}7364$	$.0^{3}7114$
−3.2	$.0^{3}6871$	$.0^{3}6637$	$.0^{3}6410$	$.0^{3}6190$	$.0^{3}5976$	$.0^{3}5770$	$.0^{3}5571$	$.0^{3}5377$	$.0^{3}5190$	$.0^{3}5009$
−3.3	$.0^{3}4834$	$.0^{3}4665$	$.0^{3}4501$	$.0^{3}4342$	$.0^{3}4189$	$.0^{3}4041$	$.0^{3}3897$	$.0^{3}3758$	$.0^{3}3624$	$.0^{3}3495$
−3.4	$.0^{3}3369$	$.0^{3}3248$	$.0^{3}3131$	[illegible]	$.0^{3}2909$	$.0^{3}2803$	$.0^{3}2701$	$.0^{3}2602$	$.0^{3}2507$	$.0^{3}241$[illegible]
−3.5	$.0^{3}2326$	$.0^{3}2241$	$.0^{3}2158$	$.0^{3}2078$	$.0^{3}2001$	$.0^{3}1926$	$.0^{3}1354$	$.0^{3}1785$	$.0^{3}1718$	$.0^{3}1653$
−3.6	$.0^{3}1591$	$.0^{3}1531$	$.0^{3}1473$	$.0^{3}1417$	$.0^{3}1363$	$.0^{3}1311$	$.0^{3}1261$	$.0^{3}1213$	$.0^{3}1166$	$.0^{3}1121$
−3.7	$.0^{3}1078$	$.0^{3}1036$	$.0^{4}9961$	$.0^{4}9574$	$.0^{4}9201$	$.0^{4}8842$	$.0^{4}8496$	$.0^{4}8162$	$.0^{4}7841$	$.0^{4}7532$
−3.8	$.0^{4}7235$	$.0^{4}6948$	$.0^{4}6673$	$.0^{4}6407$	$.0^{4}6152$	$.0^{4}5906$	$.0^{4}5569$	$.0^{4}5442$	$.0^{4}5223$	$.0^{4}5012$
−3.9	$.0^{4}4810$	$.0^{4}4615$	$.0^{4}4427$	$.0^{4}4247$	$.0^{4}4074$	$.0^{4}3908$	$.0^{4}3747$	$.0^{4}3594$	$.0^{4}3446$	$.0^{4}3304$
−4.0	$.0^{4}3167$	$.0^{4}3036$	$.0^{4}2910$	$.0^{4}2789$	$.0^{4}2673$	$.0^{4}2561$	$.0^{4}2454$	$.0^{4}2351$	$.0^{4}2252$	$.0^{4}2157$
−4.1	$.0^{4}2066$	$.0^{4}1978$	$.0^{4}1894$	$.0^{4}1814$	$.0^{4}1737$	$.0^{4}1662$	$.0^{4}1591$	$.0^{4}1523$	$.0^{4}1458$	$.0^{4}1395$
−4.2	$.0^{4}1335$	$.0^{4}1277$	$.0^{4}1222$	$.0^{4}1168$	$.0^{4}1118$	$.0^{4}1069$	$.0^{4}1022$	$.0^{5}9774$	$.0^{5}9345$	$.0^{5}8934$
−4.3	$.0^{5}8540$	$.0^{5}8163$	$.0^{5}7801$	$.0^{5}7455$	$.0^{5}7124$	$.0^{5}6807$	$.0^{5}6503$	$.0^{5}6212$	$.0^{5}5934$	$.0^{5}5668$
−4.4	$.0^{5}5413$	$.0^{5}5169$	$.0^{5}4935$	$.0^{5}4712$	$.0^{5}4498$	$.0^{5}4294$	$.0^{5}4098$	$.0^{5}3911$	$.0^{5}3732$	$.0^{5}3561$
−4.5	$.0^{5}3398$	$.0^{5}3241$	$.0^{5}3092$	$.0^{5}2949$	$.0^{5}2813$	$.0^{5}2682$	$.0^{5}2558$	$.0^{5}2439$	$.0^{5}2325$	$.0^{5}2216$
−4.6	$.0^{5}2112$	$.0^{5}2013$	$.0^{5}1919$	$.0^{5}1828$	$.0^{5}1742$	$.0^{5}1660$	$.0^{5}1581$	$.0^{5}1506$	$.0^{5}1434$	$.0^{5}1366$
−4.7	$.0^{5}1301$	$.0^{5}1239$	$.0^{5}1179$	$.0^{5}1123$	$.0^{5}1069$	$.0^{5}1017$	$.0^{6}9680$	$.0^{6}9211$	$.0^{6}8765$	$.0^{6}8339$
−4.8	$.0^{6}7933$	$.0^{6}7547$	$.0^{6}7178$	$.0^{6}6827$	$.0^{6}6492$	$.0^{6}6173$	$.0^{6}5869$	$.0^{6}5580$	$.0^{6}5304$	$.0^{6}5042$
−4.9	$.0^{6}4792$	$.0^{6}4554$	$.0^{6}4327$	$.0^{6}4111$	$.0^{6}3906$	$.0^{6}3711$	$.0^{6}3525$	$.0^{6}3348$	$.0^{6}3179$	$.0^{6}3019$
$-\infty$	0	0	0	0	0	0	0	0	0	0

TABLE 5-7.—CUMULATIVE NORMAL DISTRIBUTION FROM z = 0 to ∞

[From reference 5-2.]

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5836	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	[illegible]	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	[illegible]	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	$.9^{2}0097$	$.9^{2}0358$	$.9^{2}0613$	$.9^{2}0863$	$.9^{2}1106$	$.9^{2}1344$	$.9^{2}1567$
2.4	$.9^{2}1802$	$.9^{2}2024$	.92240	$.9^{2}2451$	$.9^{2}2656$	$.9^{2}2857$	$.9^{2}3053$	$.9^{2}3244$	$.9^{2}3431$	$.9^{2}3613$
2.5	$.9^{2}3790$	$.9^{2}3963$	$.9^{2}4132$	$.9^{2}4297$	$.9^{2}4457$	$.9^{2}4614$	$.9^{2}4766$	$.9^{2}4915$	$.9^{2}5060$	$.9^{2}5201$
2.6	$.9^{2}5339$	$.9^{2}5473$	$.9^{2}5604$	$.9^{2}5731$	$.9^{2}5855$	$.9^{2}5975$	$.9^{2}6093$	$.9^{2}6207$	$.9^{2}6319$	$.9^{2}6427$
2.7	$.9^{2}6533$	$.9^{2}6636$	$.9^{2}6736$	$.9^{2}6833$	$.9^{2}6928$	$.9^{2}7020$	$.9^{2}7110$	$.9^{2}7197$	$.9^{2}7282$	$.9^{2}7365$
2.8	$.9^{2}7445$	$.9^{2}7523$	$.9^{2}7599$	$.9^{2}7673$	$.9^{2}7744$	$.9^{2}7814$	$.9^{2}7882$	$.9^{2}7948$	$.9^{2}8012$	$.9^{2}8074$
2.9	$.9^{2}8134$	$.9^{2}8193$	$9^{2}8250$	$.9^{2}8305$	$.9^{2}8359$	$.9^{2}8411$	$.9^{2}8462$	$.9^{2}8511$	$.9^{2}8559$	$.9^{2}8605$
3.0	$.9^{2}8650$	$.9^{2}8694$	$.9^{2}8736$	$.9^{2}8777$	$.9^{2}8817$	$.9^{2}8856$	$.9^{2}8893$	$.9^{2}8930$	$.9^{2}8965$	$.9^{2}8999$
3.1	$.9^{2}0324$	$.9^{2}0646$	$.9^{3}0957$	$.9^{3}1260$	$.9^{3}1553$	$.9^{3}1863$	$.9^{3}2112$	$.9^{3}2378$	$.9^{3}2636$	$.9^{3}2886$
3.2	$.9^{3}3129$	$.9^{3}3363$	$.9^{3}3590$	$.9^{3}3810$	$.9^{3}4024$	$.9^{3}4230$	$.9^{3}4429$	$.9^{3}4623$	$.9^{3}4810$	$.9^{3}4991$
3.3	$.9^{3}5166$	$.9^{3}5355$	$.9^{3}5499$	$.9^{3}5658$	$.9^{3}5811$	$.9^{3}5959$	$.9^{3}6103$	$.9^{3}6242$	$.9^{3}6376$	$.9^{3}6505$
3.4	$.9^{3}6631$	$.9^{3}6752$	$.9^{3}6869$	$.9^{3}6982$	$.9^{3}7091$	$.9^{3}7197$	$.9^{3}7299$	$.9^{3}7398$	$.9^{3}7493$	$.9^{3}7585$
3.5	$.9^{3}7674$	$.9^{3}7759$	$.9^{3}7842$	$.9^{3}7922$	$.9^{3}7999$	$.9^{3}8074$	$.9^{3}8146$	$.9^{3}8215$	$.9^{3}8282$	$.9^{3}8347$
3.6	$.9^{3}8409$	$.9^{3}8469$	$.9^{3}8527$	$.9^{3}8583$	$.9^{3}8637$	$.9^{3}8689$	$.9^{3}8739$	$.9^{3}8787$	$.9^{3}8834$	$.9^{3}8879$
3.7	$.9^{3}8922$	$.9^{3}8964$	$.9^{4}0039$	$.9^{4}0426$	$.9^{4}0799$	$.9^{4}1158$	$.9^{4}1504$	$.9^{4}1838$	$.9^{4}2159$	$.9^{4}2468$
3.8	$.9^{4}2765$	$.9^{4}3052$	$.9^{4}3327$	$.9^{4}3593$	$.9^{4}3848$	$.9^{4}4094$	$.9^{4}4331$	$.9^{4}4558$	$.9^{4}4777$	$.9^{4}4988$
3.9	$.9^{4}5190$	$.9^{4}5385$	$.9^{4}5573$	$.9^{4}5753$	$.9^{4}5926$	$.9^{4}6092$	$.9^{4}6253$	$.9^{4}6406$	$.9^{4}6554$	$.9^{4}6696$
4.0	$.9^{4}6833$	$.9^{4}6964$	$.9^{4}7090$	$.9^{4}7211$	$.9^{4}7327$	$.9^{4}7439$	$.9^{4}7546$	$.9^{4}7649$	$.9^{4}7748$	$.9^{4}7843$
4.1	$.9^{4}7934$	$.9^{4}8022$	$.9^{4}8106$	$.9^{4}8186$	$.9^{4}8263$	$.9^{4}8338$	$.9^{4}8409$	$.9^{4}8477$	$.9^{4}8542$	$.9^{4}8605$
4.2	$.9^{4}8665$	$.9^{4}8723$	$.9^{4}8778$	$.9^{4}8832$	$.9^{4}8882$	$.9^{4}8931$	$.9^{4}8978$	$.9^{5}40226$	$.9^{5}0655$	$.9^{5}1066$
4.3	$.9^{5}1460$	$.9^{5}1837$	$.9^{5}2199$	$.9^{5}2545$	$.9^{5}2876$	[illegible]	$.9^{5}3497$	$.9^{5}3788$	$.9^{5}4066$	$.9^{5}4332$
4.4	$.9^{5}4587$	$.9^{5}4831$	$.9^{5}5065$	$.9^{5}5288$	$.9^{5}5502$	$.9^{5}5706$	$.9^{5}5902$	$.9^{5}6089$	$.9^{5}6268$	$.9^{5}6439$
4.5	$.9^{5}6602$	$.9^{5}6759$	$.9^{5}6908$	$.9^{5}7051$	$.9^{5}7187$	$.9^{5}7318$	$.9^{5}7442$	$.9^{5}7561$	$.9^{5}7675$	$.9^{5}7784$
4.6	$.9^{5}7888$	$.9^{5}7987$	$.9^{5}8081$	$.9^{5}8172$	$.9^{5}8258$	$.9^{5}834$	$.9^{5}8419$	$.9^{5}8494$	$.9^{5}8566$	$.9^{5}8634$
4.7	$.9^{5}8699$	$.9^{5}8761$	$.9^{5}8821$	$.9^{5}8877$	$.9^{5}8931$	$.9^{5}8983$	$.9^{6}0320$	$.9^{6}0789$	$.9^{6}1235$	$.9^{6}1661$
4.8	$.9^{6}2067$	$.9^{6}2463$	$.9^{6}2822$	$.9^{6}3173$	$.9^{6}3508$	$.9^{6}3827$	$.9^{6}4131$	$.9^{6}4420$	$.9^{6}4696$	$.9^{6}4958$
4.9	$.9^{6}5208$	$.9^{6}5446$	$.9^{6}5673$	$.9^{6}5889$	$.9^{6}6094$	$.9^{6}6289$	$.9^{6}6475$	$.9^{6}6652$	$.9^{6}6821$	$.9^{6}6981$
∞	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

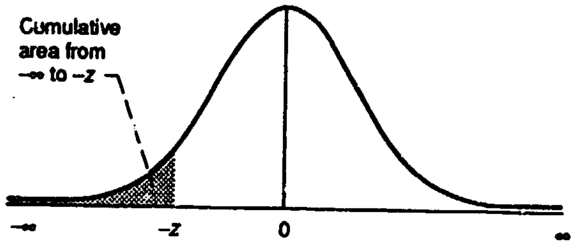


Figure 5-15.—Cumulative areas for values of z from $-\infty$ to 0.

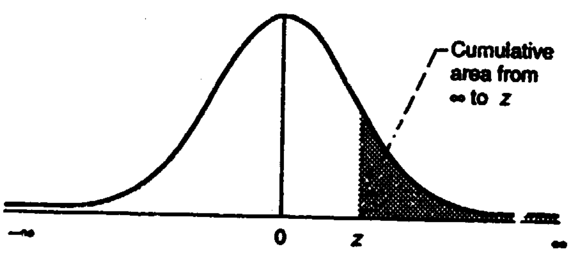


Figure 5-16.—Cumulative areas for values of z from 0 to ∞.

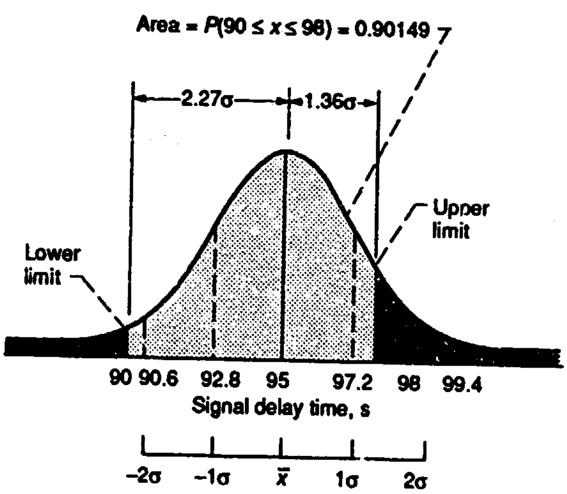


Figure 5-17.—Signal delay time.

Solution 10: Step 1—Find F(98 sec). Since the mean is given as 95 sec and the standard deviation as 2.2 sec,

$$z = \frac{\text{Limit} - \text{Mean}}{\sigma} = \frac{98 - 95}{2.2} = \frac{3}{2.2} = 1.36$$

From table 5-7

$$F(98 \text{ sec}) = F(z) = F(1.36) = 0.91309$$

Step 2—Find F(90 sec). Since the mean is 95 sec and the standard deviation is 2.2 sec,

$$z = \frac{90 - 95}{2.2} = \frac{-5}{2.2} = -2.27$$

From table 5-6

$$F(90 \text{ sec}) = F(z) = F(-2.27) = 0.01160$$

Step 3—Find $P(90 \le x \le 98)$. From steps 1 and 2

$$P(90 \le x \le 98) = F(98) - F(90) = 0.91309 - 0.01160$$

$$= 0.90149, \text{ or } 90 \text{ percent}$$

There exists, therefore, a 90-percent probability that the signal will be delayed no less than 90 sec and no more than 98 sec. This is shown in figure 5-17.

Application of Normal Distribution to Test Analyses and Reliability Predictions

This section gives two examples of how the normal distribution techniques may be applied to the analysis of test data of certain devices and how the results of the analysis may be used to estimate or predict the outcome of actual tests (ref. 5-5). Many similar examples are given in the next chapter.

Example 11: For this two-limit problem, assume that a door hinge has a pin pull-force requirement of 12±4.64 lb. Assume further that we have received 116 door hinges and have actually measured the pin pull-force required for 16 of them as part of an acceptance test. The results of the test are as shown in table 5-8 and in histogram form in figure 5-18. We now want to apply normal distribution theory and then estimate what percentage of the remaining 100 door hinges will meet the pin pull-force requirement.

Solution 11: Step 1—Solve for the mean of the test data $\bar{x}$. We have already seen that

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

where

x_i value of i^{th} measurement

n total number of measurements

TABLE 5-8.—RESULTS OF DOOR HINGE ACCEPTANCE TEST

Pull-force required, lb	Number of occurrences
8	1
10	3
12	7
14	4
16	1
Total	16

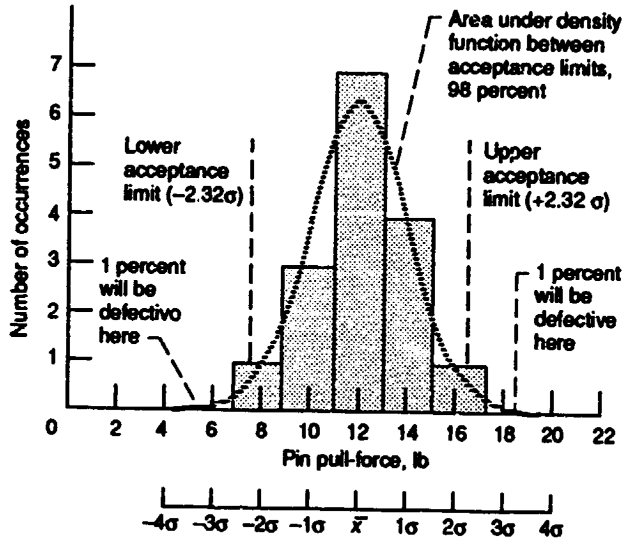


Figure 5-18.—Door hinge test results.

Let x = pound forces so that

$x_1 = 8$	$x_9 = 12$
$x_2 = 10$	$x_{10} = 12$
$x_3 = 10$	$x_{11} = 12$
$x_4 = 10$	$x_{12} = 14$
$x_5 = 12$	$x_{13} = 14$
$x_6 = 12$	$x_{14} = 14$
$x_7 = 12$	$x_{15} = 14$
$x_8 = 12$	$x_{16} = 16$

and let $n = 16$ (number of occurrences). The mean $\bar{x}$ is therefore

$$\bar{x} = \frac{\sum_{i=1}^{16} x_i}{n} = \frac{8 + 3(10) + 7(12) + 4(14) + 16}{16}$$

$$= 12 \text{ lb (rounded to two places)}$$

Step 2—Solve for the standard deviation σ. We have also seen that

$$\sigma = \left[\frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n-1}\right]^{1/2}$$

where

$\bar{x}$ observed mean

x value of i^{th} measurement

n total number of measurements

Solve for $\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2$:

$$\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2 = \sum_{i=1}^{16} \left(x_i - 12\right)^2$$

$$= (8-12)^2 + 3(10-12)^2 + 7(12-12)^2 + 4(14-12)^2 + (16-12)^2$$

$$= (-4)^2 + 3(-2)^2 + 7(0)^2 + 4(2)^2 + (4)^2$$

$$= 16 + 12 + 0 + 16 + 16 = 60$$

Then solve for $\dfrac{\sum_{i=1}^{16} \left(x_i - 12\right)^2}{n-1}$:

$$\frac{\sum_{i=1}^{16} \left(x_i - 12\right)^2}{n-1} = \frac{60}{16-1} = \frac{60}{15} = 4$$

Finally solve for σ:

$$\sigma = \left[\frac{\sum_{i=1}^{16} \left(x_i - 12\right)^2}{n-1}\right]^{1/2} = \sqrt{4} = 2 \text{ lb}$$

Step 3—With a mean of $\bar{x} = 12$ lb and a standard deviation of $\sigma = 2$ lb, figure 5-18 shows

(1) That the lower pull-force limit of 7.36 lb is $z = (7.36 - 12)/2 = -2.32$ standard deviations from the mean
(2) That the upper limit of 16.64 lb is $z = (16.64 - 12)/2 = 2.32$ standard deviations from the mean

Consequently, the percentage of door hinges that should fall within the 12 ± 4.64-lb tolerance is given by

$$P(-2.32\sigma \leq x \leq 2.32\sigma) = F(2.32) - F(-2.32)$$

$$= 0.98983 - 0.01017$$

(from tables 5-6 and 5-7)

$$= 0.97966, \text{ or } 98 \text{ percent}$$

This says that 98 percent of the door hinges should fall within the 12 ± 4.64-lb tolerance and that 2 percent should be outside

of the required tolerance. However, none of the 16 samples were outside the tolerance. So where are the 2 percent that the analysis says are defective? The answer is that the 2 percent of defective door hinges are in the 100 not tested.

We can make this statement by assuming that if we had tested all 100 door hinges, we would have expected to observe the same mean, $\bar{x}$ = 12 lb, and standard deviation, σ = 2 lb, as we did with the 16 samples. (This assumption is subject to confidence limits discussed in chapter 6.) If we accept this assumption, we would expect to find two of the 100 door hinges defective: one would have a pull-force less than 7.36 lb (the lower limit); and one, a pull-force greater than 16.64 lb (the upper limit). This is also shown in figure 5–18.

However, considering the 16 door hinges to be actually representative of all such door hinges, we could predict that only 98 percent of such door hinges produced would meet the acceptance criteria of a 12 ± 4.64-lb pin pull-force.

Example 12: In this one-limit problem, 10 power supplies are selected out of a lot of 110 and tested at increasing temperatures until all exceed a maximum permissible output of 31 V. The failure temperatures in degrees centrigrade of the 10 supplies are observed to be

$x_1 = 57$	$x_6 = 60$
$x_2 = 65$	$x_7 = 75$
$x_3 = 53$	$x_8 = 82$
$x_4 = 62$	$x_9 = 71$
$x_5 = 66$	$x_{10} = 69$

Find the probability that the remaining 100 supplies will have an output greater than 31 V at 50 °C and below.

Solution 12: Step 1—Solve for the mean $\bar{x}$.

$$\bar{x} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{57+65+53+62+66+60+75+82+71+69}{10}$$

$$= \frac{660}{10} = 66 \text{ °C}$$

Step 2—Solve for the standard deviation σ. First,

$$\sum_{i=1}^{10}\left(x_i - 66\right)^2 = (57-66)^2 + (65+66)^2 + (53-66)^2$$
$$+ (62-66)^2 + (66-66)^2 + (60-66)^2$$
$$+ (75-66)^2 + (82-66)^2 + (71-66)^2 + (69-66)^2$$
$$= 81 + 1 + 169 + 16 + 0 + 36 + 81$$
$$+ 256 + 25 + 9$$
$$= 674$$

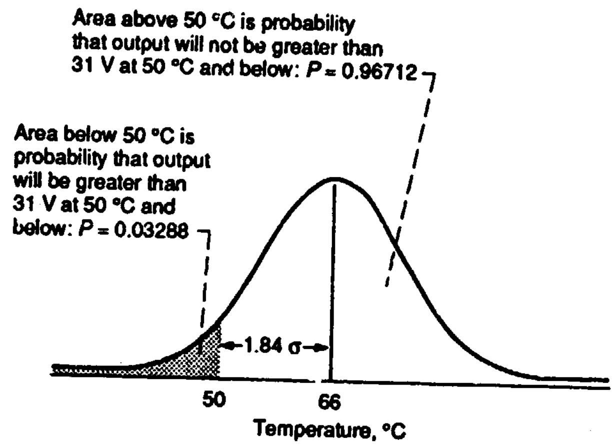


Figure 5–19.—Failure distribution of power supplies.

Then

$$\sigma = \left[\frac{\sum_{i=1}^{10}\left(x_i - 66\right)^2}{n-1}\right]^{1/2} = \left(\frac{674}{9}\right)^{1/2}$$

$$= 8.7 \text{ deg C (rounded to two places)}$$

Step 3—Solve for z = (Limit − Mean)/σ. With an observed mean of $\bar{x}$ = 66 and a standard deviation of σ = 8.7, the 50 °C limit is $z = (50-66)/8.7 = -16/8.7 = -1.84$ observation locations in standard deviations from the mean.

Step 4—Look at table 5–6 and find the cumulative area from $-\infty$ to $\sigma = -1.84$. This is given as 0.03288. Therefore, there is a 3.288-percent probability that the remaining 100 supplies will have an output greater than 31 V at 50 °C and below. This is shown in figure 5–19.

Effects of Tolerance on a Product

(1) What can tolerances do to affect the reliability of a product?
(2) How can tolerances be analyzed?
(3) What methods are available?
(4) What will affect the term P_t in the product reliability model?

These questions are important to ask because tolerances must be expected in all manufacturing processes.

Electrical circuits are often affected by part tolerances (i.e., circuit gains can shift up or down, and transfer function poles or zeros can shift into the right-hand s-plane, causing oscillations). Mechanical components may not fit together or may be so loose that excessive vibration causes trouble (refs. 5–6 to 5–8).

Notes on Tolerance Accumulation: A How-To-Do-It Guide

General.—The notation used in calculating tolerance is

T	tolerance
σ_v	standard deviation
V	dependent variable subject to tolerance accumulation
x	independent, measurable parameter
$1,2,3,n$	subscript notation for parameters
i	generalized subscript (i.e., $i = 1,2,3,\ldots,n$ for x_i)

Tolerance is usually $\pm 3\sigma$. When in doubt, find out. Note that when T is expressed in percent, always convert to engineering units before proceeding. The mean or average is $\bar{V} = f(\bar{x}_i, \bar{x}_2, \bar{x}_3, \ldots \bar{x}_n)$. The coefficient of variation is $C_v = (\sigma/\bar{V}) \times 100$ = percent.

Worst-case method—The worst-case method is as follows:

$$V = f[(\bar{x}_1 + T_1), (\bar{x}_2 + T_2), (\bar{x}_3 + T_3), \ldots, (\bar{x}_n + T_n)]$$

$$-V = f[(\bar{x}_1 - T_1), (\bar{x}_2 - T_2), (\bar{x}_3 - T_3), \ldots, (\bar{x}_n - T_n)]$$

Actually,

$$\pm V = f[(\bar{x}_1 \pm T_1), (\bar{x}_2 \pm T_2), (\bar{x}_3 \pm T_3), \ldots, (\bar{x}_n \pm T_n)]$$

where the plus or minus sign is selected for maximum V and then selected to give minimum V. If these $\pm V$ worst-case limits are acceptable, go no further. If not, try the root-sum-square method.

Root-sum-square method.—The root-sum-square method is valid only if the $f(x\text{'s})$ are algebraically additive (i.e., when V is a linear function of the x's):

$$\pm V = \bar{V} \pm 3\sigma_v$$

where

$$\sigma_v^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \ldots + \sigma_n^2$$

and

$$\sigma_i = \frac{T_i}{3} \qquad \text{if } T_i = \pm 3\sigma$$

Stated another way

$$\pm V = \bar{V} \pm \left[\sum_{i=1}^{n} \left(\frac{T_i}{3}\right)^2\right]^{1/2}$$

If these $\pm V$ root-sum-square limits are acceptable, go no further. If they are not acceptable or the $f(x\text{'s})$ involve products or quotients, try the perturbation or partial derivative methods.

Perturbation method.—The perturbation method is as follows:

$$\pm V = \bar{V} \pm 3\sigma_v$$

where

$$\sigma_v^2 = \left(\bar{V}_{\Delta x_1} - \bar{V}\right)^2 + \left(\bar{V}_{\Delta x_2} - \bar{V}\right)^2 + \ldots + \left(\bar{V}_{\Delta x_n} - \bar{V}\right)^2$$

and where

$$\bar{V}_{\Delta x_1} = f\left[(\bar{x}_1 \pm \sigma_1), (\bar{x}_2 \pm \sigma_2), (\bar{x}_3 \pm \sigma_3), \ldots, (\bar{x}_n \pm \sigma_n)\right]$$

The $\pm V$ limits are valid if $C_v = (\sigma_v/\bar{V}) \times 100 \leq 10$ percent.

Partial derivative method.—The partial derivative method is as follows:

$$\pm V = \bar{V} \pm 3\sigma_v$$

where

$$\sigma_v^2 = \left(\frac{\partial V}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial V}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \ldots + \left(\frac{\partial V}{\partial x_n}\right)^2 \sigma_{x_n}^2$$

The $\pm V$ limits are valid if $C_v = (\sigma/\bar{V}) \times 100 \leq 10$ percent.

Thus, four methods are available for estimating the effects of tolerance on a product. The worst-case method can be used on any problem. In those cases where the $\pm V$ worst-case limits are not acceptable, other methods can be tried. The root-sum-square method is usually valid if the functions are algebraically additive. The perturbation or partial derivative methods are valid only if the coefficient of variation is less than or equal to 10 percent.

Estimating Effects of Tolerance

The following examples illustrate how these tolerance equations can be used. Consider a stacked tolerance problem where the dependent variable is a linear function—three variables added to give $\bar{V}$.

$$\bar{V} = f(\bar{x}_1, \bar{x}_2, \bar{x}_3)$$

$$\bar{V} = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$$

$$T = 3\sigma$$

where

$$\bar{x}_1 = 1 \pm 0.1 \text{ mil}$$

$$\bar{x}_2 = 2 \pm 0.1 \text{ mil}$$

$$\bar{x}_3 = 3 \pm 0.1 \text{ mil}$$

Now, find $\bar{V}$ and the expected range of V.

$$\bar{V} = 1 + 2 + 3 = 6 \text{ mils}$$

Using the worst-case method, with positive tolerance

$$\bar{V}_+ = (1 + 0.1) + (2 + 0.1) + (3 + 0.1) = 6.3_+$$

and with negative tolerance

$$\bar{V}_- = (1 - 0.1) + (2 - 0.1) + (3 - 0.1) = 5.7_-$$

or

$$\bar{V}_\pm = 6 \pm 0.3 \text{ mil}$$

In the worst-case method the tolerance on $\bar{V}$ (i.e., 0.3 mil) is worse than the $3\sigma_v$ tolerance. Tolerance can and often does cause fit problems and circuit problems. Therefore, in some cases we need to know what tolerance is acceptable.

Using the root-sum-square method,

$$\bar{V} = 6 \text{ mils}$$

and

$$\sigma_1 = \frac{0.1}{3} = 0.033 = \sigma_2 = \sigma_3$$

$$\sigma_v = \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right)^{1/2} = \left(3\sigma_1^2\right)^{1/2}$$

$$\left[3(0.033)^2\right]^{1/2} = 0.0572$$

$$3\sigma_v = 0.172$$

so that

$$\bar{V}_\pm = 6 \pm 0.172 \text{ mils}$$

In the root-sum-square method, the T value of 0.172 is the 3σ tolerance on V.

As a second example, consider a volume problem that has three variables in multiplication. Find $\bar{V}$ and the expected range of V.

$$\bar{V} = \bar{L}\bar{W}\bar{H} = 10 \text{ ft} \times 5 \text{ ft} \times 2 \text{ ft} = 100 \text{ ft}^3$$

First, convert percent tolerances to engineering units:

$$\bar{L} = 10 \text{ ft} \pm 10 \text{ percent} = 10 \text{ ft} \pm 10 \text{ ft} \times 0.1 = 10 \text{ ft} \pm 1 \text{ ft}$$

$$\bar{W} = 5 \text{ ft} \pm 10 \text{ percent} = 5 \text{ ft} \pm 5 \text{ ft} \times 0.1 = 5 \text{ ft} \pm 0.5 \text{ ft}$$

$$\bar{H} = 2 \text{ ft} \pm 5 \text{ percent} = 2 \text{ ft} \pm 2 \text{ ft} \times 0.05 = 2 \text{ ft} \pm 0.1 \text{ ft}$$

$$T = \pm 3\sigma$$

Using the worst-case method,

$$V_\pm = (10 \pm 1) \times (5 \pm 0.5) \times (2 \pm 0.1) = 11 \times 5.5 \times 2.1$$

$$\text{or } 9 \times 4.5 \times 1.9 = 127 \text{ or } 77$$

The root-sum-square method cannot be used because these variables are not algebraically additive.

Using the perturbation method,

$$V = \bar{V} \pm 3\sigma_v$$

where

$$\sigma_v = \left[\left(\bar{V}_{\Delta L} - \bar{V}\right)^2 + \left(\bar{V}_{\Delta W} - \bar{V}\right)^2 + \left(\bar{V}_{\Delta H} - \bar{V}\right)^2\right]^{1/2}$$

$$= \left\{\left[\left(\bar{L} + \frac{T_L}{3}\right)\bar{W}\bar{H} - \bar{V}\right]^2 + \left[\left(\bar{W} + \frac{T_W}{3}\right)\bar{L}\bar{H} - \bar{V}\right]^2 + \left[\left(\bar{H} + \frac{T_H}{3}\right)\bar{L}\bar{W} - \bar{V}\right]^2\right\}^{1/2}$$

$$\sigma_L = \frac{T_L}{3} = \frac{1}{3} = 0.33 \text{ ft}$$

$$\sigma_W = \frac{T_W}{3} = \frac{5}{3} = 0.17 \text{ ft}$$

$$\sigma_H = \frac{T_H}{3} = \frac{0.1}{3} = 0.03 \text{ ft}$$

$$\sigma_v = \left\{[(10 + 0.33)(5)(2) - 100]^2 + [(5 + 0.17)(10)(2) - 100]^2 + [2 + 0.03)(10)(5) - 100]^2\right\}^{1/2}$$

$$= \left[(100.3 - 100)^2 + (103.4 - 100)^2 + (101.5 - 100)^2\right]$$

$$= (10.89 + 11.56 + 2.25)^{1/2} = \sqrt{25} = 5$$

$$V = \bar{V} \pm 3\sigma_v = 100 \pm 15 \text{ ft}^3$$

Checking the validity gives

$$C_v = \frac{\sigma_v}{\bar{V}} = \frac{5}{100} \times 10^2 = 5 \text{ percent}$$

which is less than 10 percent. This solution is a better estimate of the effects of tolerance on volume. Note too that various values can now be estimated for different types of problems regarding this volume because it has been represented as a normal distribution function.

Using the partial derivative method, again

$$V_{\pm} = \bar{V} \pm 3\sigma_v$$

where

$$\sigma_v = \left[\left(\frac{\partial V}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \ldots + \left(\frac{\partial V}{\partial x_n}\right)^2 \sigma_{x_n}^2\right]^{1/2}$$

$$V = LWH, \quad \frac{\partial V}{\partial L} = WH, \quad \frac{\partial V}{\partial W} = LH, \quad \frac{\partial V}{\partial H} = LW$$

$$\sigma_L = 0.33 \text{ ft}, \quad \sigma_W = 0.17 \text{ ft}, \quad \sigma_H = 0.03 \text{ ft}$$

$$\sigma_V = \left[(WH)_L^2\, \sigma_L^2 + (LH)_W^2\, \sigma_W^2 + (LW)_H^2\, \sigma_H^2\right]^{1/2}$$

$$= \left[(5 \times 2)^2(0.33)^2 + (10 \times 2)^2(0.17)^2 + (10 \times 5)^2(0.03)^2\right]^{1/2}$$

$$= \left(10.9 + 11.6 + 2.25\right)^{1/2} = \sqrt{25} = 5$$

$$V = 100 \pm 15 \text{ ft}^3$$

This method is more work and gives the same results as the perturbation method. Because the $C_v = 5$ percent, which is less than 10 percent, the method would be suitable to use.

Concluding Remarks

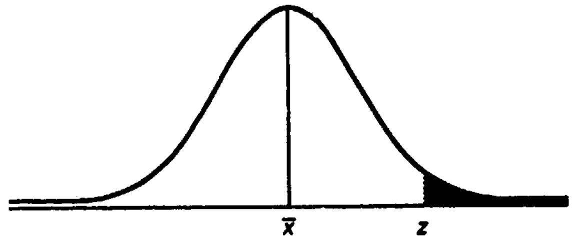
Now that you have completed chapter 5 you should have a clear understanding of the following concepts:

(1) A probability density function $p(x)$ for a random variable describes the probability that the variable will take on a certain range of values.

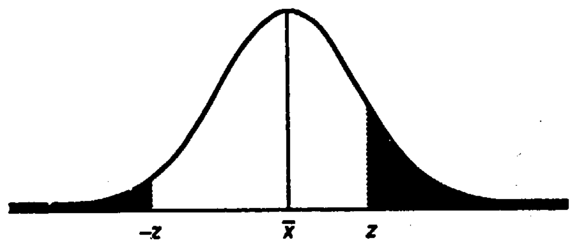
(2) The area under the density function is equal to unity, which means that the probability is 1 that the variable will be within the interval described by the density function. For example, the normal distribution describes the interval from $-\infty$ to ∞.

(3) Associated with each probability density function is a cumulative probability distribution $F(x)$ that represents the cumulative sum of the areas under the density function.

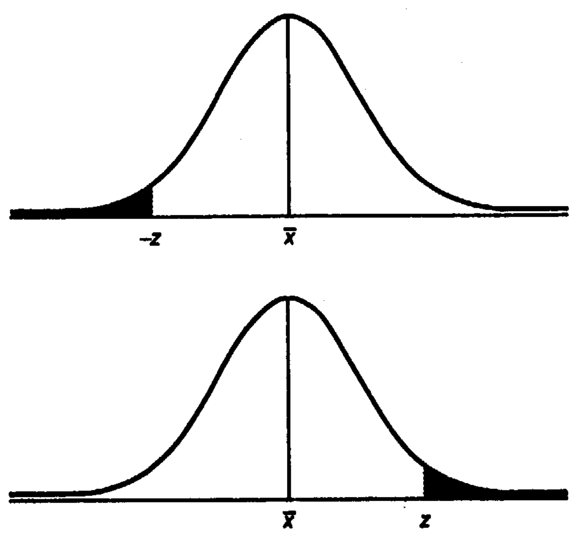
(4) The normal distribution (also called the bell curve, the Gaussian distribution, and the normal curve of error) is a probability density function. Using the normal distribution, you should be able to solve the following types of problems:



(a) Symmetrical two-limit problems, which are concerned with the probability of a variable taking on values within equal distances from both sides of the mean



(b) Nonsymmetrical two-limit problems, which are similar to (a) but within unequal distances from both sides of the mean of the density function



(c) One-limit problems, which are concerned with the probability of a variable taking on values above or below some limit represented by some distance from the mean of the density function

(5) You should be able to take data measurements of a certain device and calculate the mean of the data given by

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

and the standard deviation of the data given by

$$\sigma = \left[\sum_{i=1}^{n} \frac{\left(x_i - \bar{x}\right)^2}{n - 1} \right]^{1/2}$$

and

$$z = \frac{x_i - \bar{x}}{\sigma}$$

Using the data mean and standard deviation, you should then be able to estimate the probability of failures occurring when more of the same devices are tested or operated.

(6) The worst-case method can be used on any problem:

(a) Limits will be defined.

(b) No estimates can be made from the population distribution.

(7) The root-sum-square method only applies to algebraic variables that are additive.

(8) The perturbation or partial derivative methods are only valid if the coefficient of variation is 10 percent or less.

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Reliability Training[1]

1. A unit is required to operate at 100 °F. If tests show the mean strength of the unit is 123 °F, and the standard deviation is 9 deg F, what is the probability that the unit will operate successfully; that is, $P(x \geq 100\ °F)$?

   A. 0.5234 B. 0.2523 C. 0.9946 D. 0.9995

2. A pressure vessel (including a factor of safety) has an upper operating limit of 8000 psi. Burst tests show a mean strength of 9850 psi and a standard deviation of 440 psi. What is the probability of pressure vessel failure; that is, $P(x \leq 8000$ psi)?

   A. 0.0^4267 B. 0.0^4133 C. 0.0^4317

3. A memory drum is required to reach sink speed and stabilize in 15.5 sec at 125 °F. Five drums are tested with these stabilizing time results: 13.2 sec, 12.3 sec, 14.8 sec, 10.3 sec, and 12.9 sec.

   a. What is the mean stabilizing time?

      A. 13.1 B. 10.7 C. 12.7

   b. What is the standard deviation?

      A. 1.63 B. 1.45 C. 1.32

   c. What is the estimated percentage of drums out of specification; that is, $P(x > 15.5$ sec)?

      A. 6.7 B. 8.5 C. 4.3

4. A pyrotechnic gyro has an uncaging time requirement of 142 ± 20 msec. Six gyros were tested resulting in these uncaging times: 123, 153, 140, 129, 132, and 146 msec.

   a. What is the mean uncaging time?

      A. 133.2 msec B. 135.2 msec C. 137.2 msec

   b. What is the standard deviation?

      A. 10.2 B. 11.2 C. 11.9

   c. That is the estimated percentage of gyros within specification; that is, $P(122 \leq x \leq 162$ msec)?

      A. 89.8 B. 96.8 C. 82.6

5. A hydraulic pressure line was designed to the following stresses:

   (a) Maximum operating pressure (actual), 1500 psi

   (b) Design pressure (10 percent safety factor), 1650 psi

   Tests of the pressure line indicated a mean failure pressure of 1725 psi and a standard deviation of 45 psi.

   a. What is the reliability of the line when the design pressure limits are considered?

      A. 0.10 B. 0.90 C. 0.98

   b. What is the reliability of the line when the maximum operating pres^ure is considered?

      A. 0.99 B. 0.90 C. 0.80

[1]Answers are given at the end of this manual.

6. A communications network requires a 1300-msec watchdog delay after initiation. A sample of 10 delays were tested from a rack of 100 delays. The time delays of the circuits are as shown:

Circuit number	Delay, msec
1	1250
2	1400
3	1700
4	1435
5	1100
6	1565
7	1485
8	1385
9	1350
10	1400

a. What is the average (mean) delay time?

A. 1386 msec B. 1400 msec C. 1407 msec

b. What is the standard deviation?

A. 52.7 B. 87.1 C. 163.4

c. On the basis of this sample, what percentage of the 100 circuits will meet specifications (1300 msec or greater delay)?

A. 75 B. 80 C. 90

7. A circuit contains four elements in series. Their equivalent resistance values are

Element	Nominal resistance, R, ohm	Tolerance,[a] T, percent
A	100	± 10
B	20	± 1
C	10	± 5
D	10	± 5

[a]Where $\pm T = \pm 3\sigma$.

a. What is the nominal or mean total resistance $\bar{R}_T$?

A. 120 Ω B. 140 Ω C. 160 Ω

b. What are the worst-case R values?

A. $\begin{matrix}+131.6\\-118.7\end{matrix}\,\Omega$ B. $\begin{matrix}+176.3\\-146.2\end{matrix}\,\Omega$ C. $\begin{matrix}+151.2\\-128.8\end{matrix}\,\Omega$

c. Using the root-sum-square method, what is the probability that $\bar{R}_T \geq 135\ \Omega$?

A. 0.905 B. 0.962 C. 0.933

d. Using the perturbation method, what is the probability that $\bar{R}_T \geq 135\ \Omega$?

A. 0.905 B. 0.962 C. 0.933

8. Given power (watts) = I^2R, where $I = 0.5$ A, $T_I = \pm 5$ percent, $R = 100\ \Omega$, and $T_R = \pm 10$ percent. (Note: $\pm T = \pm 3\sigma$.)

   a. What is the nominal or mean power output P?

      A. 25 W B. 20 W C. 30 W

   b. What are the worst-case P values?

      A. $\begin{matrix}+26.6\\-18.2\end{matrix}$ W B. $\begin{matrix}+35.2\\-22.6\end{matrix}$ W C. $\begin{matrix}+30.3\\-20.3\end{matrix}$ W

   c. Using the perturbation method, what is the probability that $(23.5 \le P \le 26.5)$?

      A. 0.94 B. 0.80 C. 0.86

   d. What is the C_V (in percent) for the perturbation method used in question 8c?

      A. 12% B. 8% C. 46%

   e. Is the root-mean-square method valid for solving the probability problem 8c?

      A. Yes B. No

   f. Using the partial derivative method, what is the probability that $(23.5 \le P \le 26.5)$?

      A. 0.942 B. 0.803 C. 0.857

Chapter 6

Testing for Reliability

In chapters 3 and 4 we discussed the methods used to predict the probability that random catastrophic part failures would occur in given products and systems. These analytical techniques are well established (ref. 6–1). Yet, we should keep in mind that they are practical only when adequate experimental data are available in the form of part failure rates. In other words, their validity is predicated on large amounts of empirical information.

Such is not the case when we undertake similar analyses to determine the influence of tolerance and wearout failures on the reliability of a product. An understanding of these failure modes depends on experimental data in the form of probability density functions such as those discussed in chapter 5. In general, such data are unavailable on items at the part or system level; this kind of information must be developed empirically through reliability test methods.

Chapter 6 reviews and expands on the terms used in the reliability expression given in chapter 2 and then shows how the terms can be demonstrated or assessed through the application of attribute test, test-to-failure, and life test methods (ref. 6–2).

Demonstrating Reliability

Recall from chapter 2 that one way to define product reliability is as the probability that one or more failure modes will not be manifested (ref. 6–3). This can be written as

$$R = P_c P_t P_w (K_q K_m K_r K_\ell K_u)$$

where

- P_c probability that catastrophic part failures will not occur
- P_t probability that out-of-tolerance failures will not occur
- P_w probability that wearout failures will not occur
- K_q probability that quality test methods and acceptance criteria will not degrade inherent reliability
- K_m probability that manufacturing processes, fabrication, and assembly techniques will not degrade inherent reliability
- K_r probability that reliability engineering activities will not degrade inherent reliability
- K_ℓ probability that logistics activities will not degrade inherent reliability
- K_u probability that user or customer will not degrade inherent reliability

The term $P_c P_t P_w$ denotes inherent reliability R_i; $(K_q K_m K_r K_\ell K_u)$ are factors that affect the probability of the three modes of failure occurring during hardware manufacture and use rather than from unreliable hardware design.

First, we illustrate how the empirical value of these terms affects product reliability. Then, we discuss the particular test methods used to develop these values. Assume that a device was designed with a reliability requirement of 0.996. This means that only four out of 1000 such devices can fail. The device contains 1000 parts, it has a function to perform within a tolerance of $X \pm 2$ percent, and it must operate for a mission cycle of 1000 hours at 50 °C.

P_c Illustrated

If we know the number and types of parts in the device plus the applied stresses and part failure rates used in the exponential distribution, $e^{-t(\Sigma\lambda)}$, we can estimate the probability that no catastrophic part failure will occur during the mission cycle. Assuming, for example, that our estimate is $P_c = 0.999$ (i.e., one device in 1000 will incur a catastrophic part failure during the mission cycle), the product reliability of the device becomes

$$R = P_c P_t P_w (K \text{ factors}) = e^{-t(\Sigma\lambda)} P_t P_w (K \text{ factors})$$

$$= 0.999 P_t P_w (K \text{ factors})$$

P_t Illustrated

Suppose we now test one of the devices at 50 °C. If the functional output is greater than the specified tolerance of $X \pm 2$ percent, the reliability of that particular device is zero. It is zero because P_t is zero (i.e., $R = (0.999)(0)P_w$ (K factors) $= 0$). We can say, however, that the device will continue to

operate in an out-of-tolerance condition with a probability of no catastrophic failures equal to 0.999 just as we predicted.

To understand this better, recall that part failure rates reflect only the electrical, mechanical, and environmental stresses applied to the individual parts. For this reason a prediction on the basis of such data will neglect to indicate (1) that the parts have been connected to obtain a specified function, (2) that a tolerance analysis of the function has been performed, or (3) that the parts are packaged correctly. In other words, P_c represents only how well the individual parts will operate, not how well the combined parts will perform.

If nine more of the devices are tested at 50 °C with all the output functions remaining within the $X \pm 2$ percent tolerance, P_t becomes $9/10 = 0.9$ and the reliability of the device $R = (0.999)(0.9)P_w(K \text{ factors})$. Because the reliability requirement of the device is 0.996, it should be clear that P_t must be greater than 0.996. Let us assume then that 1000 devices are tested at 50 °C with only one tolerance failure, which produces an observed $P_t = 999/1000 = 0.999$. The reliability of the device is now

$$R = (0.999)(0.999)P_w(K \text{ factors}) = 0.998\ P_w(K \text{ factors})$$

Note that, because operating time is accumulated during original functional testing, it is possible for random catastrophic part failures to occur. Remember, however, that this type of failure is represented by P_c and not P_t.

P_w Illustrated

Now let us take another operating device and see whether wearout failures will occur within the 1000-hour mission cycle. If, as run time is accumulated, a faulty function output or catastrophic failure is caused by a *wear mechanism*, the reliability of the device again becomes zero. It is zero because P_w is zero as shown in the equation

$$R = (0.999)(0.999)(0)(K \text{ factors}) = 0$$

Note the emphasis on the words "wear mechanism." Because it is possible to experience random catastrophic part failures and even out-of-tolerance conditions during a test for wearout, it is absolutely necessary to perform physics-of-failure analyses. This is essential in ascertaining whether the failures are caused by true physical wear before including them in the P_w assessment.

So far, the first two terms, P_c and P_t, combine to yield a probability of $(0.999)(0.999) = 0.998$. As a result, the remaining terms, $P_w(K \text{ factors})$, must be no less than 0.998 if the 0.996 device requirement is to be satisfied. Therefore, we assume that we have demonstrated a P_w of 0.999, which reduces the device reliability to

$$R = P_cP_tP_w(K \text{ factors}) = (0.999)(0.999)(0.999)(K \text{ factors})$$

$$= 0.997(K \text{ factors})$$

K Factors Illustrated

Since testing obviously must be conducted on real hardware, the K factors as well as the P terms of reliability are present in every test sample. Establishing values for the K factors requires that all failures observed during a test be subjected to physics-of-failure analyses by which specific failure mechanisms are identified. Actually, the action taken to prevent the recurrence of an observed failure mechanism determines the factor that caused the failure. A failure that can be prevented by additional screening tests as part of the quality acceptance criteria is charged to the K_q factor; one that requires additional control over some manufacturing process is charged to the K_m factor, etc. Failures that require changes in documentation, design, and tolerance would be charged to the P_c, P_t, or P_w terms as applicable.

The least important aspect of testing is the ability to charge an organization or function with responsibility for a failure. More important is the need to prevent observed failures from recurring. This requires that corrective action be made a recognized part of each reliability test program.

Getting back to the illustration, we assume that one failure out of 1000 devices was caused by one of the K factors even though it could have been observed during a P_c, P_t, or P_w failure evaluation. This reduces the reliability of the device to

$$R = P_cP_tP_w(K \text{ factors}) = (0.999)(0.999)(0.999)(0.999) = 0.996$$

which indicates that the device met its requirement.

Test Objectives and Methods

The purpose of the preceding illustration was to provide a better understanding of (1) how the P terms and the K factors relate to physical hardware and (2) the techniques for demonstrating the terms through testing. Table 6-1 shows the suggested test methods. We say "suggested" because any of the test methods can be used if certain conditions are met (ref. 6-4). These conditions are pointed out as each method is discussed. Table 6-1 indicates the most efficient methods by assigning priority numbers from 1 to 3 (with 1 being the most efficient and 3 the least).

TABLE 6-1.—TEST METHOD PRIORITIES FOR DEMONSTRATING RELIABILITY

Reliability term	Suggested test method		
	Attribute tests	Tests to failure	Life tests
P_c	2	3	1
P_t	3	1	2
P_w	3	2	1
K factors	3	1	2

Test Objectives

From our discussions thus far it can be inferred that 1000 test samples are required to demonstrate a reliability requirement of 0.999. Because of cost and time considerations this is obviously an impractical approach. Furthermore, the total production of a product often may not even approach 1000 items. Because we usually cannot test the total production of a product (called product population), we must demonstrate reliability on a few samples. Thus, the main objective of a reliability test is to test an available device in such a way that the data will allow a statistical conclusion to be reached about the reliability of similar devices that will not or cannot be tested. In other words, the main objective of a reliability test is not only to evaluate the specific items tested, but also to provide a sound basis for predicting the reliability of similar items that will not be tested and that often have not yet been manufactured.

In chapter 2 we explained that to know how reliable a product is you must know how many ways it can fail and the types and magnitudes of the stresses that produce such failures. This premise leads to a secondary objective of a reliability test, which is to produce failures in the product whereby the types and magnitudes of the stresses that cause such failures are identified. It follows then that reliability tests that result in no failures provide some measure of reliability but little information about the population failure mechanisms of like devices. (There are exceptions, of course, as pointed out later.)

In the subsequent sections of this chapter, we discuss attribute test, test-to-failure, and life test methods, explain how well these methods meet the test objectives just described, show how the test results can be statistically analyzed, and introduce the subject and use of confidence limits. A good discussion of reliability testing for demonstration purposes is given in MIL-STD-785B (ref. 6-1).

Attribute Test Methods

Qualification, preflight certification, and design verification tests fall in the category of attribute tests (refs. 6-4 and 6-5). They are usually of the go/no-go type used to demonstrate that a device is good or bad without showing how good or how bad it may be. In a typical test two samples are subjected to a selected level of environmental stress, usually the maximum anticipated operational limit. If both samples pass, the device is considered qualified, preflight certified, or verified for use in the particular environment involved (refs. 6-6 and 6-7). Occasionally, such tests are called tests to success because the true objective is to have the device pass the test.

This can be illustrated by the example of two power supplies, each with an output requirement of 12 ± 0.24 V at a maximum temperature of 125 °F. If we test these items at 125 °F, we might observe an output of 12.230 V for one and 12.215 V for the other. Since the output of each supply falls within the required tolerance, we would call both qualified, or preflight certified, as the case may be. This might seem to be a declaration that all similar supplies, including any not yet built, would also pass the test and be within the tolerance limit of 125 °F. But no such statement would be valid from the results of so simple a test. The only reasonable conclusion we can reach from testing two samples to success is that these items alone are qualified.

Confidence levels.—Mr. Igor Bazovsky in his book entitled "Reliability Theory and Practice" (ref. 6-2) helps us to understand what the term "confidence" means in the business of testing:

> We know that statistical estimates are more likely to be close to the true value as the sample size increases. Thus, there is a close correlation between the accuracy of an estimate and the size of the sample from which it was obtained. Only an infinitely large sample size could give us a 100 percent confidence or certainty that a measured statistical parameter coincides with the true value. In this context, confidence is a mathematical probability relating the mutual positions of the true value of a parameter and its estimate.
>
> When the estimate of a parameter is obtained from a reasonably sized sample, we may logically assume that the true value of that parameter will be somewhere in the neighborhood of the estimate, to the right or to the left. Therefore, it would be more meaningful to express statistical estimates in terms of a range or interval with an associated probability or confidence that the true value lies within such interval than to express them as point estimates. This is exactly what we are doing when we assign confidence limits to point estimates obtained from statistical measurements.

To illustrate further the limitations of attribute test methods, we apply statistics to the test results. Figure A-4(a) in appendix A shows on the ordinate the number of events (successes) necessary to demonstrate a reliability value (abscissa) for various confidence levels (family of curves) when no failures are observed. Figures A-4(b) to (f) provide the same information when one to five failures are observed.

From the results of two devices tested with no failures, figure A-4(a) shows that we can state with 50-percent confidence that the population reliability of such devices is no less than 71 percent. Fifty-percent confidence means that there is a 50-percent chance that we are wrong and that the reliability of similar untested devices will actually be less than 71 percent. Similarly, we can also state from the same figure that we are 60 percent confident that the reliability of all such devices is 63 percent. But either way the probability of success is less than encouraging.

To gain a better understanding of figure A-4 and the theory behind it, let us stop for a moment and see how confidence levels are calculated. Recall from chapter 2 that the combination of events that might result from a test of two devices was given by

$$R^2 + 2RQ + Q^2 = 1$$

where

R^2 probability that both devices will pass

$2RQ$ probability that one device will pass and one will fail

Q^2 probability that both devices will fail

In the power supply example we observed the first event R^2 because both supplies passed the test. If we assume a 50-percent probability that both will pass, we can set $R^2 = 0.50$ and solve for the reliability of the device as follows:

$$R^2 = 0.50$$

$$R = \sqrt{0.50} = 0.71$$

We then can say with 50-percent confidence that the population reliability of the device is no less than 0.71. By assuming a 50-percent chance, we are willing to accept a 50-percent risk of being wrong, hence the term "50 percent confident." If we want only to take a 40-percent risk of being wrong, we can again solve for R from

$$R^2 = 0.40$$

$$R = \sqrt{0.40} = 0.63$$

In this case, we can be 60 percent confident that the population reliability of the devices is no less than 0.63.

Selection of the confidence level is a customer's or engineer's choice and depends on the amount of risk they are willing to take on being wrong about the reliability of the device. The customer usually specifies the risk he or she is willing to take in conjunction with the system reliability requirement. As higher confidence levels (lower risk) are chosen, the lower the reliability estimate will be. For example, if we want to make a 90-percent confidence (10-percent risk) statement based on the results of the test to success of two devices, we simply solve

$$R^2 = (1 - \text{Confidence level}) = 1 - 0.90 = 0.10$$

so that

$$R = \sqrt{0.10} = 0.316$$

Table 6–2 illustrates how the reliability lower bound changes with various confidence levels. The curves in figure A–4 are developed in a similar manner. In figure A–4(b), which is used when one failure is observed, for 10 samples tested with one observed failure the statistically predicted or demonstrated reliability at 90-percent confidence is 0.66. This answer is found by solving

$$R^{10} + 10R^9Q = 1 - 0.90$$

$$R = 0.663$$

which agrees with the figure to two places.

TABLE 6-2.—RELIABILITY AND CONFIDENCE LEVEL FOR TWO-SAMPLE ATTRIBUTE TEST WITH NO FAILURES

Confidence level, percent	Reliability, R	Risk, percent
10	0.95	90
50	.71	50
60	.63	40
70	.55	30
80	.45	20
90	.32	10
99	.10	1

Application.—The discussion thus far has underscored the shortcomings of attribute tests when sample sizes are small. Tests involving only two or three samples may reveal gross errors in hardware design or manufacturing processes, but when relied on for anything more, the conclusions become risky (refs. 6–8 and 6–9).

Attribute tests can be useful in testing for reliability when a sufficient sample size is used. For example, 10 samples tested without failure statistically demonstrate a population reliability of 0.79 at 90-percent confidence; 100 tests without failure demonstrate a population reliability of 0.976 at 90-percent confidence. To understand better the application of attribute tests and the use of figure A–4, consider the following examples:

Example 1: During the flight testing of 50 missiles, five failures are observed. What confidence do we have that the missile is 80 percent reliable?

Solution 1: From figure A–4(f) the answer is read directly to be a 95-percent confidence level. The a posteriori reliability of these 50 missiles, or that derived from the observed facts, is still 45/50 = 90 percent. Thus, future flights will be at least 80 percent reliable with a 5-percent risk of being wrong.

Example 2: An explosive switch has a reliability requirement of 0.98. How many switches must be fired without a failure to demonstrate this reliability at 80-percent confidence?

Solution 2: From figure A–4(a) the answer is read directly as 80 switches.

Example 3: A test report states that the reliability of a device was estimated to be 0.992 at 95-percent confidence based on a test of 1000 samples. How many failures were observed?

Solution 3: In figure A–4(d) the 95-percent confidence curve crosses the 1000-event line at $R = 0.992$. Therefore, three failures were observed.

In these examples the population reliability estimates may represent any of the P terms or the K factors in the expression for product reliability, depending on the definition of failure used to judge the test results. For a device that is judged only on its capability to remain within certain tolerances, the reliability would be the P_t term. Had catastrophic failures been included, we would have demonstrated the P_cP_t terms.

In general, attribute tests include all failure modes as part of the failure definition and, consequently, the associated reliability is product reliability with both the P terms and the K factors included.

Attribute test/safety margin slide rule.—A special-purpose slide rule has been developed to facilitate determining attribute test/safety margin confidence levels. A slide rule should be in the back of this manual. Take it out and use it as you go over the following examples:

Examples 4 (confidence level for attribute test): Attribute tests are tests to success. The objective is for a selected number of samples, called tests on the slide rule, to operate successfully at some predetermined stress level. Some tests, however, may fail. This slide rule handles combinations of up to 1000 tests and up to 500 failures. The answer is a direct population reliability reading of the untested population at a selected confidence level. Six confidence levels from 50 to 90 percent are available. (The statistical basis for this rule is the χ^2 approximation of binomial distribution.)

Example 4a: Fifteen items are tested with one failure observed. What is the population reliability at 70-percent confidence level?

Solution 4a: Set one failure on the movable slide above the 70-percent confidence level index. Read from TOTAL NUMBER OF TESTS the tests for a population reliability of 0.85 at 70-percent confidence level. By setting one failure at successive levels of confidence this example gives these population reliabilities: 0.710 at 95-percent confidence level, 0.758 at 90 percent, 0.815 at 80 percent, 0.873 at 60 percent, and 0.895 at 50 percent.

Example 4b: A population reliability of 0.9 at 95-percent confidence level is desired. How many tests are required to demonstrate this condition?

Solution 4b: Set zero failures at the 95-percent confidence level index. From TOTAL NUMBER OF TESTS read 29 tests directly above 0.90 population reliability. Therefore, 29 tests without failure will demonstrate this combination. If, however, one failure occurs, set one failure at 95 percent. Then 46 others must pass the test successfully. Progressively more observed failures such as 10 (set of 10 at 95 percent) require 170 successes (160 + 10).

Examples 5 (confidence level for safety margins): Safety margin S_M indicates the number of standard deviations σ_M between some preselected reliability boundary R_b and the mean of the measured sample failure distribution. Thus, $S_M = (\bar{X}_M - R_b) \div \sigma_M$, where $\bar{X}_M$ and σ_M are the measured mean and standard deviation of the samples under test. The larger the sample size, the more nearly the measured S_M approaches the safety margin of the untested population S_D. This rule equates S_M for six levels of confidence for sample sizes N between 5 and 80. (Statistical basis for this rule: noncentral t distribution.)

Example 5a: Ten items are tested to failure with an observed or measured S_M of 5.8. What is the lower expected safety margin of the untested population at 90-percent confidence?

Solution 5a: Set 5.8 on the movable slide at the top window for the S_M value. Under $N = 10$ on the 90-percent window, read $S_D \geq 3.9$. Without moving the slide, for successive levels of confidence, 4.45 at 80 percent, 4.85 at 70 percent, 5.21 at 60 percent, and 5.57 at 50 percent.

Example 5b: Six samples are available for test. What S_M is required to demonstrate a population safety margin of 4.0 or greater at 90-percent confidence level?

Solution 5b: Using the 90-percent window, set $S_D = 4.0$ opposite $N = 6$. At S_M read 7.1. Therefore, test results of 7.1 or greater will demonstrate $S_D \geq 4.0$ at a 90-percent confidence level. If 25 samples are available for test, set $S_D = 4.0$ opposite $N = 25$ on the 90-percent window. An S_M of only 5.0 or greater would demonstrate 4.0 or greater safety margin at 90-percent confidence.

Sneak circuits.—During attribute testing the flight hardware may sometimes not work properly because of a sneak circuit. A sneak circuit is defined for both hardware and software as follows (ref. 6–10):

(1) Hardware: A latent condition inherent to the system design and independent of component failure that inhibits a desired function or initiates an undesired function (path, timing, indication, label)

(2) Software: An unplanned event with no apparent cause-and-effect relationship that is not dependent on hardware failure and is not detected during a simulated system test (path, timing, indication, label)

Each sneak circuit problem should be analyzed, a cause determined, and corrective action implemented and verified. References 6–10 to 6–12 give a number of examples on how this can be done:

(1) Reluctant Redstone—making complex circuitry simple
(2) F-4 example
(3) Trim motor example
(4) Software example

A few minutes spent with one of these references should solve any sneak circuit problem.

Attribute test summary.—In summary, four concepts should be kept in mind:

(1) An attribute test, when conducted with only a few samples, is not a satisfactory method of testing for reliability. But it can identify gross design and manufacturing problems.

(2) An attribute test is an adequate method of testing for reliability only when sufficient samples are tested to establish an acceptable level of statistical confidence.

(3) Some situations dictate attribute tests or no tests at all (e.g., limited availability or the high cost of samples, limited time for testing, test levels that exceed the limits of test equipment, and the need to use the test samples after testing).

(4) Confidence, a statistical term that depends on supporting statistical data, reflects the amount of risk we are willing to take when stating the reliability of a product.

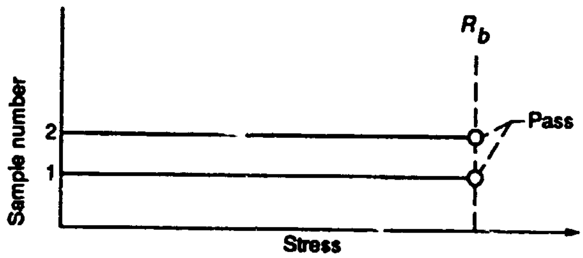


Figure 6-1.—Samples tested to success at reliability boundary.

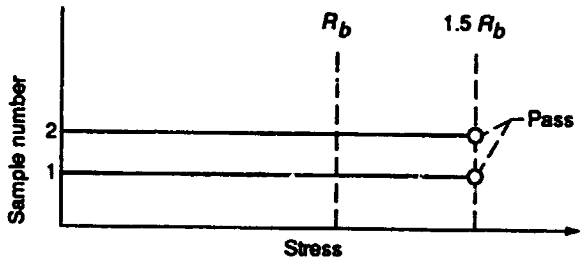


Figure 6-2.—Samples tested to success at 1.5 times reliability boundary.

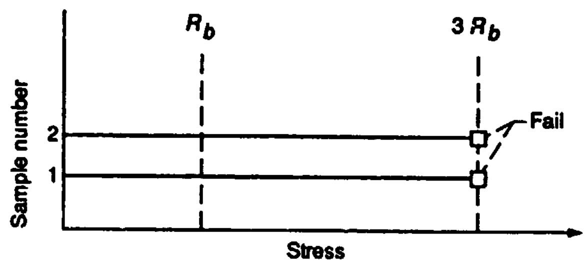


Figure 6-3.—Samples tested to failure at 3 times reliability boundary.

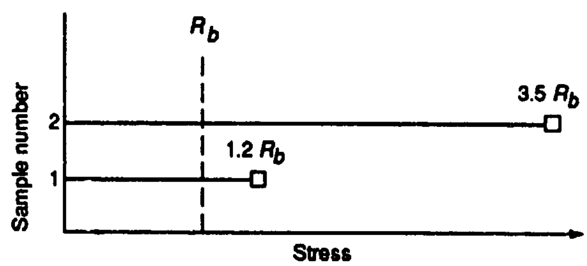


Figure 6-4.—Samples failing at different stress levels above reliability boundary.

Test-To-Failure Methods

Let us return momentarily to the problem of interpreting the result of two samples tested to success at a maximum anticipated stress, or qualification level. This is the reliability boundary R_b, above which a sample is not required to operate or survive. This test result is shown in figure 6-1.

As indicated earlier, such attribute tests tell only whether gross defects exist in the devices tested; they tell nothing about similar devices that will not be tested. To obtain better results, we can test the two samples at a higher stress level, such as

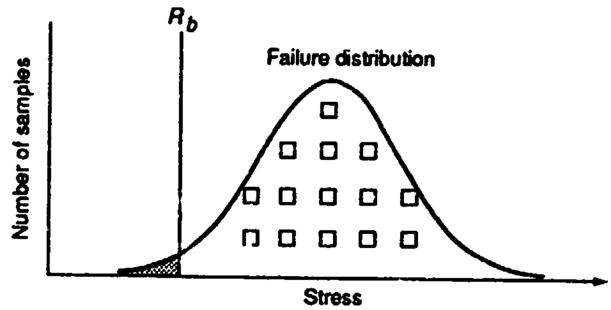


Figure 6-5.—Testing to develop failure distribution.

1.5 times the reliability boundary. If both samples pass at this level, we will certainly feel more confident that similar devices will pass the R_b. Statistically, however, we are no better off than before. This result is shown in figure 6-2.

We can also continue to increase the stress level until both samples fail. If they fail at the same level, such as three times the R_b (as shown in fig. 6-3), we can call the device qualified and infer that all similar devices will survive at stress levels up to the R_b.

But what if one sample fails at 1.2 times the R_b and the other at 3.5 times the R_b (as shown in fig. 6-4)? What then could we say about the point at which a third sample might fail? Would it fail at the R_b, at 2 times the R_b, or below the R_b? Clearly, this type of test result casts some doubt upon the qualification status of the device even though no failure occurs at or below the R_b.

Thus, it is desirable to test enough samples for the failure distribution or density function to be established, as shown in figure 6-5. Afterwards, we can determine the proportion of the product that is expected to fail at or below the R_b. We do this by applying the density function and the cumulative distribution theory discussed in chapter 5.

This method of testing to determine failure distributions is called test to failure. Its purpose is to fail the device under test, instead of passing it as in the attribute test.

Application.—As mentioned before, the purpose of the test-to-failure technique is to develop failure distribution for a product under one or more types of stress. The results are used to calculate the demonstrated reliability of the device for each stress. In this case the demonstrated population reliability will usually be the P_t or P_w product reliability term. Before going further, however, three terms must be understood.

Reliability boundary.—The reliability boundary, which is the maximum anticipated operating stress level, may be represented in two ways:

(1) As a single point, such as 30 g's, 125 °F, −25 °F, or 10 W. When the R_b is presented this way, we assume that the equipment will be operated at the level indicated 100 percent of the time. Because this is usually not done, this method represents a worst-case situation.

(2) As a point in a stress-density function. For example, the g force reliability boundary for a missile autopilot during

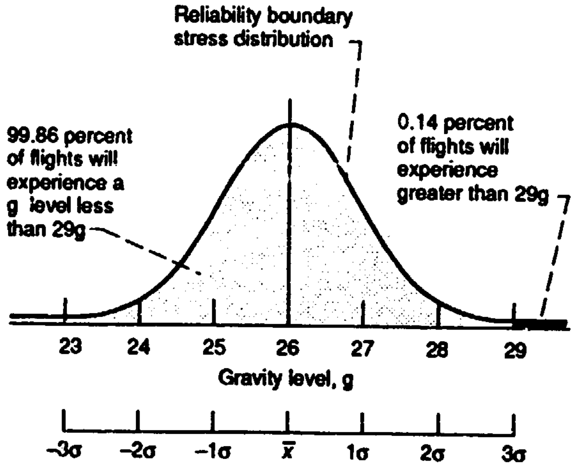


Figure 6-6.—Gravity level during missile flight.

a flight could be expressed as a 3σ limit of a normal distribution—say 29 g's—indicating that a stress of 29 g's or more would be experienced only 0.14 percent of the time. This is shown in figure 6-6.

This method obviously represents a truer picture than method (1) of what stress levels to expect. But this type of stress information is usually hard to obtain. Subsequent sections demonstrate the difference this method makes in design philosophy and the resultant reliability values.

Failure (or strength) distribution.—The failure density function reflects the failure distribution of a device under a specific stress (refs. 6-8 and 6-9). The data used to develop a failure distribution, also called a strength distribution, represent failure points obtained through test-to-failure methods. Figure 6-7 shows such a distribution for a composition resistor at high temperatures, which we interpret just as discussed in chapter 5. For example, we can say that 50 percent of the resistors will fail at 160 °C and below, 84 percent at 170 °C and below, etc.

Safety margin.—The safety margin S_M of a device is defined as the number of standard deviations of the strength distribution σ_s that lie between the reliability boundary and the mean strength $\bar{x}_s$. This is stated mathematically as

$$S_M = \frac{R_b - \bar{x}_s}{\sigma_s}$$

Thus, S_M is the same as the x/σ value calculated in chapter 5 from

$$\frac{x}{\sigma} = \frac{\text{Limit} - \text{Mean}}{\sigma}$$

when the limit is R_b. (The minus sign is ignored.)

As an illustration, assume a reliability boundary of −25 °F for a hydraulic system. Through test-to-failure exposure at low temperatures we are able to define a failure distribution that has a mean of $\bar{x}_s = -37$ °F and a standard deviation of $\sigma_s = 4$ deg F. The safety margin of the system in reference to the −25 °F boundary is given by

$$S_M = \frac{R_b - \bar{x}_s}{\sigma_s} = \frac{-25 - (-37)}{4} = \frac{12}{4} = 3$$

as shown in figure 6-8.

Having calculated a safety margin, we can solve for the percentage of these systems that will lie above or below the reliability boundary. For this we use the technique described in chapter 5 under "One-Limit Problems." In our illustration a safety margin of 3 indicates (from table 5-7 in chapter 5) that 0.998650 of the systems will not fail until the reliability boundary of −25 °F is exceeded. If the failure distribution represents an out-of-tolerance condition, the safety margin of 3 indicates a P_t of 0.998650 at low temperatures.

Test procedure and sample size.—Devices that are not automatically destroyed upon being operated are normally not expended or destroyed during a functional test. Electronic equipment usually falls into this category. For such equipment a minimum sample size of five is necessary, with each sample being subjected to increasing stress levels until failure occurs

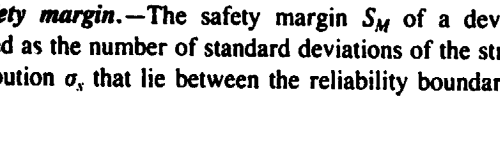


Figure 6-7.—Failure, or strength, distribution of resistor in high temperature.

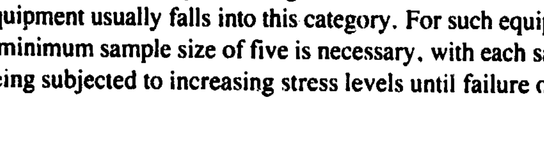


Figure 6-8.—Safety margin of device in low temperature.

or the limits of the testing facility are reached. In the latter case no safety margin calculation is possible because no failures are observed. Here, we must rely on intuition in deciding the acceptability of the device.

Test-to-failure procedure and sample size requirements for one-shot devices are different because a one-shot device is normally expended or destroyed during a functional test. Ordinance items such as squib switches fall into this category. For such devices at least 20 samples should be tested, but 30 to 70 would be more desirable. At least 12 failures should be observed during a test. In a typical one-shot test, of which there are many variations, a sample is tested at the reliability boundary and, if it passes, a new sample is tested at predetermined stress increments until a failure occurs. Then, the next sample is tested at one stress increment below the last failure. If this sample passes, the stress is increased one increment for the next sample. This process, depicted in figure 6-9, continues until at least 12 failures have been observed.

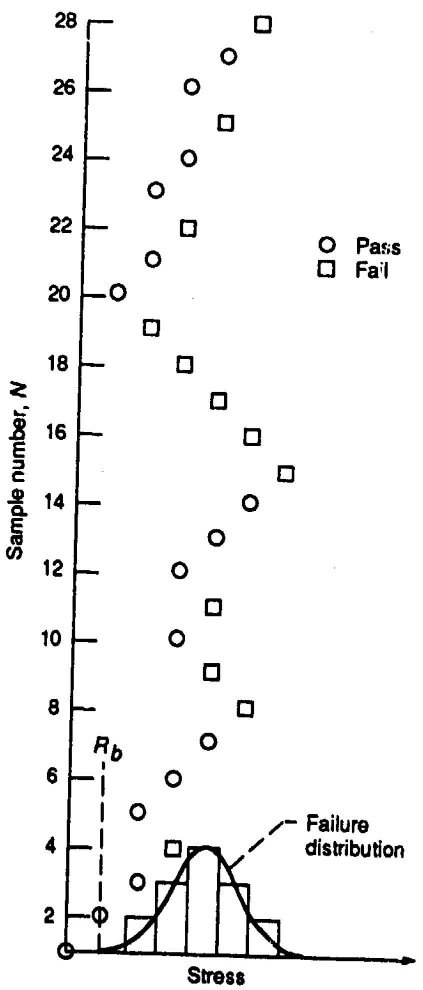


Figure 6-9.—Example of one-shot test-to-failure procedure.

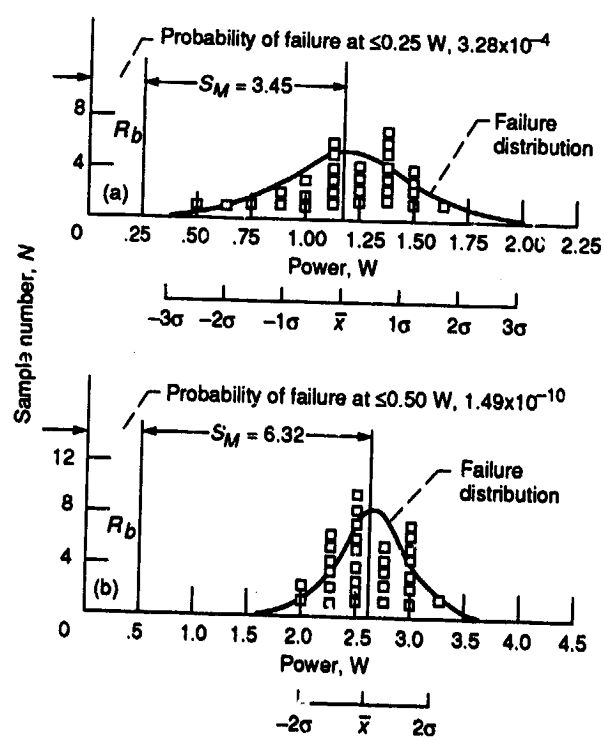
Safety margins for single failure modes.—For devices that exhibit a single failure mode during a test-to-failure exposure, the safety margin and the reliability are calculated by the technique just discussed in the definition of safety margin. The following examples further illustrate the method and show the practical results.

Example 6: A test was conducted on a vendor's 0.25- and 0.50-W film resistors to evaluate their ability to operate reliably at their rated power levels. Thirty samples of each type were tested by increasing the power dissipation until the resistance change exceeded 5 percent. The results are shown in figure 6-10, from which the following points are noteworthy:

(1) The mean strength of the 0.25-W resistor was less than half the mean strength of the 0.50-W resistor: $\bar{x}_{0.25} = 1.19$ W compared with $\bar{x}_{0.50} = 2.6$ W. This was to be expected, since the 0.50-W resistor was larger, had more volume, and could dissipate more energy.

(2) The standard deviation of the 0.25-W resistor was almost the same as that for the 0.50-W resistor: $\sigma_{0.25} = 0.272$ W; $\sigma_{0.50} = 0.332$ W. This was also expected because both resistors were made by the same manufacturer and subjected to the same process controls and quality acceptance criteria.

(3) The 0.50-W resistor, because of its higher mean strength, had a safety margin of 6.32 in reference to its rated power dissipation of 0.50 W. According to table 5-5, this



(a) 0.25-W resistor, $\bar{x}_x = 1.19$ W; $\sigma_x = 0.272$ W.
(b) 0.50-W resistor, $\bar{x}_x = 2.6$ W; $\sigma_x = 0.332$ W.

Figure 6-10.—Test-to-failure results for 0.25- and 0.50-W resistors.

means that only 0.0^9149 resistors would exceed a 5-percent resistance change when applied at 0.50 W. The 0.25-W resistor, because of its lower mean strength, had a safety margin of only 3.45 in reference to its rated power of 0.25 W. According to table 5-5 again, this means that 0.0^3337 resistors would exceed a 5-percent resistance change when applied at 0.25 W. Derating the 0.25 W to 0.125 W increased the safety margin to 3.92 and decreased the expected number of failures to 0.0^4481, an improvement factor of 7.5. This, of course, is the reason for derating components, as discussed in chapter 4. Although we have indicated that a safety margin of 6.32 has statistical meaning, in practice a population safety margin of 5 or higher indicates that the applicable failure mode will not occur unless, of course, the strength distribution deviates greatly from a normal distribution.

Example 7: A fiberglass material to be used for a flame shield was required to have a flexural strength of 15 000 psi. The results of testing 59 samples to failure are presented in figure 6-11. The strength distribution of the material was calculated to have a mean of 19 900 psi and a standard deviation of 4200 psi. The safety margin was then calculated as

$$S_M = \frac{15\,000 - 19\,900}{4200} = 1.17$$

Because, from table 5-7, $S_M = \bar{x}_s/\sigma_s = 1.17$ indicates that 87.9 percent of the samples will fail at reliability boundaries above 15 000 psi, we can see that 12.1 percent will fail at boundaries below 15 000 psi. This analysis is optimistic in that 11/59 = 18.7 percent actually did fail below 15 000 psi. The test also shows that the reliability of the flame shield could be improved by either selecting another type of material to obtain a higher mean strength or changing the fabrication processes to reduce the large strength deviation.

Example 8: Samples of transistors from two vendors were tested to failure under high temperatures. Failure was defined as any out-of-tolerance parameter. The results, shown in figure 6-12, indicate that vendor B's materials, design, and process control were far superior to vendor A's as revealed by the large differences in mean strength and standard deviation. With an S_M of 1.41, 7.9 percent of vendor *A*'s transistors would fail at the 74 °C reliability boundary; with an S_M of 8.27, vendor B's transistors would not be expected to fail at all. It is unlikely that an attribute test would have identified the better transistor.

Example 9: Squib switch samples were tested to failure under vibration in accordance with the procedure for testing one-shot items. The results are shown in figure 6-13, where the mean and standard deviations of the failure distribution have been calculated from the failure points observed. As shown, $\bar{x}_s$ = 14 g's and σ_s = 1.04 g's to produce a safety margin of 3.84 in reference to the reliability boundary of 10 g's.

The preceding examples have shown how the P_t product reliability term can be effectively demonstrated through test-to-failure methods. This has been the case because each example except the squib switch involved a tolerance problem. The examples also show that the K_m factor plays an important role in product reliability and that control over *K* factors can ensure a significant increase in reliability.

Multiple failure modes.—Most products perform more than one function and have more than one critical parameter for each function. In addition, most products are made up of many types of materials and parts and require many fabrication processes during manufacture. It follows then that a product can exhibit a variety of failure modes during testing.

In the conduct of a test to failure each failure mode detected must be evaluated individually; that is, a failure distribution

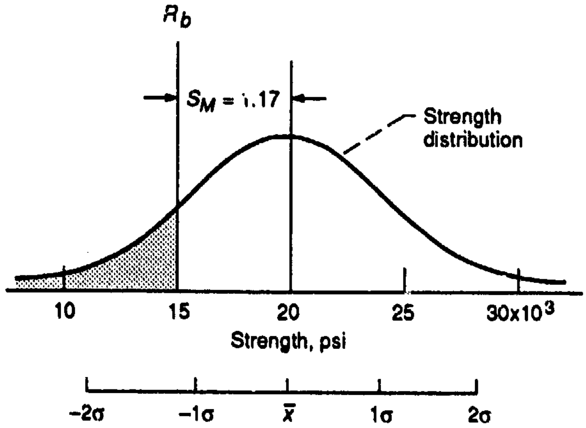
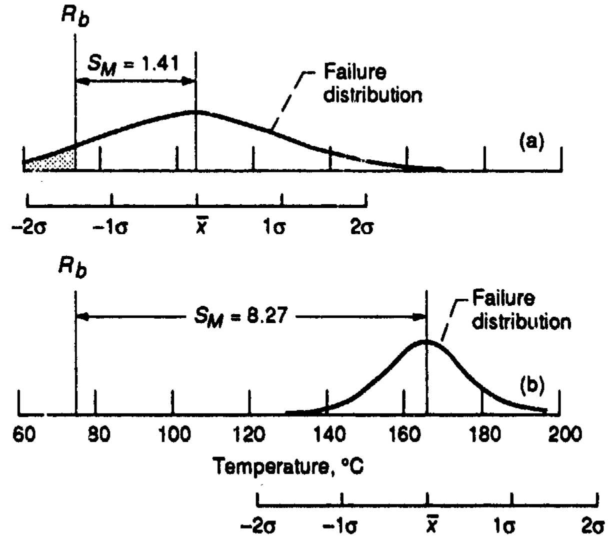


Figure 6-11.—Strength distribution in fiberglass material. $\bar{x}_s$ = 19 900 psi; σ_s = 4200 psi.



(a) Vendor A. $\bar{x}_s$ = 105 °C; σ_s = 22 deg C.
(b) Vendor B. $\bar{x}_s$ = 165 °C; σ_s = 11 deg C.

Figure 6-12.—Test-to-failure results for two transistors.

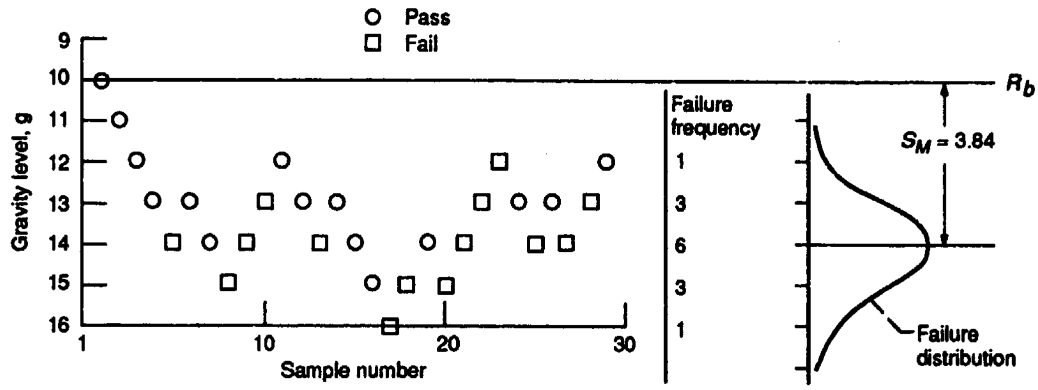


Figure 6-13.—Vibration test-to-failure results of one-shot device (squib switch). $\bar{x}_s$ = 14 g's; σ_s = 1.04 g's.

must be developed for each failure mode and safety margins must be calculated for each individual failure distribution. Moreover, as mentioned before, at least five samples or failure points are needed to describe each failure mode distribution.

To see this more clearly, consider the test results shown in figure 6-14. Here, each of the three failure modes observed is described in terms of its own failure distribution and resulting safety margin with reference to the same reliability boundary. If these failure modes are independent and each represents an out-of-tolerance P_f condition, the P_f of the test device is given by

$$P_{f,\mathrm{total}} = P_{f,1}(S_M = 3.5)P_{f,2}(S_M = 2.1)P_{f,3}(S_M = 7.6)$$

$$= (0.9998)(0.9821)(1.00) = 0.9819$$

This also shows that the independent evaluation of each failure mode identifies the priorities necessary to improve the product. For example, the elimination of failure mode 2, either by increasing $P_{f,2}$ to 1 or by eliminating the mode altogether, increases $P_{f,\mathrm{total}}$ from 0.9819 to 0.9998.

When stress distribution is known.—When safety margins are calculated in reference to a single point or a fixed reliability boundary, the resulting reliability estimate is conservative because it is assumed that the equipment will always be operated at the reliability boundary. As an illustration, figure 6-15 shows the stress distribution for the operating temperature of a device and the maximum anticipated operating limit (145 °F), which is given in the device specifications and would normally be considered the reliability boundary.

Figure 6-16 shows the strength distribution of the device for high temperatures and also that a safety margin for the device, when referenced to the 145 °F reliability boundary, is 1.54, or a reliability of 93.8 percent. We know, however, that the 145 °F limit is the 3σ limit of the stress distribution and will occur only 0.135 percent of the time. The question is, How does this affect the estimated reliability of the device in the temperature environment?

If we select random values from the stress and strength distribution and subtract the stress value from the strength value, a positive result indicates a success—the strength exceeds the stress. A negative result indicates a failure—the

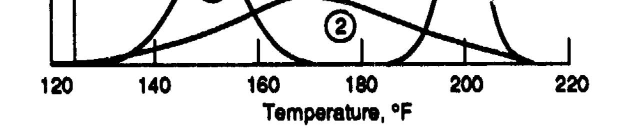


Figure 6-14.—Test-to-failure results when multiple failure modes are observed.

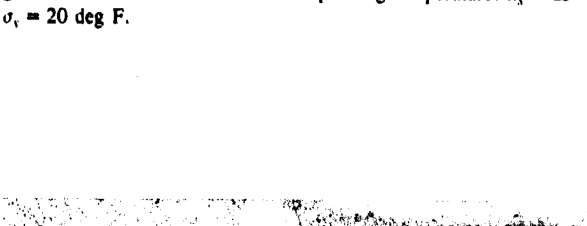


Figure 6-15.—Stress distribution for operating temperature. $\bar{x}_s$ = 85 °F; σ_s = 20 deg F.

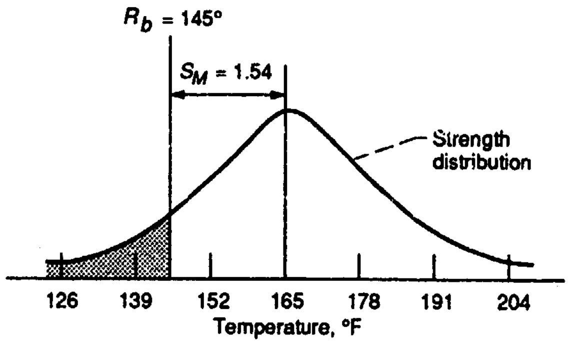


Figure 6-16.—Strength distribution for operating temperature. $\bar{x}_s$ = 165 °F, σ_s = 13 deg F.

stress exceeds the strength. With this knowledge we can calculate a *difference distribution* and, through the application of the safety margin technique, solve for the probability of the strength being greater than the stress (i.e., success). This difference distribution is also distributed normally and has the following parameters:

$$\bar{x}_{\text{difference}} = \bar{x}_s - \bar{x}_{\text{stress}}$$

$$\sigma_{\text{difference}} = (\sigma_s^2 - \sigma_{\text{stress}}^2)^{1/2}$$

From the strength and stress distribution parameters given in the preceding example (figs. 6-15 and 6-16),

$$\bar{x}_{\text{difference}} = 165 - 85 = 80\ ^\circ\text{F}$$

$$\sigma_{\text{difference}} = \left(20^2 + 13^2\right)^{1/2} = 24 \text{ deg F}$$

This distribution is shown in figure 6-17.

Because positive numbers represent success events, we are interested in the area under the difference distribution that includes only positive numbers. This can be calculated by using zero as the reliability boundary and solving for the safety margin from

$$S_M = \frac{0 - \bar{x}_s}{\sigma_s} = \frac{0 - 80}{24} = 3.33$$

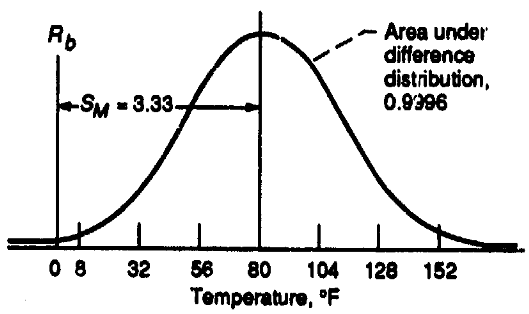


Figure 6-17.—Strength and stress difference distribution. $\bar{x}_s$ = 80 °F; σ_s = 24 deg F.

This 3.33 safety margin gives a reliability of 0.9996 when the stress distribution is considered. Comparing this result with the estimated reliability of 0.938 when the reliability boundary point estimate of 145 °F was used shows the significance of knowing the stress distribution when estimating reliability values.

Confidence levels.—As discussed before, the main objective in developing a failure distribution for a device by test-to-failure methods is to predict how well a population of like devices will perform. Of course, such failure distributions, along with the resulting safety margins and reliability estimates, are subject to error. Errors result from sample size limitations in much the same way that the demonstrated reliability varies with sample size in attribute testing. Specifically, the mean and the standard deviations of the strength distribution must be adjusted to reflect the sample size used in their calculation. Tables A-3 to A-5 in appendix A have been developed for this purpose by using the noncentral t distribution. Table 6-3 shows the applicable appendix A tables for selected confidence levels and sample sizes, and the examples that follow illustrate their use.

Example 10: Upon being tested to failure at high temperatures, 10 devices were found to have a failure distribution of $\bar{x}_s$ = 112.7 °C and σ_s = 16 deg C. The reliability boundary was 50 °C. Find the safety margin and reliability demonstrated at 90-percent confidence.

Solution 10: Step 1—Solve first for the observed safety margin.

$$S_M = \frac{R_b - \bar{x}_s}{\sigma_s} = \frac{50 - 112.7}{16} = 3.92$$

From table 5-7 the observed reliability is 0.99996.

Step 2—Now refer to table A-5(a) in appendix A, which deals with 90-percent confidence limits for safety margins, and follow across to column N = 10, the number of samples. The values under the N headings in all of the tables listed in table 6-3 represent the observed safety margins for sample sizes as calculated from raw test data. The S_M column lists corresponding population safety margins for the observed safety margins shown under the N headings. Finally, corre-

TABLE 6-3.—CONFIDENCE LEVEL TABLES FOR VARIOUS SAMPLE SIZES

Confidence level, percent	Sample size			
	5-12	13-20	21-29	30-100
	Confidence level tables			
99	A-3(a)	A-3(b)	A-3(c)	A-3(d)
95	A-4(a)	A-4(b)	A-4(c)	A-4(d)
90	A-5(a)	A-5(b)	A-5(c)	A-5(d)

sponding population reliability estimates are shown under the P_x headings, which may represent P_t or P_w as applicable.

Step 3—Proceed down the $N = 10$ column to 3.923, the observed safety margin derived in step 1.

Step 4—Having located $S_M = 3.923$ with 10 samples, follow horizontally to the left to find the demonstrated population safety margin in the S_M column. This is 2.6.

Step 5—With a population S_M of 2.6, follow the same line to the right to find the population reliability estimate under the P_x heading. This value is 0.9953. Recall that the observed safety margin was 3.923 and the observed reliability, 0.99996.

Example 11: Twelve gyroscopes were tested to failure by using time as a stress to develop a wearout distribution. The wearout distribution was found to have a $\bar{x}_s$ of 5000 hours and a σ_s of 840 hours. Find the P_w demonstrated at 95-percent confidence with a reliability boundary of 1000 hours.

Solution 11: Step 1—The sample safety margin is

$$S_M = \frac{1000 - 5000}{840} = 4.76$$

Step 2—The population safety margin at 95-percent confidence with a 12-sample safety margin of 4.76 is read directly from table A-4(a) to be 3.0.

Step 3—For a population S_M of 3.0, the corresponding P_w under the P_x column is 0.9986. Thereby 99.86 percent of the gyroscopes will not wear out before 1000 hours have been accumulated.

Safety factor.—This section is included in the discussion of test-to-failure methods because the term "safety factor" is often confused with safety margin. It is used widely in industry to describe the assurance against failure that is built into structural products. There are many definitions for safety factor S_F, with the most common being the ratio of mean strength to reliability boundary:

$$S_F = \frac{\bar{x}_s}{R_b}$$

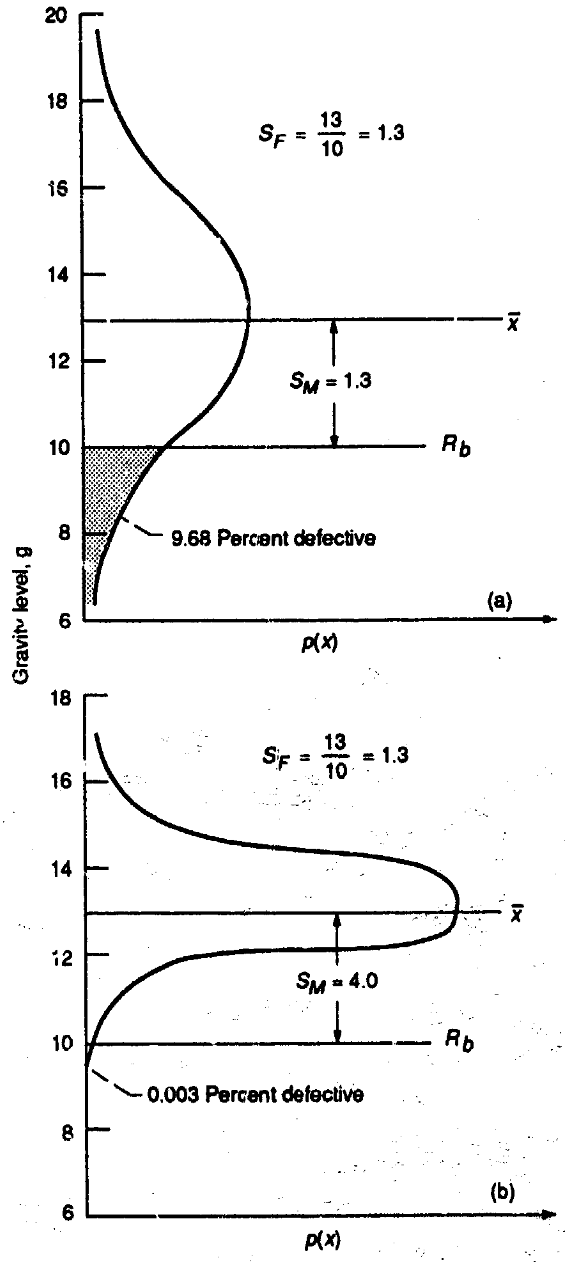
When dealing with materials with clearly defined, repeatable, and "tight" strength distributions, such as sheet and structural steel or aluminum, using S_F presents little risk. However, when dealing with plastics, fiberglass, and other metal substitutes or processes with wide variations in strength or repeatability, using S_M provides a clearer picture of what is happening (fig. 6-18). In most cases, we must know the safety margin to understand how accurate the safety factor may be.

Test-to-failure summary.—In summary, you should understand the following concepts about test-to-failure applications:

(1) Developing a strength distribution through test-to-failure methods provides a good estimate of the P_t and P_w product reliability terms without the need for the large samples required for attribute tests.

(2) The results of a test-to-failure exposure of a device can be used in predicting the reliability of similar devices that cannot or will not be tested.

(3) Testing to failure provides a means of evaluating the failure modes and mechanisms of devices for improvement purposes.



(a) Structure A.
(b) Structure B.

Figure 6-18.—Two structures with identical safety factors (S_F = 13/10 = 1.3) but different safety margins.

(4) It allows confidence levels to be applied to the safety margins and to the resulting population reliability estimates.

(5) To know how accurate a safety factor may be, we must also know the associated safety margin.

Life Test Methods

Chapters 3 and 4 introduced the "bathtub" curve used to illustrate how the failure rate of a typical system or complex subsystem varies during its operating life. In association with this curve we identified three traditional failure rate regions: the debugging or burn-in region, the intrinsic-failure-rate region, and the wearout region. This curve is presented again in figure 6–19, but this time with data that indicate when the failure rate regions occur.

This illustration shows that the greatest reduction in failure rate during the debugging or burn-in region (as great as 10 to 1) occurs before 600 to 1200 hours of operation. The curve also shows that electronic failure rates continue to decrease through as much as 26 000 hours, or 3 years, of continuous operation without signs of a wearout region. Items of equipment with true inherent wear mechanisms usually enter the wearout region at 3000 or more hours.

It should be obvious that such data provide valuable guidelines for controlling product reliability. They figure prominently in the establishment of burn-in requirements, predictions of spare part requirements, and an understanding of the need or lack of need for a system overhaul program. Such data are obtained through laboratory life tests or from the normal operation of a fielded system. In either case collecting and assessing life data are vital in testing for reliability.

Application.—Although life test data are derived basically for use in evaluating the failure characteristics of a product, byproducts of the evaluation may serve many other purposes. Four of the most frequent are

(1) To serve as acceptance criteria for new hardware. For example, a product may be subjected to a life test before it is accepted for delivery to demonstrate that its failure rate is below some predetermined value. Examples of such applications are burn-in or debugging tests and group B life tests conducted on electronic parts. Some manufacturers of communications satellites subject all electronic parts to a 1200-hour burn-in test and use only the ones that survive.

(2) To identify product improvement methods. Here, life tests serve a dual purpose by providing hardware at essentially no cost for physics-of-failure analyses. In turn, these analyses identify failure mechanisms and the action needed to reduce effectively a product's failure rate. In the past 10 years this has resulted in significant part failure rate reductions. In fact, the failure rates of some components have been reduced so far that accelerated life tests (life tests at elevated stress levels) and test-to-failure techniques must be employed to attain reliability improvements in a reasonable timeframe.

(3) To establish preventive maintenance policies. Products with known or suspected wear mechanisms are life tested to determine when the wearout process will begin to cause undesirable failure rate trends. Once the wearout region is established for a product, system failures can be reduced by implementing a suitable preventive maintenance plan or overhaul program. This is effectively illustrated in figure 6–20, which shows the failure rate trend in a commercial jet aircraft subsystem. Here, the upward trend after 4000 hours of operation was revealed to be caused by a servomechanism that required lubrication. By establishing a periodic lubrication schedule for the mechanism, further failures were eliminated. Note that this subsystem also exhibited burn-in and intrinsic-failure-rate regions.

(4) To assess reliability. Here, tests are performed or life data collected from fielded systems to establish whether contractual reliability requirements are actually being met. In cases of noncompliance and when the field failures are analyzed, one of the preceding methods is employed to improve the product, or else a design change is implemented. The effectiveness of the corrective action is then evaluated from additional life data. Because life-test-observed failure rates include catastrophic, tolerance, wearout, and K factor failures, life tests usually demonstrate product reliability.

Test procedure and sample size.—Conducting a life test is fairly straightforward. It involves only the accumulation of equipment operating time. Precautions must be taken, however, when the test is conducted in a laboratory. Operating conditions must include all of the factors that affect failure

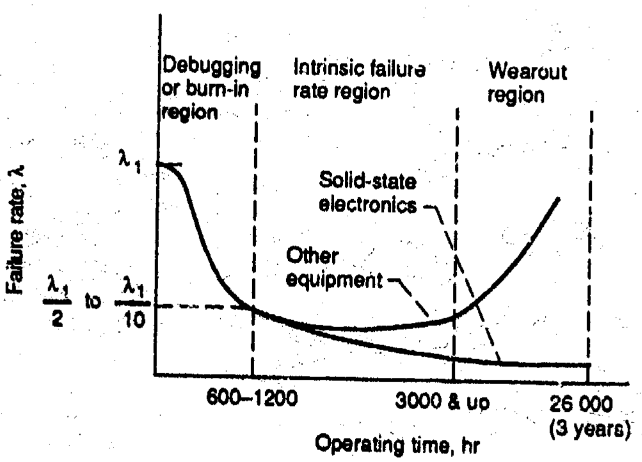


Figure 6–19.—Failure rate versus operating time for typical systems and complex subsystems.

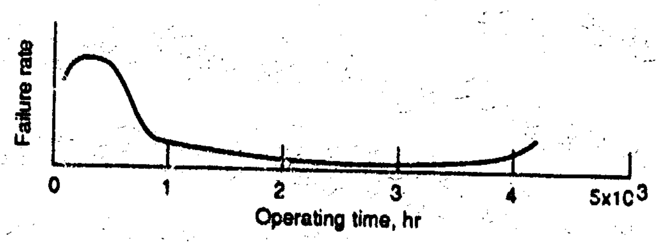


Figure 6–20.– Failure rate characteristics of commercial jet electronic subsystem.

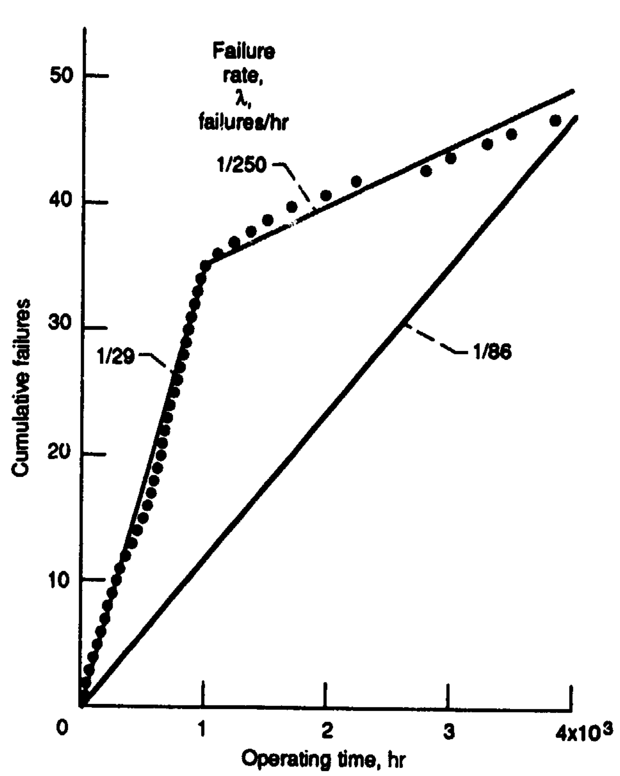


Figure 6-21.—Results of complex electronic system life test.

rates when the device is operated tactically. Major factors are environment, power-on and power-off times, power cycling rates, preventive maintenance, operator tasks, and field tolerance limits. Ignoring any of these factors may lead to an unrealistic failure rate estimate.

When accelerated life tests are conducted for screening purposes, stress levels no greater than the inherent strength of the product must be chosen. The inherent strength limit can be evaluated through test-to-failure methods before the life tests are conducted.

Experience with nonaccelerated life tests of military standard electronic parts for periods as long as 5000 hours indicates that an average of one to two failures per 1000 parts can be expected. For this reason life tests will not provide good reliability estimates at the part level except when quantities on the order of 1000 or more parts are available. On the other hand, life tests are efficient at the system level with only one sample as long as the system is fairly complex (includes several thousand parts).

Life tests intended to reveal the wearout characteristics of a device may involve as few as five samples, although from 20 to 30 are more desirable if a good estimate of the wearout distribution is to be obtained.

Analyzing life test data.—Recall from chapter 3 that an empirical definition of mean time between failures (MTBF) was given as

$$\text{MTBF} = \frac{\text{Total test hours}}{\text{Total observed failures}}$$

Remember also that, because this expression neglects to show when the failures occur, it assumes an intrinsic failure rate and therefore an intrinsic mean time between failures, or MTBF. The assumption of an intrinsic failure rate may not be valid in some cases, but life test results have traditionally been reported this way.

To see this illustrated, consider the results of a 4000-hour life test of a complex (47 000 parts) electronic system as shown in figure 6-21. This graph plots cumulatively in terms of the times the 47 failures are observed, so that the slopes of the lines represent the failure rate. The solid line shows the system failure rate that resulted from assuming an intrinsic failure rate, which was

$$\lambda = \frac{\text{Total failures}}{\text{Total operation time}} = \frac{47}{4000} = 1 \text{ failure/86 hours}$$

From the plotted test data, it is obvious that this intrinsic failure rate was not a good estimate of what really happened. The plotted data indicate that there were two intrinsic-failure-rate portions: one from 0 to 1000 hours and the other from 1000 to 4000 hours. In the 0- to 1000-hour region the actual failure rate was

$$\lambda = \frac{35}{1000} = 1 \text{ failure/29 hours}$$

or about 3 times higher than the total average failure rate of 1/86 hours; in the 1000- to 4000-hour region the actual failure rate was

$$\lambda = \frac{12}{3000} = 1 \text{ failure/250 hours}$$

or about 2.9 times lower than the average.

This illustration establishes the desirability of knowing when failures occur, not just the number of failures. The results of analyzing data by regions can be used to evaluate burn-in and spare parts requirements. The burn-in region was identified to be from 0 to 1000 hours because after this time the failure rate decreased by a factor of 8.6.

This result also has a significant effect on logistics. For example, if we assume that the system will accumulate 1000 hours per year, we can expect during the first year to replace 35 parts

$$\left(\frac{1 \text{ failure}}{29 \text{ hours}} \times 1000 \text{ hours}\right)$$

whereas during the next and subsequent years we can expect to make only four replacements

$$\left(\frac{1 \text{ failure}}{250 \text{ hours}} \times 1000 \text{ hours}\right)$$

Using the average failure rate of 1 failure/86 hours, we would have to plan, however, for 28 replacements every year. Obviously, the cost impact of detailed analysis can be substantial.

Running averages.—When system failure rates are irregular or when there is need to evaluate the effect of different operating conditions on a system, running average analyses are useful. This can best be illustrated through the example presented in figure 6-22. A 300-hour running average in 50-hour exposures is shown for a complex system during an engineering evaluation test. (Running averages are constructed by finding the failure rate for the first 300 hours of operation, then dropping the first 50 hours and picking up the 300- to 350-hour interval and calculating the new 300-hour regional failure rate, and then repeating the process by dropping the second 50 hours of data and adding the next 50 hours for the total test period.) From the resultant curve you can readily see (1) the effects of the debugging test, (2) the increase in failure rate during the high-temperature test and the decrease after that test, (3) another increase during low-temperature exposure and the subsequent decrease, (4) a slight increase caused by vibration, and (5) a continuously decreasing rate as the test progressed. The curve indicates that the system is the most sensitive to high temperature and that, because the failure rate continued to decrease after high-temperature exposure, exposure to high temperatures is an effective way to screen defective parts from the system. Because the failure rate continued to decrease after the tests were completed, neither low temperature nor vibration caused permanent damage to the system.

At the end of the 3000-hour period the failure rate was 3.3 failures per 1000 hours. This reflected a tenfold decrease from the initial failure rate during debugging, typical of the results observed for many complex systems. An example of a running average failure rate analysis that identifies a system wearout region is shown in figure 6-23. The increasing failure rate after 3000 hours was caused by relay failures (during approximately 10 000 cycles of operation). This type of information can be used to establish a relay replacement requirement as part of a system preventive maintenance plan.

Confidence levels.—As discussed in chapter 4, failure rates are statistical. Consequently, they are subject to confidence levels just as attribute and test-to-failure results are influenced by such factors. Confidence levels for intrinsic failure rates are calculated by using table A-2 in appendix A.

To use this table, first calculate the total test hours accumulated from

$$t = \sum_{i=1}^{n} N_i t_i$$

where

N_i i^{th} unit tested

t_i test time of N_i

n total units tested

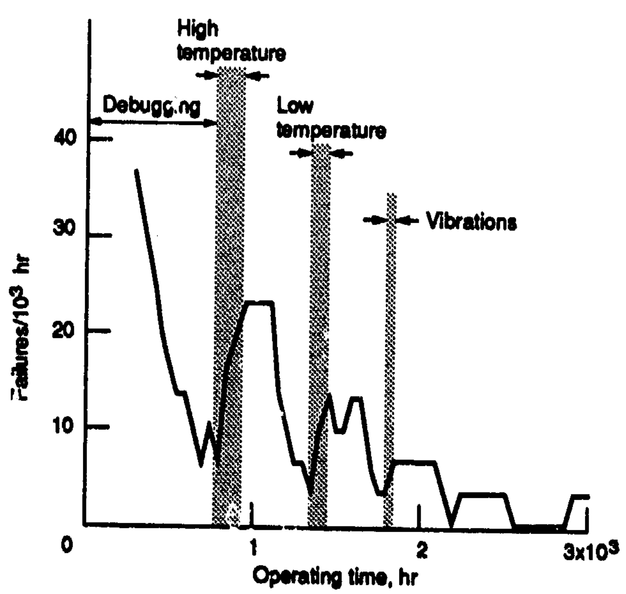


Figure 6-22.—Running average failure rate analysis of life test data (300-hr running average in 50-hr increments).

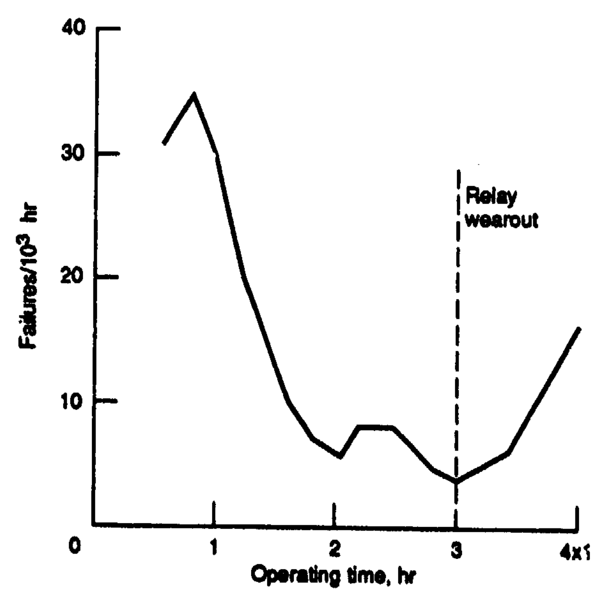


Figure 6-23.—Running average failure rate analysis of life test data identifying wearout region (600-hr running average in 200-hr increments).

Then find under the number of failures observed during the test the tolerance factor for the desired confidence level. The lower limit for the MTBF at the selected confidence level is then found from

$$\text{MTBF} = \frac{t}{\text{Tolerance factor}}$$

and the upper limit for failure rate from

$$\lambda = \frac{\text{Tolerance factor}}{t}$$

Example 13: A system was life tested for 3000 hours, during which six failures were observed. What is the demonstrated 80-percent-confidence MTBF?

Solution 13: Step 1—Solve for the total test hours.

$$t = \sum_{i=1}^{n} N_i t_i = 1 \times 3000 = 3000$$

Step 2—From table A-2 find the tolerance factor for six failures at 80-percent confidence to be 9.0.

Step 3—Solve for the demonstrated MTBF.

$$\text{MTBF} = \frac{t}{\text{Tolerance factor}} = \frac{3000}{9} = 333 \text{ hours}$$

in contrast to the observed MTBF of 3000/6 = 500 hours.

Example 14: Had four of the six failures in example 13 been observed in the first 1000 hours, what would be the demonstrated MTBF at 80-percent confidence in the region from 1000 to 3000 hours?

Solution 14: Step 1—The total test time is given as $t = 2000$ hours.

Step 2—From table A-2 find the tolerance factor for two failures at 80-percent confidence to be 4.3.

Step 3—Find the demonstrated MTBF at 80-percent confidence after 1000 to 3000 hours.

$$\text{MTBF} = \frac{2000}{4.3} = 465 \text{ hours}$$

Example 15: It is desired to demonstrate an 80-hour MTBF on a computer at 90-percent confidence. How much test time is required on one sample if no failures occur?

Solution 15: Step 1—From table A-2 find the tolerance factor for no failures at 90-percent confidence to be 2.3.

Step 2—Because the desired 90-percent-confidence MTBF is given as 80 hours and the tolerance factor is known, calculate the total test time required from

$$t = (\text{MTBF})(\text{Tolerance factor}) = (80)(2.3) = 184 \text{ hours}$$

to prove that 184 hours with no failures demonstrates an 80-hour MTBF at 90-percent confidence.

A good discussion of fixed time and sequential tests is given in MIL-STD-781D (ref. 6-3).

Life test summary.—In summary, the following concepts are reiterated:

(1) Life tests are performed to evaluate product failure rate characteristics.

(2) If "failures" include all causes of system failure, the failure rate of the system is the only true factor available for evaluating the system's performance.

(3) Life tests at the part level require large sample sizes if realistic failure rate characteristics are to be identified.

(4) Laboratory life tests must simulate the major factors that influence failure rates in a device during field operations.

(5) The use of running averages in the analysis of life data will identify burn-in and wearout regions if such exist.

(6) Failure rates are statistics and therefore are subject to confidence levels when used in making predictions.

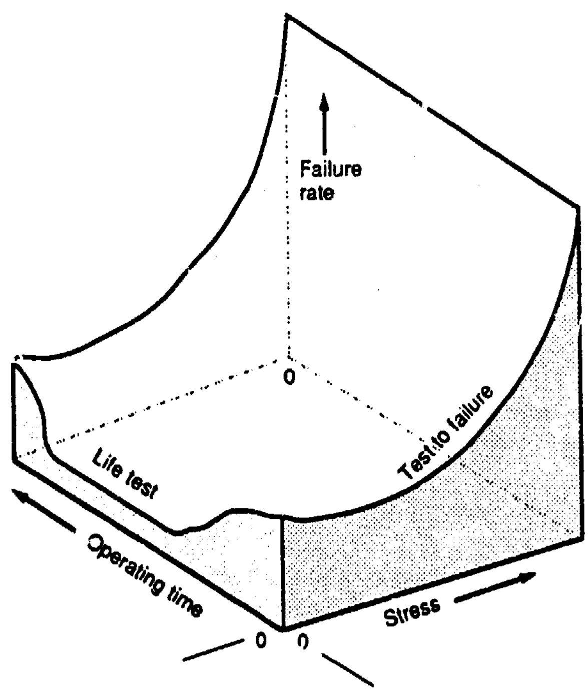


Figure 6-24.—Product failure surface.

Concluding Remarks

To summarize our discussion of test methods, figure 6-24 is presented to illustrate what might be called a failure surface for a typical product. This drawing shows system failure rate versus operating time and environmental stress. These three parameters therefore describe a surface in such a way that, given an environmental stress and an operating time, the failure rate is a point on the surface.

Test-to-failure methods generate lines on the surface parallel to the stress axis; life tests generate lines on the surface parallel to the time axis. Therefore, these tests provide a good description of the failure surface and, consequently, the reliability of a product.

Attribute tests result only in a point on the surface if failures occur and a point somewhere within the volume if failures do not occur. For this reason attribute testing is the least desirable method for ascertaining reliability, as indicated in table 6-1. Of course, in the case of missile flights or other events that produce go/no-go results, an attribute analysis is the only way to determine product reliability.

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C-2

Reliability Training[1]

1. Seven hydraulic power supplies were tested in a combined high-temperature and vibration test. Outputs of six of the seven units tested were within limits.

   a. What is the observed reliability $\hat{R}$ of the seven units tested?

      A. 0.825 B. 0.857 C. 0.913

   b. What is the predicted population reliability R at 89-percent confidence?

      A. 0.50 B. 0.75 C. 0.625

   c. How many tests (with one failure already experienced) are needed to demonstrate $R = 0.88$ at 80-percent confidence?

      A. 24 B. 15 C. 30

2. A vibration test was conducted on 20 autopilot sensing circuits with these results: Mean $\bar{x}_s = 7.8$ g's; standard deviation $\sigma_s = 1.2$ g's; reliability boundary $R_b = 6$ g's.

   a. What is the observed safety margin S_M?

      A. 2.0 B. 1.0 C. 1.5

   b. What is the observed reliability R?

      A. 0.900 B. 0.935 C. 0.962

   c. What is the predicted population safety margin S_M at 80-percent confidence?

      A. 1.19 B. 2.19 C. 3.19

   d. What is the predicted population reliability R at 80-percent confidence?

      A. 0.75 B. 0.95 C. 0.88

3. Twenty-five low-pressure hydraulic line samples were tested to destruction. These lines are rated to carry 30 psia (R_b); $\bar{x}_s = 31.5$ psia; $\sigma_s = 0.75$ psia.

   a. What is the observed S_M of these test items?

      A. 1.0 B. 2.0 C. 3.0

   b. What is the predicted population safety margin S_M at 90-percent confidence?

      A. 0.95 B. 1.25 C. 1.51

   c. The design requirement calls for an $S_M \geq 4.0$ at 90-percent confidence. After discussing the problem with the designer, it was learned that the 30-psia rating included a 2.5-psia "pad." Using the corrected R_b of 27.5 psia, now what are the S_M and S_D at 90-percent confidence?

      i. S_M (observed) = ?

         A. 4.22 B. 5.33 C. 6.44

      ii. S_D (predicted) = ?

         A. 4.275 B. 3.75 C. 4.80

[1]Answers are given at the end of this manual.

Chapter 7
Software Reliability

Software reliability management is highly dependent on how the relationship between quality and reliability is perceived. For the purposes of this manual, quality is closely related to the process, and reliability is closely related to the product. Thus, both span the life cycle.

Before we can stratify software reliability, the progress of hardware reliability should be briefly reviewed. Over the past 25 years the industry has observed (1) the initial assignment of "wizard status" to hardware reliability for theory, modeling, and analysis, (2) the growth of the field, and (3) the final establishment of hardware reliability as a science. One of the major problems was aligning reliability predictions and field performance. Once that was accomplished, the wizard status was removed from hardware reliability. The emphasis in hardware reliability from now to the year 2000, as discussed in chapter 1, will be on system failure modes and effects.

Software reliability has reached classification as a science for many reasons. The difficulty in assessing software reliability is analogous to the problem of assessing the reliability of a new hardware device with unknown reliability characteristics. The existence of 30 to 50 different software reliability models indicates the organization in this area. As discussed in chapter 1, hardware reliability started at a few companies and later was focused on by the AGREE reports. The field then logically progressed through different models in sequence over the years. Along the same lines numerous people and companies have simultaneously entered the software reliability field in their major areas; namely, cost, complexity, and reliability. The difference is that at least 100 times as many people are now studying software reliability as initially studied hardware reliability. The existence of so many models and their purports tends to mask the fact that several of these models have shown excellent correlations between software performance predictions and actual software field performance; for instance, the Musa model as applied to communications systems and the Xerox model as applied to office copiers. There are also reasons for not accepting software reliability as a science, and they are briefly discussed here.

One impediment to the establishment of software reliability as a science is the tendency toward programming development philosophies such as (1) "do it right the first time" (a reliability model is not needed), or (2) "quality is a programmer's development tool", or (3) "quality is the same as reliability and is measured by the number of defects in a program and not by its reliability." All of these philosophies tend to eliminate probabilistic measures because the managers consider a programmer as a software factory whose quality output is controllable, adjustable, or both. In actuality, hardware design can be controlled for reliability characteristics better than software design can. Design philosophy experiments that failed to enhance hardware reliability are again being formulated for software design. (Some of the material in this chapter is reprinted with permission from ref. 7-1.) Quality and reliability are not the same. Quality is characteristic and reliability is probabilistic. Our approach draws the line between quality and reliability because quality is concerned with the development process and reliability is concerned with the operating product. Many models have been developed and a number of the measurement models show great promise. Predictive models have been far less successful partly because a data base (such as MIL-HDBK-217E (ref. 7-2) for hardware) is not yet available for software. Software reliability often has to use other methods; it must be concerned with the process of software product development.

Models

The development of techniques for measuring software reliability has been motivated mainly by project managers, who need not only ways of estimating the personpower needed to develop a software system with a given level of performance, but also techniques for determining when this level of performance has been reached. Most software reliability models presented to date are still far from satisfying these two needs.

Most models assume that the software failure rate will be proportional to the number of implementation and design errors in the system, without taking into account that different kinds of errors may contribute differently to the total failure rate. Eliminating one significant design error may double the mean time to failure, whereas eliminating 10 minor implementation errors (bugs) may have no noticeable effect. Even assuming that the failure rate is proportional to the number

of bugs and design errors in the system, no model considers the fact that the failure rate will then be related to the system workload. For example, doubling the workload without changing the distribution of input data to the system may double the failure rate.

Software reliability models can be roughly grouped into four categories: time domain, data domain, axiomatic, and other.

Time Domain Models

Models formulated in the time domain attempt to relate software reliability (characterized, for instance, by a mean-time-to-failure (MTTF) figure under typical workload conditions) to the number of bugs present in the software at a given time during its development. Typical of this approach are the models presented by Shooman (ref. 7-3), Musa (ref. 7-4), and Jelinsky and Moranda (ref. 7-5). Removing implementation errors should increase MTTF, and correlating bug removal history with the time evolution of the MTTF value may allow the prediction of when a given MTTF will be reached. The main disadvantages of time domain models are that bug correction can generate more bugs and that software unreliability can be due not only to implementation errors but also to design (specification) errors, characterization, and simulation during testing of the typical workload.

The Shooman model (ref. 7-3) attempts to estimate the software reliability—that is, the probability that no software failure will occur during an operating time interval $(0,t)$—from an estimate of the number of errors per machine-language instruction present in a software system after T months of debugging. The model assumes that at system integration there are E_i errors present in the system and that the system is operated continuously by an exerciser that emulates its real use. The hazard function after T months of debugging is assumed to be proportional to the remaining errors in the system. The reliability of the software system is then assumed to be

$$R(t) = e^{-CE(r,T)}$$

where $E(r,T)$ is the remaining number of errors in the system after T months of debugging and C is a proportionality constant. The model provides equations for estimating C and $E(r,T)$ from the results of the exerciser and the number of errors corrected.

The Jelinsky-Moranda model (ref. 7-5) is a special case of the Shooman model. The additional assumption is made that each error discovered is immediately removed, decreasing the remaining number of errors by one. Assuming that the amount of debugging time between error occurrences has an exponential distribution, the density function of the time of discovery of the i^{th} error, measured from the time of discovery of the $(i-1)^{th}$ error is

$$p(t_i) = \lambda(i)e^{-\lambda(i)t_i}$$

where $\lambda(i) = f(N - i + 1)$ and N is the number of errors originally present. The model gives the maximum likelihood estimates for N and f.

The Jelinsky-Moranda model has been extended by Wolverton and Schick (ref. 7-6). They assume that the error rate is proportional not only to the number of errors but also to the time spent in debugging, so that the chance of discovery increases as time goes on. Thayer, Lipow, and Nelson (ref. 7-7) give another extension in which more than one error can be detected in a time interval, with no correction being made after the end of this interval. New maximum likelihood estimators of N and f are also given.

All the models presented so far attempt to predict the reliability of a software system after a period of testing and debugging. In a good example of an application of this type of model, Miyamoto (ref. 7-8) describes the development of an on-line, real-time system for which a requirement is that the mean time between software errors (MTBSE) has to be longer than 30 days. The system will operate on a day-by-day basis, 13 hours a day. (It will be loaded every morning and reset every evening.) The requirement is formulated so that the value of the reliability function $R(t)$ for $t = 13$ hours has to be greater than $e^{(-13/\mathrm{MTBSE})} = 0.9672$. Miyamoto also gives the variations in time of the MTBSE as a function of the debugging time. The MTBSE remained low for most of the debugging period, jumping to an acceptable level only at the end. The correlation coefficient between the remaining number of errors in the program and the failure rate was 0.77, but the scatter plot shown is disappointing and suggests that the correlation coefficient between the failure rate and any other system variable could have given the same value. In the same paper Miyamoto describes in detail how the system was tested.

None of the models above takes into account that in the process of fixing a bug, new errors may be introduced in the system. The final number given is usually the mean time between software errors, but only Miyamoto points out that this number is valid only for a specific set of workload conditions.

Other models for studying the improvement in reliability of a software item during its development phase exist, such as Littlewood (ref. 7-9), where the execution of a program is simulated with continuous-time Markov switching among smaller programs. This model also demonstrates that under certain conditions in the software system structure, the failure process will be asymptotically Poisson. Trivedi and Shooman (ref. 7-10) give another Markov model, where the most probable number of errors that will have been corrected at any time t is based on preliminary modeling of the error occurrence and repair rates. The model also predicts the system's availability and reliability at time t. Schneidewind (ref. 7-11) describes a model which assumes that the failure process is described by a nonhomogeneous Poisson process. The rate of error detection in a time interval is assumed to be proportional to the number of errors present during that

interval. This leads to a Poisson distribution with a decreasing hazard rate.

Data Domain Models

Another approach to software reliability modeling is studying the data domain. The first model of this kind is described by Nelson (ref. 7-12). In principle, if sets of all input data upon which a computer program can operate are identified, the reliability of the program can be estimated by running the program for a subset of input data. Thayer, Lipow, and Nelson (ref. 7-7) describe data domain techniques in more detail. Schick and Wolverton (ref. 7-13) compare the time domain and data domain models. However, different applications will tend to use different subsets of all possible input data, yielding different reliability values for the same software system. This fact is formally taken into account by Cheung (ref. 7-14), where software reliability is estimated from a Markov model whose transition probabilities depend on a user profile. Cheung and Ramamoorthy (ref. 7-15) give techniques for evaluating the transition probabilities for a given profile.

In the Nelson model (ref. 7-12) a computer program is defined as a computable function F defined on the set $E = (E_i, i = 1, \ldots, N)$, where E includes all possible combinations of input data. Each E_i is a sample of data needed to make a run of the program. Execution of a program produces, for a given value of E_i, the function value $F(E_i)$.

In the presence of bugs or design errors a program actually implements F'. Let E_e be the set of input data such that $F'(E_e)$ produces an execution failure (execution terminates prematurely, or fails to terminate, or the results produced are not acceptable). If N_e is the quantity of E_i leading to failure F_e,

$$p = \frac{N_e}{N}$$

is the probability that a run of the program will result in an execution failure. Nelson defines the reliability R as the probability of no failures or

$$R = 1 - p = 1 - \frac{N_e}{N}$$

In addition, this model is further refined to account for the fact that the inputs to a program are not selected from E with equal apriori probability but are selected according to some operational requirement. This requirement may be characterized by a probability distribution $(P_i, i = 1, \ldots, N)$, P_i being the probability that the selected input is E_i. If we define the auxiliary variables Y_i to be 0 if a run with E_i is successful, and 1 otherwise,

$$p = \sum_{i=1}^{N} P_i Y_i$$

where p is again the probability that a run of the program will result in an execution failure.

A mathematical definition of the reliability of a computer program is given as the probability of no execution failures after n runs.

$$R(n) = R^n = (1 - p)^n$$

The model elaborates on how to choose input data values at random for E according to the probability distribution P_i to obtain an unbiased estimator of $R(n)$. In addition, if the execution time for each E_i is also known, the reliability function can be expressed in terms of the more conventional probability of no failure in a time interval $(0, t)$.

Chapter 6 in Thayer, Lipow, and Nelson (ref. 7-7) extends the previous models to take into account how the testing of input data sets should be partitioned. Also discussed are the uncertainty in predicting reliability values, the effect of removing software errors, and the effect of program structure.

Axiomatic Models

The third category includes models in which software reliability (as well as software quality in general) is postulated to obey certain universal laws (Ferdinand and Sutherla, ref. 7-16; Fitzsimmons and Love, ref. 7-17). Although such models have generated great interest, their general validity has never been proven and, at most, they only give an estimate for the number of bugs present in a program.

The best-known axiomatic model is the so-called software science theory developed by Halstead (see ref. 7-17). Halstead used an approach similar to thermodynamics to provide quantitative measures of program level, language level, algorithm purity, program clarity, effect of modularization, programming effort, and programming time. In particular, the estimated number of bugs in a program is given by the expression

$$B = K\left(\frac{V}{E_0}\right)$$

where

K proportionality constant

E_0 mean number of mental discriminations between errors made by programmer

V volume of algorithm implementation, $N \log_2(n)$

where

N program length

n size of vocabulary defined by language used

More specifically,

$$N = N_1 + N_2$$
$$n = n_1 + n_2$$

TABLE 7-1.—CORRELATION OF EXPERIENCE TO SOFTWARE BUG PREDICTION BY AXIOMATIC MODELS

Reference	Correlation coefficient between predicted and real number of bugs
Funami and Halstead (ref. 7-33)	0.98, 0.83, 0.92
Cornell and Halstead (ref. 7-34)	0.99
Fitzsimmons and Love (ref. 7-17):	
System A	0.81
System B	.75
System C	.75
Overall	.76

where

N_1 total number of occurrences of operators in a program

N_2 total number of occurrences of operands in a program

n_1 number of distinct operators appearing in a program

n_2 number of distinct operands appearing in a program

and E_0 has been empirically estimated as approximately 3000.

Many publications have either supported or contradicted the results proposed by the software science theory, including a special issue of the IEEE Transactions on Software Engineering (ref. 7-18). Though unconventional, the measures proposed by the software science theory are easy to compute, and in any case it is an alternative for estimating the number of bugs in a software system. Table 7-1 shows a correlation coefficient between the real number of bugs found in a software project and the number predicted by the software science theory for several experiments. There are significant correlations with error occurrences in the programs, although the data reported by Fitzsimmons and Love (ref. 7-17) (obtained from three General Electric software development projects totaling 166 280 statements) show weaker correlation than the original values reported by Halstead.

Other Models

The model presented by Costis, Landrault, and Laprie (ref. 7-19) is based on the fact that for well-debugged programs a software error results from conditions on both the input data set and the logical paths encountered. We can then consider these events random and independent of the past behavior of the system (i.e., with constant failure rate). Also, because of their rarity, design errors or bugs may have the same effect as transient hardware faults.

The model is built on the following assumptions:

(1) The system initially possesses N design errors or bugs that can be totally corrected by N interventions of the maintenance team.

(2) The software failure rate is constant for a given number of system design errors.

(3) The system starts and continues operation until a fault is detected; it then passes to a repair state. If the fault is due to a hardware transient, the system is put into operation again after a period of time for which the probability density function is assumed to be known. If the fault is due to a software failure, maintenance takes place, during which the error may be removed, more errors may be introduced, or no modifications may be made to the software.

The model computes the availability of the system as a function of time by using semi-Markovian theory. That is, the system will make state transitions according to the transition probabilities matrix, and the time spent in each state is a random variable whose probability density function is either assumed to be known or is measurable. The main result presented by Costis, Landrault, and Laprie (ref. 7-19) is how the availability of the system improves (when all the design errors have been removed) as the design errors are being removed under some restrictive conditions. They show that the minimum availability depends only on the software failure rate at system integration, and not on the order of occurrence of the different types of design errors. The presence of different types of design errors only extends the time necessary to approach the asymptotic availability.

The mathematics of the model is complex, requiring numerical computation of inverse Laplace transforms for the transition probabilities matrix, and it is not clear that the parameters needed to simulate a real system accurately can be easily measured from a real system.

Finally, some attempts have been made to model fault-tolerant software through module duplication (Hecht, ref. 7-20) and warnings about how not to measure software reliability (Littlewood, ref. 7-21).

None of the preceding models characterizes system behavior accurately enough to give the user a guaranteed level of performance under general workload conditions. They estimate the number of bugs present in a program but do not provide any accurate method of characterizing and measuring operational system unreliability due to software. There is a large gap between the variables that can be easily measured in a running system and the number of bugs in its software. Instead, a cost-effective analysis should allow precise evaluation of software unreliability from variables easily measurable in an operational system, without knowing the details of how the software has been written.

Trends and Conclusions

With software reliability being questioned as a science, programming process control appears to be the popular answer to both software reliability and software quality. Measurements of the programming process are supposed to ensure the generation of an "error free" programming product, if such an achievement is possible. Further, quality and productivity measurements combined with select leading process indicators

are supposed to fulfill the control requirements for developing quality software. This so-called answer is similar to a philosophy that failed in attempts to develop hardware reliability control. Reliability *should* be used to predict field performance. Especially with real-time communications and information management systems, the field performance requirements vastly overshadow the field defect level requirements. How can we change the present popular trend (toward programming process control) to one that includes a probabilistic reliability approach? The answer is not a simple one; these models must be finely balanced so that a clear separation of reliability and quality can be achieved.

The trends for reliability tasks in the large-scale integrated circuit (LSI) and very large-scale integrated circuit (VLSI) hardware areas are in the failure modes and effects analysis and the control of failures. The same emphasis can be placed on software (programming bugs or software errors). Once this is done, reliability models can reflect system performance due to hardware and software "defects" because their frequency of occurrence and the effects of their presence in the operation will be known. This philosophy focuses on the complete elimination of critical defects and the specified tolerance level of minor defects. Normally, minor defects are easier to find and more numerous than the most critical defects and therefore dominate a defect-removal-oriented model.

We conclude that the proper method for developing quality programming products combines quality, reliability, and a selective measurements program. In addition, a redirection of the programming development process to be based in the future on the criticality of defects, their number, and their budgeting at the various programming life-cycle phases is the dominant requirement. A reliability growth model will monitor and control the progress of defect removal for the design phases and prove a direct correlator to actual system field performance. With such an approach a system can be placed in operation at a customer site at a preselected performance level as predicted by the growth model.

Software

We have discussed software models before describing software for several reasons. The reader should not be biased or led to a specific type of software. Few papers on software reliability make a distinction between product software, embedded software, applications software, and support software. In addition, the models do not distinguish between vendor-acquired software and in-house software and combinations of these.

Categories of Software

According to Electronic Design Magazine, the United States supports at least 50 000 software houses, each grossing approximately $500 000 per year. It is projected that software sales in the United States will surpass hardware sales and reach the $60 billion range. International competition will eventually yield error-free software.

In-house and vendor-acquired software can be put into four categories as follows:

(1) Product software
(2) Embedded software
(3) Applications software
(4) Support software

Product software.—This categorization is from the viewpoint of the software specialist. Communications digital switching systems software is included as "product software" along with the software for data packet switching systems, text systems, etc.

Embedded software.—This category of software comprises programming systems embedded in physical products to control their operational characteristics. Examples of products are radar controllers, boiler controls, avionics, and voice recognition systems.

Applications software.—This category of software is usually developed to service a company's internal operations. The accounting area of this category covers payroll systems, personnel systems, etc. The business area includes reservations systems (car, motel), delivery route control, manufacturing systems, and on-line agent systems.

Support software.—This category consists of the software tools needed to develop, test, and qualify other software products or to aid in engineering design and development. The category includes compilers, assemblers, test executives, error seeders, and development support systems.

Vendor-acquired software.—This software can be absorbed by the previous four categories and is only presented here for clarification. It includes FORTRAN compilers, COBOL compilers, assemblers, the UNIX operating system, the ORACLE data base system, and application packages.

Processing Environments

Software can usually be developed in three ways; namely, (1) interactive, (2) batch, and (3) remote job entry. In the operational environment the ways expand to include real time. Real-time development can be characteristic of both product software and embedded software. However, because product software and embedded software differ greatly in their requirements and their development productivity and quality methodologies, they should not be combined (e.g., avionics has size, weight, and reliability requirements resulting in dense software of a type that a communications switching system does not have).

Severity of Software Defects

We must categorize and weigh the effects of failures. The following four-level defect severity classification is presented in terms of typical software product areas:

(1) System unusable (generic: frequent system crashes)

(a) Management information system (MIS) software defects: inability to generate accounts payable; inability to access data base; improper billing

(b) Computer-aided design (CAD), manufacturing (CAM), and engineering (CAE) defects: inability to use systems; CAD produces incorrect designs

(c) Telephone switching defects: frequent service outages; loss of emergency communications service

(d) Data communications defects: loss of one or more signaling channels; unrecoverable errors in transmission; erratic service

(e) Military system defects: success of mission jeopardized; inability to exercise fire control systems; loss of electronic countermeasure capabilities

(f) Space system defects: success of space mission jeopardized; risk of ground support team or flight crew life; loss of critical telemetry information

(g) Process control defects: waste of labor hours, raw materials, or manufactured items; loss of control resulting in contamination or severe air and water pollution

(2) Major restrictions (generic: loss of some functions)

(a) MIS software defects: loss of some ticket reservation centers or loss of certain features such as credit card verification

(b) CAD/CAM/CAE defects: loss of some features in computer-aided design such as the update function; significant operational restrictions in CAM or CAE areas; faults produced for which there is no workaround

(c) Telephone switching defects: loss of full traffic capability; loss of billing

(d) Data communications defects: occasional loss of consumer data; inability to operate in degraded mode with loss of equipment

(e) Military system defects: significant operational restrictions; loss of intermediate fast frequency function in detection systems; loss of one or more antijamming features

(f) Space system defects: occasional loss of telemetry data and communications; significant operational or control restrictions

(g) Process control defects: process cannot consistently handle exceptions; inability to complete all process control functions

(3) Minor restrictions (generic: loss of features; inability to effectively modify program)

(a) MIS software defects: mishandling of records; system occasionally cannot handle exceptions

(b) CAD/CAM/CAE defects: occasional errors produced in design system; faults produced for which there are workarounds

(c) Telephone switching defects: loss of some support feature, such as call forwarding or conferencing

(d) Data communications defects: occasional inability to keep up with data rate or requests; occasional minor loss of data transmitted or received

(e) Military system defects: loss of some operational modes such as tracking history, monitor or slave model of operation, multiple option selection

(f) Space system defects: occasional loss of update information or frame; occasional loss of subframe synchronization or dropouts of some noncritical measurements

(g) Process control defects: problems that require a workaround to be implemented; minor reductions in rate or throughput; manual intervention at some points in the process

(4) No restrictions (generic: cosmetic; misleading documentation; inefficient machine/person interface)

Software Bugs Compared With Software Defects

Software bugs are not necessarily software defects: the term "defect" implies that removal or repair is necessary, and the term "bug" implies removal, some degree of correction, or a certain level of toleration. A recent example of bug toleration from the telecommunications industry is contained in reference 7-22; "It is not technically or economically feasible to detect and fix all software problems in a system as large as No. 4 Electronic Switching System (ESS). Consequently, a strong emphasis has been placed on making it sufficiently tolerant of software errors to provide successful operation and fault recovery in an environment containing software problems."

Various opinions exist in the industry about what constitutes a software failure. Definitions range from a software failure being classed as any software-caused processor restart or memory reload to a complete outage. One argument against assigning an MTBF to software-caused processor restarts or memory reloads is that, if the system recovers in the proper manner by itself, there has not been a software failure, only a software fault or the manifestation of a software bug. From a systems reliability viewpoint, if the system recovers within a reasonable time, the event is not to be classed as a software failure.

Hardware and Software Failures

Microprocessor-based products have more refined definitions. Four types of failure may be considered: (1) hardware catastrophic, (2) hardware transient, (3) software catastrophic, and (4) software transient. In general, the catastrophic failures require a physical or remote hardware replacement, a manual or remote unit restart, or a software program patch. The transient failure categories can result in either restarts or reloads for the microprocessor-based systems, subsystems, or individual units and may or may not require further correction. A recent reliability analysis of such a system assigned ratios

for these categories. Hardware transient faults were assumed to occur at 10 times the hardware catastrophic rate, and software transient faults were assumed to occur at 100 to 500 times the software catastrophic rate.

The time of day is of great concern in reliability modeling and analysis. Although hardware catastrophic failures occur at any time of the day, they often manifest themselves during busier system processing times. On the other hand, hardware transient failures generally occur during the busy hours as do software transient failures. The availability of restart times is also critical and in the example presented in reference 7-23, the system downtime is presented as a function of the MTBF of the software and the reboot time. When a system's predicted reliability is close to the specified reliability, such a sensitivity analysis must be performed.

Reference 7-24 presents a comprehensive summary of developed models and methods that encompass software life-cycle costs, productivity, reliability and error analysis, and complexity and the data parameters associated with these models and methods. The various models and methods are compared in reference 7-24 on a common basis, and the results are presented in matrix form.

Manifestations of Software Bugs

Many theories, models, and methods are available for quantifying software reliability. Nathan (ref. 7-25) has stated, "It is contrary to the definition of reliability to apply reliability analysis to a system that never really works. This means that the software which still has bugs in it really has never worked in the true sense of reliability in the hardware sense." This statement agrees with reference 7-22, which says that large, complex software programs used in the communications industry are usually operating with some software bugs. Thus, a reliability analysis of such software is different from a reliability analysis of established hardware. Software reliability is not alone in the need for establishing qualitative and quantitative models. Reference 7-26 discusses the "bathtub curve" and the effect of recent data on electronic equipment failure rate, and reference 7-27 discusses the effects of deferred maintenance and nonconstant software and hardware fault rates.

In the early 1980's work was done on a combined hardware/software reliability model. Reference 7-28 states, "The use of steady-state availability as a reliability/maintainability measure is shown to be misleading for systems exhibiting both hardware and software faults." The authors develop a theory for combining well-known hardware and software models in a Markov process and they consider the topic of software bugs and errors based on their experience in the telecommunications field. To synthesize the manifestations of software bugs, we must note some of the hardware trends for these systems:

(1) Hardware transient failures increase as integrated circuits become denser.

(2) Hardware transient failures tend to remain constant or increase slightly with time after the "infant mortality" phase.

(3) Hardware (integrated circuit) catastrophic failures decrease with time after the "infant mortality" phase.

These trends affect the operational software of communications systems. If the transient failures increase, the error analysis and system security software are called into action more often. This increases the risk of misprocessing a given transaction in the communications system. A decrease in the catastrophic failure rate of integrated circuits can be significant, as described in reference 7-13, which predicts an order-of-magnitude decrease in the failure rate of 4K memory devices between the first year and the twentieth year. We also tend to oversimplify the actual situations. Even with five vendors of these 4K devices, the manufacturing quality control person may have to set up different screens to eliminate the defective devices from different vendors. Thus, the system software will see many different transient memory problems and combinations of them in operation.

Central control technology has prevailed in communications systems for 25 years. The industry has used many of its old modeling tools and applied them directly to distributed control structures. Most modeling research was performed on large duplex processors. With an evolution through forms of multiple duplex processors and load-sharing processors and on to the present forms of distributed processing architectures, the modeling tools need to be verified. With fully distributed control systems the software reliability model must be conceptually matched to the software design in order to achieve valid predictions of reliability.

The following trends can be formulated for software transient failures:

(1) Software transient failures decrease as the system architecture approaches a fully distributed control structure.

(2) Software transient failures increase as the processing window decreases (i.e., less time allowed per function, fast timing mode entry, removal of error checking, removal of system ready checks, etc.)

A fully distributed control structure can be configured to operate as its own error filter. In a hierarchy of processing levels each level acts as a barrier to the level below and prevents errors or transient faults from propagating through the system. Central control structures cannot usually prevent this type of error propagation.

If the interleaving of transaction processes in a software program is reduced, such as with a fully distributed control architecture, the transaction processes are less likely to fail. This is especially true with nonconsistent user interaction as experienced in communications systems. Another opinion on software transient failures is that the faster a software program runs, the more likely it is to cause errors (such as encountered in central control architectures). Some general statements can be formulated:

(1) In large communications systems software transient failures tend to remain constant, and software catastrophic failures tend to decrease with time.

TABLE 7-2.—CRITICALITY INDEX

Bug manifestation rate	Defect removal rate	Level of criti-cality	Failure type	Failure characteristic
4 per day	1 per month	5	Transient	Errors come and go
3 per day	1 per week	4	Transient	Errors are repeated
2 per week	1 per month	3	Transient or catastrophic	Service is affected
1 per month	2 per year	2	Transient or catastrophic	System is partially down
1 per two years	1 per year	1	Catastrophic	System stops

(2) In small communications systems software transient failures decrease with time.

(3) As the size of the software program increases, software transient failures decrease and hardware failures increase.

A "missing link" needs further discussion. Several methods can be used to quantify the occurrence of software bugs. However, manifestations in the system's operations are detrimental to the reliability analysis because each manifestation could cause a failure event. The key is to categorize levels of criticality for bug manifestations and estimate their probability of occurrence and their respective distributions. The importance of this increases with the distribution of the hardware and software. Software reliability is often controlled by establishing a software reliability design process. Reference 7-22 presents techniques for such a design process control. The final measure is the system test, which includes the evaluation of priority problems and the performance of the system while under stress as defined by audits, interrupts, reinitialization, and other measurable parameters. The missing link in quantifying software bug manifestations needs to be found before we can obtain an accurate software reliability model for measuring tradeoffs in the design process on a predicted performance basis. If a software reliability modeling tool could additionally combine the effects of hardware, software, and operator faults, it would be a powerful tool for making design tradeoff decisions. Table 7-2 is an example of the missing link and presents a five-level criticality index for defects. Previously, we discussed a four-level defect severity classification with level four not causing errors. These examples indicate the flexibility of such an approach to criticality classification.

Software reliability measurement and its applications are discussed in reference 7-29 for two of the leading software reliability models, Musa's execution time model and Littlewood's Bayesian model. Software reliability measurement has made substantial progress and continues to progress as additional projects collect data. The major hurdle of establishing a software reliability measurement tool for use during the requirement stage is under way.

Comparing references 7-30 and 7-29 yields an insight into the different methods of achieving software reliability. The method described in reference 7-30 concentrates on the design process meeting a present level of reliability or performance at the various project design stages. When the system meets its final software reliability acceptance criteria, the process is complete. Reference 7-29 describes a model that provides the design process with a continuous software reliability growth prediction. The Musa model can compare simultaneous software developments and can be used extensively in making design process decisions. An excellent text on software reliability based on extensive data gathering was published in 1987 (ref. 7-31).

We can choose a decreasing, constant, or increasing software bug removal rate for systems software. Although each has its application to special situations and systems, a decreasing software bug removal rate will generally be encountered. Systems software also has advantages in that certain software defects can be temporarily patched and the permanent patch postponed to a more appropriate date. Thus, this type of defect manifestation is treated in general as one that does not affect service, but it should be included in the overall software quality assessment. The missing link concerns software bug manifestations. As described in reference 7-32, until the traditional separation of hardware and software systems is overcome in the design of large systems, it will be impossible to achieve a satisfactory performance benchmark. This indicates that software performance modeling has not yet focused on the specific causes of software unreliability.

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Reliability Training[1]

1. In-house and vendor-acquired software can be classified into what four categories?

   A. Product, embedded, applications, and error-free software
   B. Useful, embedded, applications, and harmful software
   C. Product, embedded, applications, and support software

2. Name the four categories of software reliability models.

   A. Time domain, data axiom, corollary, and many
   B. Time domain, data domain, axiomatic, and other
   C. Time axiom, data domain, frequency domain, and corollary

3. Can the bug manifestation rate be

   A. Equal to the defect removal rate?
   B. Greater than the defect removal rate?
   C. Less than the defect removal rate?
   D. All of the above?

4. What are the various software processing environments?

   A. Interactive, batch, remote job entry, and real time
   B. Hyperactive, batch, close job entry, and compressed time
   C. Interactive, batch, real job entry, and remote time

5. Name the four levels of severity for software defect categorizations.

   A. Generic system, functional, category restrictions, and working
   B. System unusable, major restrictions, minor restrictions, and no restrictions
   C. System unusable, system crashes, loss of features, and minor bugs

6. An on-line, real-time system has a mean time between software errors of 15 days. The system operates 8 hours per day. What is the value of the reliability function? Use the Miyamoto model.

   A. 0.962 B. 0.999 C. 0.978

7. Is it always necessary to remove every bug from certain software products?

   A. Yes B. No C. Don't know

8. Name the four types of hardware and software failure.

   A. Hardware part, hardware board, software module, software plan
   B. Hardware plan, hardware build, software cycle, software type cycle,
   C. Hardware catastrophic, hardware transient, software catastrophic, software transient

[1]Answers are given at the end of this manual.

Chapter 8
Software Quality Assurance

Concept of Quality

Let us first look at the concept of quality before going on to software quality. The need for quality is universal. The concepts of "zero defects" and "doing it right the first time" have changed our perspective on quality management. We changed from measuring defects per unit and acceptable quality levels to monitoring the design and cost reduction processes. The present concepts indicate that quality is not free. One viewpoint is that a major improvement in quality can be achieved by perfecting the process of developing a product. Thus, we would characterize the process, implement factors to achieve customer satisfaction, correct defects as soon as possible, and then strive for total quality management. The key to achieving quality appears to have a third major factor in addition to product and process. This third factor is the environment. People are important. They make the process or the product successful. Figure 8-1 represents the union of these three factors.

The term "software quality" is defined and interpreted differently by the many companies involved in producing programming products. To place the subject in perspective, we present principles and definitions for software quality from several source materials:

(1) The purpose of software quality assurance is to assure the acquisition of high-quality software products on schedule, within cost, and in compliance with the performance requirements (ref. 8-1).

(2) The developer of a methodology for assessing the quality of a software product must respond to various needs. There can be no single quality metric (ref. 8-2).

(3) The process of assessing the quality of a software product begins when specific characteristics and certain of the metrics are selected (ref. 8-3).

(4) Software quality can be defined as (a) the totality of features and characteristics of a software product that bear on its ability to satisfy needs (e.g., conform to specifications), (b) the degree to which software possesses a desired combination of attributes, (c) the degree to which a customer or user perceives that software meets his or her expectations, and (d) the composite characteristics of software that determine the degree to which the software in use will meet the expectations of the user.

We can infer from these statements and other source materials that software quality metrics (e.g., defects per 1000 lines of code per programmer year, 70 percent successful test cases for the first 4 weeks, and zero major problems at the preliminary design review) may vary more than hardware quality metrics (e.g., MTBF or errors per 1000 transactions). In addition, software quality management has generally focused on the process, and software reliability management has focused on the product. Since processes differ for different software products, few comparative benchmarks are available. For hardware, in general, benchmarks have been available for a long time (i.e., MIL-HDBK-217E series (ref. 8-4) for reliability). Recently, Rome Air Development Center (RADC), the sponsor of MIL-HDBK-217E, has sponsored a survey of software reliability. It was intended to give software quality the same status as hardware quality.

The next step is to discuss what the process of achieving quality in software consists of and how quality management is involved. The purpose of quality management for programming products is to ensure that a preselected software quality level has been achieved, on schedule, in a cost-effective manner. In developing a quality management system the programming product's critical life-cycle phase reviews provide the reference base for tracking the achievement of quality objectives. The International Electrotechnical Commission (IEC) system life-cycle phases presented in their guidelines for reliability and maintainability management are as follows.

(1) Concept and definition phase, in which the need for the product is decided and its basic requirements defined, usually in the form of a product specification, which is agreed upon between manufacturer and user.

(2) Design and development phase, in which the product hardware and software are created to perform the functions described in the product specification. This phase will normally include the assembly and testing of a prototype product under laboratory simulated conditions or in actual field trial conditions and the formulation of detailed manufacturing specifications and instructions for operation and maintenance.

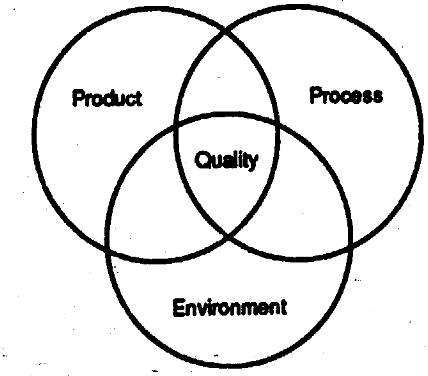

Figure 8-1.—Quality diagram.

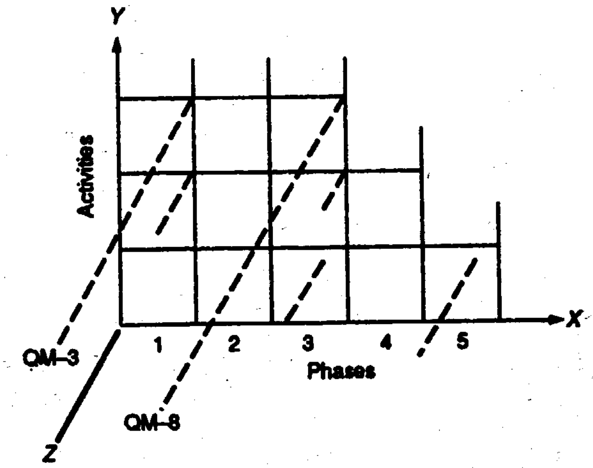

Figure 8-2.—Programming quality measurement map.

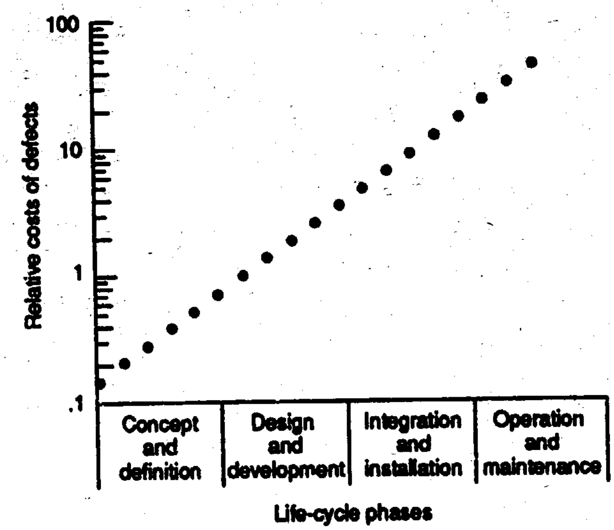

Figure 8-3.—Increasing costs of programming defects.

(3) Manufacturing, installation, and acceptance phase, in which the design is put into production. In the case of large, complex products the installation of the product on a particular site may be regarded as an extension of the manufacturing process. This phase will normally conclude with acceptance testing of the product before it is released to the user.

(4) Operation and maintenance phase, in which the product is operated for the period of its useful life. During this phase, essential preventive and corrective maintenance actions are taken along with product enhancements, and product performance is monitored. The useful life of a product ends when its operation becomes uneconomic because of increasing repair costs or other factors or the product becomes technically obsolete.

(5) Disposal phase, in which the product reaches the end of its planned useful life or the requirement no longer exists for the product, and it is disposed of, destroyed, or, if economically feasible, modernized.

The quality of the programming product can be controlled in the first three life-cycle phases in order to achieve the expected level of performance of the final product. Once the fourth phase has been entered, the operation and maintenance phase, the quality of the software is generally fixed. With these five life-cycle phase boundaries in place, we can conceptualize what can be implemented as "programming quality measurement." If the phases and activities are the X and Y coordinates, the individual quality metrics can be placed on the Z axis as shown in figure 8-2.

Without stating the specific activities for each phase, we can discuss the generalities of software quality and its cost. The cost of implementing quality increases with distance along the X axis. Activities can be arranged along the Y axis so that the cost of quality increases with distance along the Y axis. With this arrangement we can establish rigorous quality standards for the individual quality metrics as a function of cost effectiveness (e.g., error seeding—the statistical implanting and removal of software defects—may be expensive). Other quality metrics (e.g., test case effectiveness) may cost significantly less and could be selected.

In general, for a programming product the higher the level of quality, the lower the costs of the product's operation and maintenance phase. This fact produces an incentive for implementing quality metrics in the early design phases. The programming industry has traditionally required large maintenance organizations to correct programming product defects. A typical phase-cost curve presented in figure 8-3 shows the increased costs of correcting programming defects in the later phases of the programming product's life cycle. Note that the vertical axis is nonlinear.

Software Quality

The next step is to look at specific software quality items. Software quality is defined in reference 8-4 as "the achievement of a preselected software quality level within the costs, schedule, and productivity boundaries established by management." However, agreement on such a definition is often difficult to achieve. In practice, the quality emphasis can change with respect to the specific product application environ-

ment. Different perspectives of software product quality have been presented over the years. However, in todays' literature there is general agreement that the proper quality level for a particular software product should be determined in the concept and definition phase and that quality managers should monitor the project during the remaining life-cycle phases in order to ensure the proper quality level.

The developer of a methodology for assessing the quality of a software product must respond to the specific characteristics of the product. There can be no single quality metric. The process of assessing the quality of a software product begins with the selection of specific characteristics, quality metrics, and performance criteria.

The specifics of software quality can now be addressed. Several areas of interest are

(1) Software quality characteristics
(2) Software quality metrics
(3) Overall software quality metrics
(4) Software quality standards

Areas (1) and (2) are applicable during both the design and development phase and the operation and maintenance phase. In general, area (2) is used during the design and development phase before the acceptance phase for a given software product. Each of these four areas is now addressed in detail.

Software Quality Characteristics

A software quality characteristic tree is presented in reference 8–5. The authors assume that different software products require different sets of quality characteristics. A product that has a rigorous constraint on size may sacrifice the maintainability characteristic of the software in order to meet its operational program size goals. However, this same product may need to be highly portable for use on several different processors. In general, the primary software quality characteristics are

(1) Maintainability
(2) Portability
(3) Reliability
(4) Testability
(5) Understandability
(6) Usability
(7) Freedom from error

Management's view of software quality is the quality characteristics. Established criteria for these characteristics will provide the level of quality desired. The quantitative measures (metrics) place the quality at the achieved level. This concept is shown in figure 8–4.

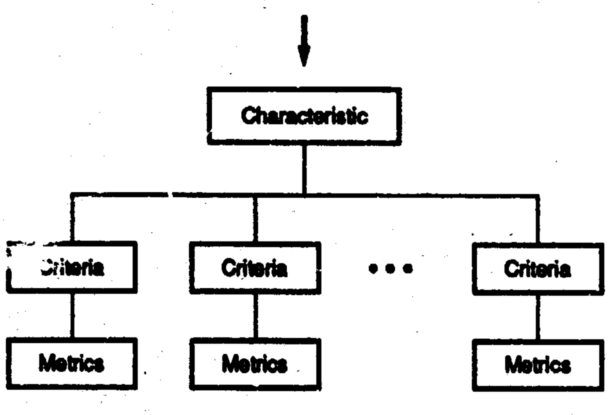

Figure 8-4.—Management's view of quality.

Software quality criteria and metrics are directly related to the specific product. Too often, establishing the characteristic and the metric in the early life-cycle phases without the proper criteria leads to defective software. An example of the characteristics and their importance for various applications is presented in table 8–1.

TABLE 8-1.—APPLICATION-DEPENDENT SOFTWARE QUALITY CHARACTERISTICS

Characteristic	Application	Importance
Maintainability	Aircraft	High
	Management information systems	Medium
	Test beds	Low
Portability	Spacecraft	Low
	Test beds	High

Software Quality Metrics

The entire area of software measurements and metrics has been widely published and discussed. Two textbooks (refs. 8–6 and 8–7) and the establishment of the Institute for Electrical and Electronics Engineers (IEEE) Computer Society's working group on metrics, which has developed a guide for software reliability measurement, are three examples of such activity. Software metrics cannot be developed before the cause and effect of a software defect have been established for a given product with relation to its product life cycle.

Table 8–2 is a typical cause-and-effect chart for a software product. It includes the process indicator concept. At the testing stage of product development the evolution of software quality levels can be assessed by characteristics such as freedom from error, successful test case completion, and estimate of the software bugs remaining. These process indicators can be used to predict slippage of the product delivery date, the inability to meet original design goals, etc.

When the programming product enters the qualification, installation, and acceptance phase and continues into the maintenance and enhancements phase, the concept of performance is important in the quality characteristic activity. This concept is shown in table 8–3, where the 5 IEC system life-cycle phases have been expanded into 10 software life-cycle phases:

(1) Conceptual planning phase, in which the functional, operational, and economic context of the proposed software is understood and documented in a product proposal
(2) Requirements definition phase, in which a product proposal is expanded into specific product requirements

TABLE 8-2.—MEASUREMENT OF SOFTWARE QUALITY CHARACTERISTICS

Characteristic	Software life-cycle phase				
	3	4	5	7	9
	Product definition	Top-level design	Detailed design	Testing and integration	Maintenance and enhancements
Maintainability	---	(a)	(a)	--------	(b)
Portability				---	---
Reliability	(a)			(b)	(b)
Testability:	---				
Test case completion	---				
Estimate of bugs remaining	---				
Understandability	(a)		↓		
Usability	(a)	↓	-------	↓	
Freedom from error	---	---	(a), (c)	(a), (c)	↓

[a]Where quality characteristic should be measured.
[b]Where impact of poor quality is realized.
[c]Metric can take form of process indicator.

TABLE 8-3.—MEASUREMENTS AND PROGRAMMING PRODUCT LIFE CYCLE

System life-cycle phase	Software life-cycle phase	Order of precedence	
		Primary	Secondary
Concept and definition	Conceptual planning (1) Requirements definition (2) Product definition (3)	---------------- ---------------- Quality metrics[a]	---------------- ---------------- ----------------
Design and development	Top-level design (4) Detailed design (5) Implementation (6)	Quality metrics Quality metrics Process indicators[b]	Process indicators Process indicators Quality metrics
Manufacturing and installation	Testing and integration (7) Qualification, installation, and acceptance (8)	Process indicators Performance measures[c]	Performance measures Quality metrics
Operation and maintenance	Maintenance and enhancements (9)	Performance measures	----------------
Disposal	Disposal (10)	----------------	----------------

[a]Metrics—qualitative assessment, quantitative prediction, or both.
[b]Indicators—month-by-month tracking of key project parameters.
[c]Measures—quantitative performance assessment.

and the requirements, such as performance and functional capabilities, are analyzed and translated into unambiguous developer-oriented terms

(3) Product definition phase, in which software engineering principles, technical information, and creativity are used to describe the architecture, interfaces, algorithms, and data that will satisfy the specified requirements

(4) Top-level design phase, in which the functional, operational, and performance requirements are analyzed and designs for system architecture, software architecture, interfaces, and data are created and documented to satisfy requirements

(5) Detailed design phase, in which the functional, operational, and performance requirements are analyzed and designs for system architecture, software architecture, components, interfaces, and data are further created, documented, and verified to satisfy requirements

(6) Implementation phase, in which the software product is created or implemented from the software design and the faults are detected and removed

(7) Testing and integration phase, in which software elements, hardware elements, or both are combined into an overall system or an element of a system and the elements are tested in an orderly process until the entire system has been evaluated, integrated, and tested

(8) Qualification, installation, and acceptance phase, in which a software product is formally tested to ensure the customer or customer's representative that the product

meets its specified requirements. This phase includes all steps necessary to deliver, install, and test a specific release of the system software and its deliverable documentation.

(9) Maintenance and enhancements phase, in which the product is ready for or serving its designated function, is monitored for satisfactory performance, and is modified as necessary to correct problems or to respond to changing requirements

(10) Disposal phase, in which the product reaches the end of its planned useful life or the requirement no longer exists for the product and it is disposed of, destroyed or, if economically feasible, modernized

Overall Software Quality Metrics

Several overall software quality metrics have been put into practice and have effectively indicated software quality. Jones (ref. 8–8) presents an overall quality metric called defect removal efficiency. The data collected for the overall quality metric are simplified to the more practical expression of "defects per 1000 lines of source code."

A second overall quality metric is based on the concept of quality prisms (refs. 8–9 and 8–10), which considers the extent of effort with which a given quality characteristic has been implanted into a product and the degree of effort for quality that has occurred in each life-cycle phase. An example of the extent and degree of effort is presented in table 8-4 for any given quality characteristic.

As table 8–4 shows,

(1) Each quality characteristic can have a matrix similar to this with a specific quality program tailored to a company's products.

(2) The quality effort is extended to each of the product's life-cycle phases to the degree desired by the company.

(3) For each level, as the complexity and difficulty of a characteristic requirement increase, the intensity of the test and verification program effort increases.

(4) This matrix will change for each characteristic in accordance with company emphasis.

(5) Traditionally, the quality levels of a product correspond to degrees of effort. However, this matrix extends the effort to all phases of the product's life cycle.

As an example of using the matrix shown in table 8–4, a characteristic such as reliability may be targeted to reach service level 2. Then throughout planning, design, testing,

TABLE 8-4.—QUALITY CHARACTERISTIC DEGREE/EXTENT MATRIX

Product phase	Service level					
	0	1	2	3	4	
Planning	No activity	General high level required	Specific detailed requirements definition	Highly complex required definition and support model	Difficult or complex required definition and prototype	Extent of effort
Design and test	No activity	General architecture consideration; general test and measurement program	Detailed architecture structure impact; language impact; test program extended	Extensive architecture and structure consideration; tailored language, operating system, man-machine interface impact, etc.; code walkthroughs; detailed documentation	Separate quality teams to verify design; detailed test facility; extensive qualification test plans and procedure	
Integration and installation	No activity	General quality management program; acceptance test; nominal change control quality program	Extensive qualification test plans and procedure to verify characteristics; above-nominal-quality-requirement verification testing	Quality teams formed; detailed quality configuration control release program; extensive data collection, verification, and analysis	Specialized quality integration, manufacturing, and installation programs to ensure achievement of quality characteristics by separate quality organization	
Service	No activity	General quality tracking and redesign program to achieve quality objectives and requirements	Formal data collection and analysis program to verify quality objectives; quality redesign effort	Detailed measurements, data analysis, and modeling program to verify high-level quality objectives; extensive redesign to obtain quality	Extensive measures and modeling, vigorous data analysis, and specialized tests to ensure high-level achievement of detailed quality requirements; extensive change program	
	No quality	First level of quality	Second level of quality	Third level of quality	Fourth level of quality	
Degree of effort						

integration, and installation, the reliability should achieve at least level 2. These indicators are tied to the proper major phase review points of a product's life cycle. For most characteristics the planning level should be achieved after the preliminary design review (PDR); the design level, after the development phase or at the critical design review (CDR); the integration level, after integration at the qualification testing; and the service level, during the operational service reviews.

Now quality management can apply this matrix to each characteristic in a manner depending on how critical it is to ensure achievement of the characteristic. For example, the reliability goal for a key system may be 10 or fewer mishandled calls per week, but the reliability goal for a private branch exchange (PBX) may be only 5 mishandled calls per month. These objectives may cause quality management to define a planning 2, design 2, integration 2, and service 2 program for the key system and a more demanding planning 4, design 3, integration 3, and service 3 program for the PBX.

In this manner the quality characteristics are clearly identified by detailed criteria that set the scope of and limit the required objectives. Once these objectives are identified, a quality program can be determined that defines the specific required definition, design, test, and measurement efforts. No longer are nebulous measurements made against vague objectives in the service phase of a product's life cycle in a last-minute attempt to improve quality.

The program for pursuing quality characteristics must be established early. If a particular quality characteristic is not pursued to a reasonable extent in the planning and design phases, a maximum degree of effort (4) may not realistically be achieved in the service phase. Conversely, the more uniformly and consistently a quality characteristic is pursued, the more achievable and figuratively stable is the characteristic. This is graphically reflected for a single characteristic in figures 8-5 to 8-7, where the quality item is shown as either stable, unstable, or extremely costly to stabilize.

In figure 8-5 an optimum tradeoff of stability and productivity is portrayed. The base of the prism is secure, supporting the platform by properly balancing quality versus cost. In figure 8-6 schedule pressures have established an unstable prism to support the platform. In this example the decision was made to send the product into the field at service level 1 even though it initially had reached a more extensive degree of quality (3) in the planning phase (considerable effort to define quality objectives in the planning phase but no followup). Figure 8-7 presents the extremely costly view of upgrading a programming product in the field to service level 4 (after passing the first three phases only to the first degree). Note the increasing amount of time and effort to achieve service levels 1, 2, or 3. Service level 4 in this example is usually extremely difficult and expensive, if not impossible, to achieve. The measured productivity of such a product will most likely be low.

An excellent example of the need for this type of quality management process occurred many years ago. The lessons

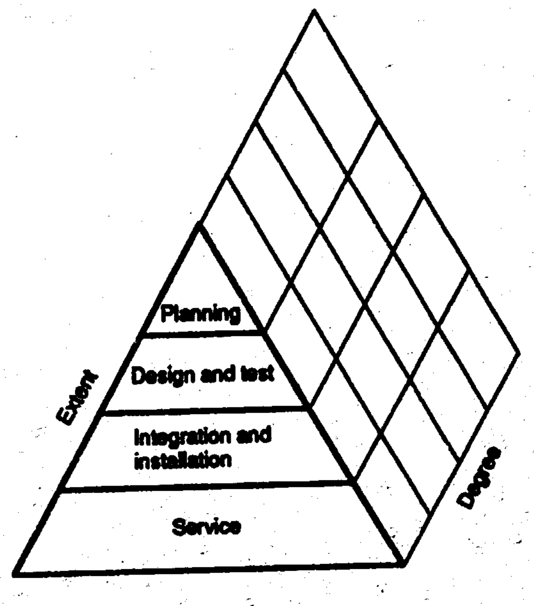

Figure 8-5.—Stability in quality and cost.

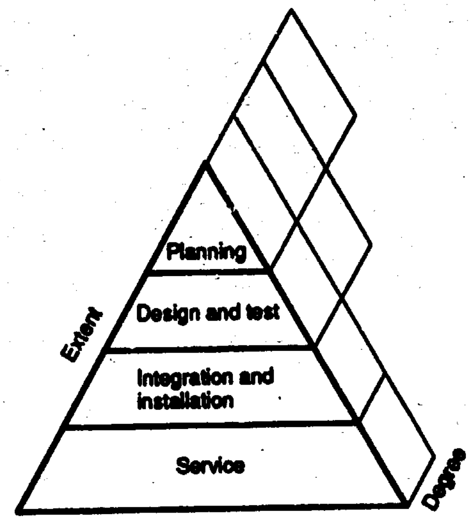

Figure 8-6.—Instability due to scheduling decisions.

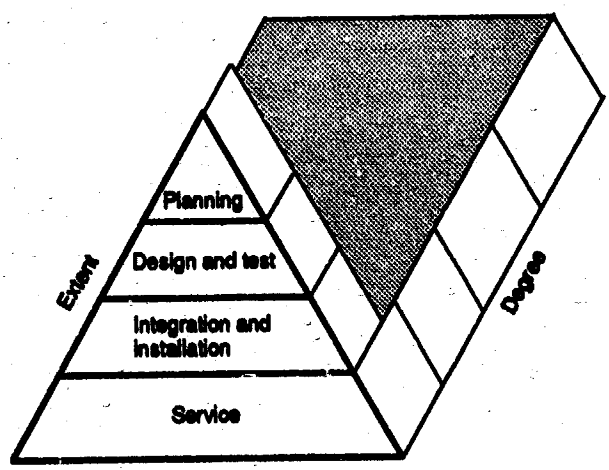

Figure 8-7.—Extremely costly programming products.

still apply today. An automated program was proposed to generate, from 160 fields of input data per customer, a centralized data base that would control a table-driven wired-logic system. It was estimated that 13 weeks of design time would be required to construct this table generator using a nominal amount of computer support time. A representative of the design group was assigned to define the input and output requirements for the support program and verify its operation. The program was initially written in assembly language. It was later redesigned and split into three separate programs written in a high-level language. These programs could then be separately designed, verified, and maintained. The main consideration became the verification process. An input and output test was written to check the extensive program paths. The project dragged along for a year as verification testing attempted to meet a zero-defect objective (imposed after the initial design had been completed). Costs increased and the schedule became critical as the customer became impatient (fig. 8–7). As the program began to function more successfully, deciding the degree of testing required for verification became a serious problem. Confrontation developed between the design and marketing departments over the commercial release of the program. The testing continued without agreement on the required degree of effort. Eventually, the customer became disillusioned and turned to another firm to provide the table generator.

Had a clear quality management decision been made in the planning phase and tracked throughout the development on the degree of error-free "verified" operation, the quality characteristic objectives for its design architecture and structure, the language required for changes, etc., a more realistic projection (and control) of schedule and people could have been achieved. Several releases to the customer may have been required as the program designs and operation were verified to a predetermined extent within the various life-cycle phases. Had this procedure been followed, both the customer and the supplier would have been more satisfied.

This example offered an excellent opportunity to first determine the type and degree of quality desired. Then management could have constructed a quality process, in terms of the extent and degree of each desired characteristic, with a elastic compromise between the schedule, resources, and design activity needed to achieve it. In this case many of the "ilities," such as changeability, usability, maintainability, and reliability, were subsequently more critically identified. These considerations could have been translated into the initial requirements for structural design, program segmentation, extensive documentation, and type of language as well as the amount of code walkthrough, the number of subfunctional tests, the amount of error acceptable at first release, the depth of verification reviews, etc. From this form of planning, the "quality prisms" could have been established to define the extent and degree (such as service level 2, 3, or 4) to which each of these characteristics should have been pursued in terms of project cost restraints, depending on user willingness to pay and wait for a quality product.

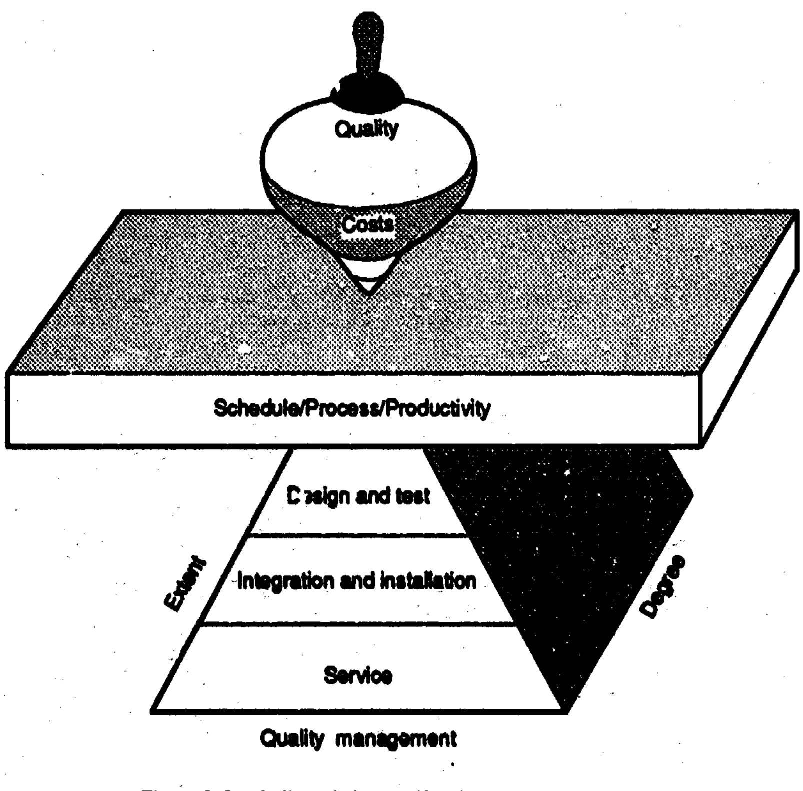

Figure 8-8.—Delicate balance—planning complete.

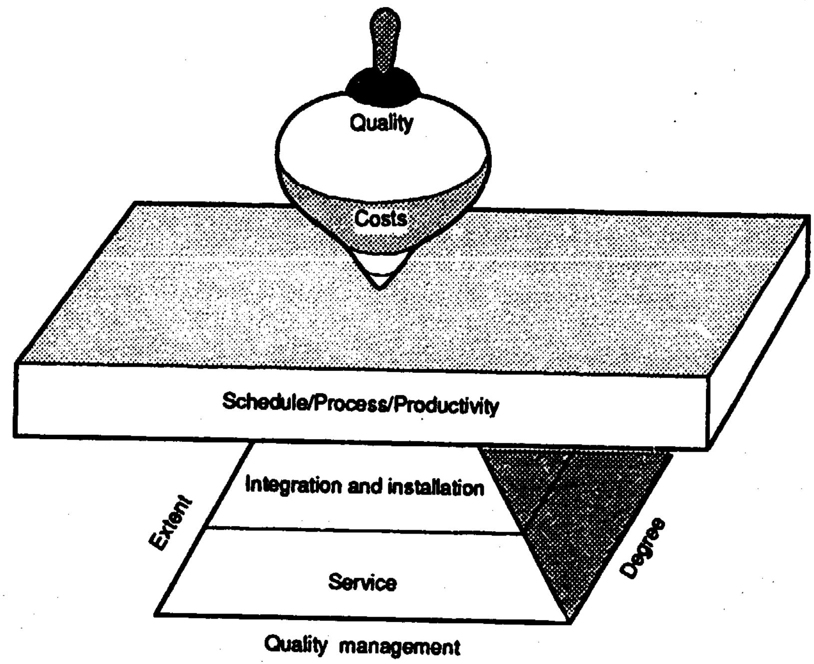

Figure 8-9.—Delicate balance—design and testing complete.

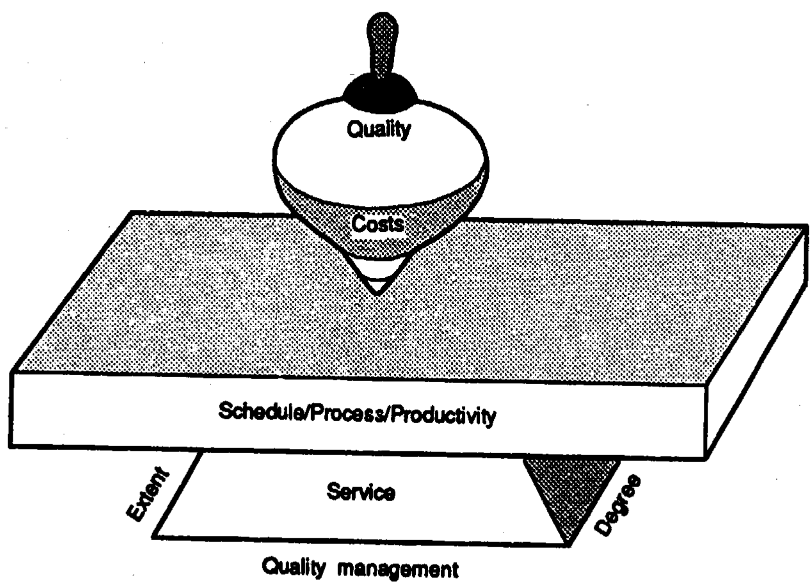

Figure 8-10.—Delicate balance—integration and installation complete.

A figuratively secure prismatic base for the programming product is presented in figure 8-5. This security is developed through execution of an extensive quality program, as progressively shown in figures 8-8 to 8-10. A product's quality objective is usually composed of more than one characteristic. Previously, those have tentatively been noted as maintainability, portability, reliability, testability, understandability, usability, and freedom from error. Thus, quality management can extend the support prismatic structure to a greater depth than to just one quality characteristic. In practice, several quality prisms will be placed together to achieve a firm quality base.

It may be desirable to have a product developed that has reached service level 4 for all of the forementioned quality characteristics. However, realistic schedules and productivity goals must be considered in terms of cost. These considerations establish the need for vigorous quality management over all life-cycle phases to selectively balance the various possibilities. It would be nonsupportive, expensive, and time consuming if quality management established the structural combination of individual characteristic quality prisms graphically presented in figure 8-11. Unfortunately, this is the case for too many products. Quality management would do better to establish a more consistent support structure, like that represented in

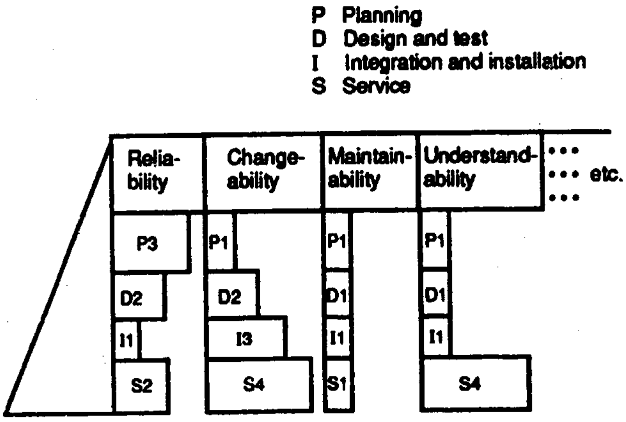

Figure 8-11.—Example of poor quality management.

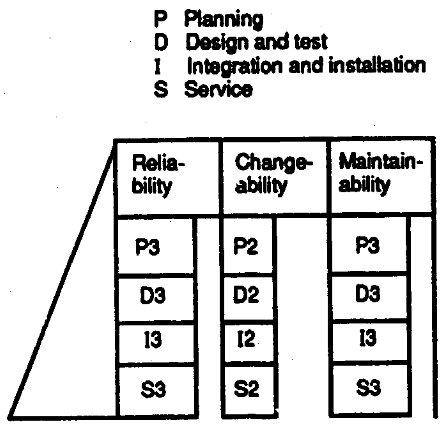

Figure 8-12.—Example of good quality management.

figure 8-12. The figurative result of this consistent effort is shown in the solid cost-effective base of figure 8-13.

If quality characteristics are established, monitored, measured, and verified throughout the life cycle, a realistic balance can successfully be achieved between quality costs, schedule, and productivity. However, it will require an active quality management process to establish and track these indicators. An example of such a quality management process matrix is presented in table 8-5 to quantify the extent and degree of effort needed to achieve a desired level of quality. This table can be used as a programming product quality worksheet, as well as both the characteristic survey data collection instrument and part of the final quality prisms planning document.

As discussed, a quality management team must establish the degree of quality that a particular quality characteristic must reach throughout its life cycle. It may use specialized support tools, measurement systems, and specific product quality standards in pursuing its quality objectives. A point system can give a quantitative reference for the pursuit of quality. The point system can become the basis for trading time versus cost to reach specific quality goals. Of course, a firm's quality management will define their own point system. However, the

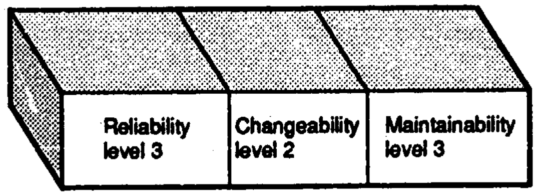

Figure 8-13.—Example of solid quality base.

following example point system will serve as an illustration for discussion purposes.

If a single characteristic's quality effort has progressed through all four levels, as well as through each level's maximum degree, it has accumulated a maximum of 4 + 4 + 4 + 4 = 16 points. If another characteristic's effort has moved through the levels only at one-half of its maximum degree, it has accumulated 2 + 2 + 2 + 2 = 8 points. If it reached three-quarters of the maximum degree of effort on all levels, it has 3 + 3 + 3 + 3 = 12 points. Management can now assign a reference value to the pursuit of quality for a programming product. This is shown in the simplified example in table 8-6. For this example the total is 8 + 12 + 13 = 33 points out of a possible 16 + 16 + 16 = 48 points, or 69 percent. (In more general terms, this can also be referred to as an overall level 3 quality effort in the 50 to 75 percent range.) Note that the real indication of the quality objectives will be the magnitude of the *X*/*Y* (33/48) values. The greater the *X* and *Y* values, the deeper the degree to which the characteristics have been pursued. The greater the *X* value, the more stable the structure has become and the more quality objectives the programming product has achieved.

If this type of analysis is carried over all eight characteristics (8 × 16), a maximum of 128 points is possible. Products that approach this level of effort will have a considerably more stable structure than those that are only based upon a 16-point single-character structure. The *X* percent quality reference number should also be qualified by a factor to note how many characteristics were actually used. This could be shown as 69 percent/C3 or 33/48/C3.

Finally, some characteristics will be more complex and require greater costs to achieve than others. Thus, a weighting

TABLE 8-5.—EXAMPLE OF QUALITY MANAGEMENT PROCESS MATRIX

[Number in circle denotes degree of quality selected by a quality management process.]

Product phase	Quality characteristic		
	Reliability	Changeability	Maintainability
Planning	1 2 3 4 (2 circled)	1 2 3 4 (4 circled)	1 2 3 4 (3 circled)
Design and test	1 2 3 4 (2 circled)	1 2 3 4 (4 circled)	1 2 3 4 (3 circled)
Integration and installation	1 2 3 4 (2 circled)	1 2 3 4 (2 circled)	1 2 3 4 (4 circled)
Service	1 2 3 4 (2 circled)	1 2 3 4 (2 circled)	1 2 3 4 (3 circled)

Extent of quality ↓

Degree of quality →

TABLE 8-6.—EXAMPLE OF PURSUIT OF QUALITY

Product phase	Quality characteristic		
	Reliability	Changeability	Maintainability
Planning	2	4	3
Design and test		4	3
Integration and installation		2	4
Service		2	3
Total points/ available points	8/16 (50%)	12/16 (75%)	13/15 (81%)
Total	(33/48)/C3, or (69%)/C3		

multiplier (WM) can be used to equalize the quality characteristics. Weighting multipliers for the preceding example are demonstrated in table 8-7. For this example the total of 10 + 28 + 19 = 57 points out of a possible 20 + 40 + 24 = 84 points is 57/84/C3, or 68 percent/C3. This three-part programming quality ratio (e.g., 57/84/C3) can be used for reviewing quality across programming products within a corporation as a more quantitative cross reference of quality costs to quality objectives.

A quality management process matrix (table 8-5) has been presented for pursuing quality throughout a programming product's life cycle. It relates the pursuit of quality characteristics to the planning, design and testing, integration and installation, and service phases. In practice, actual implementation of this approach will require the selection of languages, code walkthroughs, type of testing, etc., to be specifically defined for reaching service quality level 2, 3, or 4. From this matrix the impact on schedule and the cost of quality can be projected and monitored.

This process will also help management to compare the extent and degree of quality for products of competing companies or internal corporate divisions. Of course, until such

TABLE 8-7.—EXAMPLE OF USE OF WEIGHTING MULTIPLIERS (WM)

Product phase	Quality characteristic		
	Reliability	Changeability	Maintainability
	Level × WM	Level × WM	Level × WM
Planning	2 × 1	4 × 2	3 × 2
Design and test	2 × 1	4 × 2	3 × 1.5
Integration and installation	2 × 1	2 × 3	4 × 1
Service	2 × 2	2 × 3	3 × 1.5
Total points/ available points	10/20 (50%)	28/40 (70%)	19/24 (79%)
Total	(57/84)/C3, or (68%)/C3		

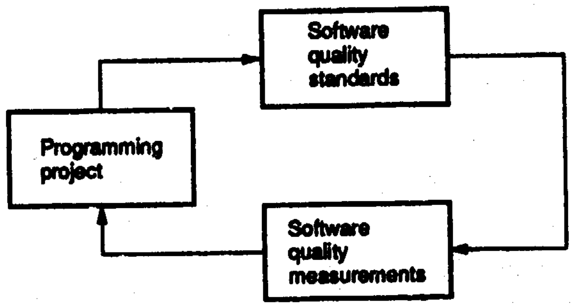

Figure 8-14.—Relationship of measurements and standards.

a standard is developed, the quality management team will subjectively assign values and multipliers as noted in table 8-5 and relate them to their own acceptable degree of documentation, code walkthrough, module tests, etc. These subjective values are extremely useful in establishing individual product quality effort goals, by translating the concept of quality prisms to planning, design, and test considerations that balance schedule and cost against quality objectives. However, management will now have a more reasonable opportunity to pursue and successfully achieve the extent and degree of desired quality for their products.

The ability to specify an overall software quality metric has been addressed. Overall quality measurements can be normalized, as in the quality prisms concept, for purposes of comparison. The quality prisms concept can be used to compare the software of two or more different projects within the same company or between different companies even if the software products have unique applications or utilize different programming languages. Quality prisms can also be used to combine hardware quality and software quality into an assessment of the quality of the whole system.

Software Quality Standards

The relationship of software quality standards and software quality measurements is depicted in figure 8-14. Measurements and standards must agree. If a set of quality standards is established (i.e., zero defects) and quality measurement cannot prove it (i.e., through exhaustive testing, error seeding, etc.), the software development project must realistically set a goal so that both quality standards and measurements can be developed. The IEEE has published many articles on and general guides for formulating goal criteria. In addition, many technical papers are available on specific goals both on a life-cycle basis and on a per-delivered software product basis.

Concluding Remarks

This chapter has presented a snapshot of where software quality assurance is today and has indicated future directions. A base for software quality standardization was issued by the IEEE (ref. 8-11). Research is continuing into the use of overall software quality metrics and better software prediction tools

for determining the defect population. In addition, simulators and code generators are being further developed so that high-quality software can be produced.

Several key topics have been discussed:

(1) Life-cycle phases
(2) Software quality characteristics
(3) Software quality metrics
(4) Overall software quality metrics
(5) Software quality standards

In addition, table 8-3 presented the topics

(6) Process indicators
(7) Performance measures

Process indicators are closely tied to the software quality effort and some people include them as part of the software development effort. In general, there are measures such as (1) test cases completed versus test cases planned, and (2) the number of lines of code developed versus the number expected. Such process indicators can also be rolled up (all software development projects added together) to give an indication of overall company or corporate progress toward a quality software product. Too often, personnel are moved from one project to another and thus the lagging projects improve but the leading projects decline in their process indicators. The life cycle for programming products, as shown in table 8-3, should not be disrupted.

Performance measures, which include such criteria as the percentage of proper transactions, the number of system restarts, the number of system reloads, and the percentage of uptime, should reflect the user's viewpoint. The concept of recently proposed performability (ref. 8-12) combines performance and availability from the customer's perspective.

In general, the determination of applicable quality measures for a given software product development is viewed as a specific task of the software quality assurance function. The determination of the process indicators and performance measures is a task of the software quality standards function.

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Reliability Training[1]

1. What are the three entities that determine quality software?

   A. Process, material, and vibration
   B. Process, product, and environment
   C. Planning, product, and shock
   D. All of the above

2. What does software quality consist of?

   A. Various aspects of producing programming products
   B. Bar charts for process control
   C. Statistical analysis of software bugs
   D. All of the above

3. How is the term "software quality" defined?

   A. To assure the acquisition of high-quality software products on schedule, within cost, and in compliance with the performance requirements
   B. To ignore various needs
   C. To develop specifications, develop attributes, perceive customer needs, and meet the user's expectations
   D. All of the above

4a. What are the 10 software life-cycle phases?

   A. Conceptual; requirements; product definition; design; implementation; testing; vibration; prototypes; installation; and disposal
   B. Planning; definition; design; manufacturing; testing; acceptance; debugging; and repair
   C. Conceptual planning; requirements definition; product definition; top-level design; detailed design; implementation; testing and integration; qualification, installation, and acceptance; maintenance and enhancements; and disposal
   D. All of the above

4b. What are the IEC system life-cycle phases?

   A. Concept and research; design and plan; manufacture and debug; operation and maintenance; and wearout
   B. Concept and definition; design and development; manufacturing and installation; operation and maintenance; and disposal
   C. Research and development; design and breadboard; manufacturing and testing; operation and maintenance; and disposal
   D. All of the above

4c. How can the 10 software life-cycle phases be combined to fit in the IEC system life-cycle phases?

   A. Concept and definition: conceptual planning; requirements definition; and product definition
   B. Design and development: top-level design and detailed design
   C. Manufacturing and installation: implementation; testing and integration; qualification; and installation and acceptance
   D. Operations and maintenance: maintenance and enhancement
   E. Disposal: disposal
   F. All of the above

[1]Answers are given at the end of this manual.

5. Can there be different degrees of a quality characteristic for different life-cycle phases?

   A. Yes B. No C. Do not know

6a. The definition of lack of software quality is

   A. The lack of proper planning in early life-cycle phases
   B. The application of dependent software quality characteristics
   C. Poorly developed software that lacks proper criteria in life-cycle phases
   D. All of the above

6b. Three example characteristics of software quality are

   A. Testing, integration, and portability
   B. Maintainability, portability, and reliability
   C. Design, implementation, and reliability
   D. All of the above

7. Seven software quality characteristics are

   A. Maintainability, portability, reliability, testability, understandability, usability, and freedom from error
   B. Planning, definition, reliability, testing, software, hardware, usability
   C. Design, implementation, integration, qualification, acceptance, enhancement, maintenance
   D. All of the above

8. Management has decided that quality engineering should measure four characteristics of the *XYZ* software: maintainability, portability, reliability, and testability. The desired goals set at the beginning of the program by management for the characteristic effort were maintainability, 3.5; portability, 3.0; reliability, 3.9; and testability, 3.5. The overall goal was thus 87 percent/C4 for the extent of quality. The 2-year program gave the following results:

	Planning	Design and test	Integration	Service
Maintainability	4.0	3.5	3.4	3.4
Portability	4.0	3.0	3.1	3.1
Reliability	3.5	3.6	3.9	3.9
Testability	4.0	3.1	3.5	3.6
Total	15.5	13.2	13.9	14.0

   a. The actual extent of quality was

      A. (87.5%)/C4 B. (88.4%)/C4 C. (88.8%)/C4 D. None of the above

   b. Have the management objectives been achieved?

      A. Yes B. No C. Do not know

Chapter 9
Reliability Management

Roots of Reliability Management

Over the past few years the term "reliability management" has been raised to a high level of awareness. Previously, the management of reliability was concerned with eliminating failure by testing to prove reliability, and it generally complemented the design function. Quality management, on the other hand, focused on quality control and generally aligned itself with manufacturing and production. The picture began to change with the focus on customer reliability and quality concerns. Specifically, the usage and standardization by companies of reliability growth models established that the new concept of reliability management is replacing the old concept of the management of reliability. New stress is being placed on enlarging the area of reliability concern to all phases of the life cycle. It is felt that all aspects of management operations and functions must be integrated into the reliability concept and program. Thus, reliability in the manufacturing or production phase is as important as reliability in the design phase (ref. 9–1), as shown in figure 9–1.

Planning a Reliability Management Organization

In planning a reliability management organization the reliability function must report to a high enough level to be effective. If the reporting level does not involve top management in reliability issues, the reporting level is too low. For example, many successful programs today encompass 3 to 6 hours per month at vice-presidential staff meetings. Each company must find the level that makes reliability a real issue to be addressed. A guide to reliability management is reference 9–2.

A functional organization forms groups performing similar generic tasks such as planning, design, testing, and reliability. Often this type of organization gets muddled down with too many levels of management, and specific product priorities are often different in the many task groups. However, many benefits accrue from the concentration of talent and constant technical peer review. With today's time-to-market pressures, building such a large centralized reliability organization is often

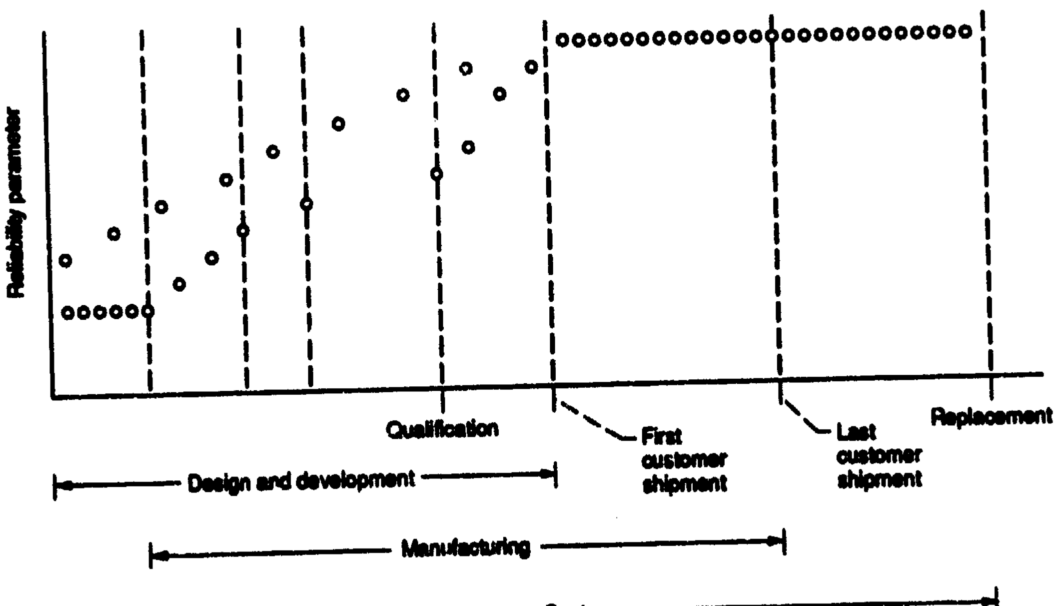

Figure 9–1.—Life cycle reliability growth, with two different parts to first customer shipment.

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not the best choice. The team approach, distributed reliability, is often selected over functional organization.

A team organization forms teams of people often with diverse talents and backgrounds. Quality circles and reliability circles are based on the same organizational approach. Even though peer review is not constantly in place, the cross-technology knowledge of today's personnel appears to fully compensate for the lack of constant peer review. In the software development world, several types of team organization exist. For instance, the first type of typical team organization is the project team organization. This is a hierarchical organization in which programmers with less experience are assigned to work for programmers with more experience. The project team organization is designed to fit the company organization rather than to fit project requirements. The second type is the chief programmer team, which employs a highly skilled person who performs most of the programming while providing technical direction. A third type is the Weinberg programming team, which is composed of groups of 10 or fewer programmers with complementary skills. Group consensus and leadership role shifts are characteristic of this type of team organization. Each of these team organizations has advantages depending on the size of the project, the newness of the technology being implemented, etc.

The fourth type of team organization, matrix organization, is a hybrid approach that can be a reliability disaster especially if time-to-market pressures exist. Often the technology is masked by middle management procedural meetings. The matrix organization combines functional talent to put teams together. These teams report to one manager. Individual contributors are added to work on one or more tasks of a given project or product development. These projects usually report to middle management.

A fifth possibility is based on the theory stated in reference 9-3 that reliability is actively pursued by involvement starting on the vice-presidential level and is organization wide. This new style of reliability involves establishing a reliability council, dedicating a full-time diagnostic person or team, and generally making an upward change in the reliability reporting level. Figure 9-2 presents this concept. The reliability council's responsibilities are

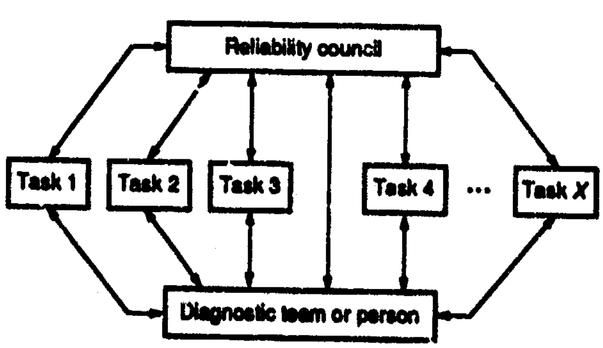

Figure 9-2.—Reliability organization.

(1) To endorse the annual reliability plan
(2) To regularly review reliability status
(3) To approve reliability improvement projects
(4) To set priorities on resources
(5) To assign tasks
(6) To regularly review tasks
(7) To participate in reliability improvement awards

The reliability council membership may consist of

(1) The vice-president of the company or division as chairman
(2) The vice-president's staff
(3) The vice-president's business partners
(4) The corporate engineering director
(5) The corporate manufacturing director
(6) The corporate customer services director

The diagnostic team's or person's functions are

(1) To review the internal reliability status
(2) To review reliability as perceived by customers
(3) To recommend tasks to the reliability council
(4) To diagnose problems
(5) To design experiments
(6) To collect and analyze data

The diagnostic team's or person's concerns include

(1) Reliability, quality, and statistics
(2) Engineering and manufacturing engineering
(3) Product development and process optimization
(4) Product assembly and test strategies
(5) Customer perception

This is a new dynamic approach for establishing reliability management at the proper level in a corporation while optimizing its effectiveness.

General Management Considerations

Program Establishment

In order to design for successful reliability and continue to provide customers with a reliable product, the following steps are necessary:

(1) Determine the reliability goals to be met.
(2) Construct a symbolic representation (e.g., block diagram or Petri net, ref. 9-4).
(3) Determine the logistics support and repair philosophy.
(4) Select the reliability analysis procedure.
(5) Select the source or sources of the data for failure rates and repair rates.
(6) Determine the failure rates and the repair rates.
(7) Perform the necessary calculations.
(8) Validate and verify the reliability.
(9) Measure reliability until customer shipment.

This section will address the first three steps in detail.

Goals and Objectives

Goals must be placed into proper perspective. They are often examined by using models that the producer develops. However, one of the weakest links in the reliability process is the modeling. Dr. John D. Spragins, an editor for the IEEE Transaction on Computers, places this fact in context (ref. 9–3) with the following statement:

> Some standard definitions of reliability or availability, such as those based on the probability that all components of a system are operational at a given time, can be dismissed as irrelevant when studying large telecommunication networks. Many telecommunication networks are so large that the probability they are operational according to this criterion may be very nearly zero; at least one item of equipment may be down essentially all of the time. The typical user, however, does not see this unless he or she happens to be the unlucky person whose equipment fails; the system may still operate perfectly from this user's point of view. A more meaningful criterion is one based on the reliability seen by typical system users. The reliability apparent to system operators is another valid, but distinct, criterion. (Since system operators commonly consider systems down only after failures have been reported to them, and may not hear of short self-clearing outages, their estimates of reliability are often higher than the values seen by users.)

Reliability objectives can be defined differently for various systems. An example from the telecommunications industry (ref. 9–5) is presented in table 9–1. We can quantify the objectives, for example, for a private automatic branch exchange (PABX) (ref. 9–5) as shown in table 9–2. Table 9–2 presents the reliability specification for a wide variation of PABX sizes (from fewer than 120 lines to over 5000 lines).

Symbolic Representation

Chapter 3 presents reliability diagrams, models that are the symbolic representations of the analysis. The relationship of operation and failures can be represented in these models.

TABLE 9-1.—RELIABILITY OBJECTIVES FOR TELECOMMUNICATIONS INDUSTRY

Module or system	Objective
Telephone instrument	Mean time between failures
Electronic key system	Complete loss of service Major loss of service Minor loss of service
PABX	Complete loss of service Major loss of service Minor loss of serivce Mishandled calls
Traffic service position system (TSPS)	Mishandled calls System outage
Class 5 office	System outage
Class 4 office	Loss of service
Class 3 office	Service degradation

Redundancy (simple and compound) is also discussed in chapter 3. Performance estimates and reliability predictions are now being performed simultaneously by using symbolic modeling concepts such as Petri nets.

Twenty-five years ago, Carl Adam Petri published a mathematical technique for modeling known as a Petri net. A Petri net is a tool for analyzing systems and their projected behavior. In 1987, Carl Petri delivered the keynote address at the international workshop on Petri nets and performance models (ref. 9–7). Many applications were discussed at the workshop including the use of timed models for determining the expected delay in complex sequences of actions, methods used to determine the average data throughput of parallel computers, and the average failure rates of fault-tolerant computer designs. Correctness analysis and flexible manufacturing techniques were also described. Timed Petri nets show promise for analyzing throughput performance in computer and communications systems.

TABLE 9-2.—RELIABILITY SPECIFICATION FOR PABX

	Number of lines							
	<120	200	400	600	800	1200	3000	5000
Common control performance:								
Mean time between catastrophic failures, yr	10	---	---	---	---	----	----	----
System outage time per 20 yr, hr	----	---	---	---	---	1	1	1
Mean time between outages, yr	----	---	---	---	---	>5	>5	>5
Mean time between complete losses of service, yr	5	10	40	40	40	----	----	----
Service level:								
Mean time between major losses of service, days	200	400	300	200	150	365	365	----
Mean time between minor losses of service, days	60	60	50	40	30	30	15	----
Degradation of service, hr/yr	---	---	---	---	---	---	----	1
Mishandled calls, percent	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.02

TABLE 9-3.—SPARES POLICY

Subsystem	On-site spares ?	Subdepot spares ?	Turnaround time[a] of subdepot spares, days	Depot spares ?	Turnaround time[a] of depot spares, days,
Common control and memory	Yes	Yes	2	Yes	15
Network	No	↓	↓	↓	30
Line and trunk units	Yes	↓	↓	↓	30
Peripheral equipment	No	↓	↓	↓	30
Test equipment	No	No	---	↓	5

[a]For replacing spares.

A Petri net is an abstract and formal graphical model used for systems that exhibit concurrent, asynchronous, or nondeterministic behavior. The Petri net model provides accurate system information when the model is a valid representation of the system and the solution of the model is correct. A Petri net is composed of four parts: a set of places, a set of transitions, an input function, and an output function. The input function and the output function relate to transitions and places. In general, graphics are used to represent the Petri net structures and show the concepts and the problems. A circle represents a place, a bar represents a transition, and directed arcs connect transitions to places or places to transitions. The state of a Petri net is called the PN marking and is defined by the number of "tokens" contained in each place. A place is an input to a transition if an arc exists from the place to the transition and an output if an arc exists from the transition to the place. Enabled transitions can be "fired" by removing one token from each input place and adding one token to each output place. The firing of a transition causes a change of state and produces a different PN marking. Reference 9–8 contains additional information. Petri nets are a useful reliability modeling tool.

Logistics Support and Repair Philosophy

The logistics support plan is normally based on criteria such as (1) failure rates and repair rates of replaceable units, (2) system maturity, (3) whether or not the sites can be served by depots or subdepots, and (4) the rate at which additional sites are added to the depot responsibility. Since spares are the key to support, this chapter will examine them further.

The size of the spares stock depends on (1) the criticality of the replaceable unit to the system, (2) the necessary spare adequacy level, (3) the number of systems served, (4) whether the area served is rural, suburban, or urban, and (5) whether the repair facility is on site or remote. A typical spares policy for a telecommunications system (ref. 9–9) is presented in table 9–3.

Policies can be formulated for families of systems or for multifamily geographical areas. The turnaround time depends on the replaceable units failure rate, the repair location, the repair costs, etc. A specific spares policy can be tailored to a given geographical area. Note that subsystems have different spares policies owing to the criticality of their failures in contrast to a blanket spares assignment without regard to functionality or survivability.

Even though the spares location and turnaround time are the same for two different subsystems, the spares adequacy can be different. Some spares adequacy levels for a telecommunications systems are presented in table 9–4.

Spares provisioning is an important part of a spares plan. Requirements must be clearly stated or they can lead to over- or undersparing. For example, a spares adequacy of 99.5 percent can be interpreted in two ways. First, six spares might be needed to guarantee that spares are available 99.5 percent of the time. Alternatively, if one states that when a failure occurs a spare must be available 99.5 percent of the time, it will be necessary to supply $6 + 1 = 7$ spares.

The establishment of depot and subdepot sparing, rather than only individual site sparing, has proven to be cost effective. As an example, table 9–5 presents the depot effectiveness for a typical digital PABX. This table indicates that a 14.5-percent spares level would be required if only per-site sparing was used; however, when one depot serves 100 sites, the required spares level is less than 1 percent.

A centralized maintenance base (CMB) (ref. 9–10) is essential to a deferred maintenance concept. Deferred maintenance can be available on a real-time basis. When a failure occurs

TABLE 9-4—SPARES ADEQUACY

Subsystem	On-site spares?	Subdepot spares	Depot spares
		Adequacy[a]	
Common control and memory	Yes	0.9995	0.9995
Network	No	.995	.995
Line and trunk units	Yes	.999	.999
Peripheral equipment	No	.99	.99
Test equipment	No	------	.95

[a]Probability of having spares available.

TABLE 9-5.—DEPOT EFFECTIVENESS FOR TYPICAL DIGITAL PABX

Foreign branch part	Control automatic trunk	Printed wiring cards for n systems					Spare printed wiring cards for n systems				
		1	2	10	50	100	1	2	10	50	100
15002	6	65	130	650	3 250	6 500	2	2	5	13	20
15003	5	16	32	160	800	1 600	1	1	2	5	7
15004	6	14	28	140	700	1 400	1	1	4	5	8
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.
20703	8	28	56	280	1 400	2 800	2	2	4	10	15
20705	16	153	206	1 530	7 650	15 300	7	11	29	106	196
	Total	1058	2116	10 580	52 900	105 800	153	173	287	658	1001
						Spares, percent of total	14.5	8.2	2.7	1.2	0.95

at an unattended site, the CMB would receive information on a display as to the criticality of the failure and the deferred maintenance action taken if imposed and would receive a projection indicating impending problems. The CMB would analyze the situation for the specific site configuration, the processing level in the system, and the site's failure-repair history.

Input data could consist of items such as the last similar occurrence, the next planned visit to the site, the criticality of the site to the operating network, the cumulative site failures for the last 3 months, and the probability of additional failures occurring. The data would be analyzed with a maintenance-prediction computer program to generate a table based on system loading, such as table 9-6. Often the suggested maintenance deferral time is recommended to be the next maintenance visit (NMV). The NMV will vary with the amount of equipment on site and the projected failure frequency (ref. 9-10).

The combination of deferred maintenance and a centralized maintenance base dictates the needs for an efficient spares program. Spares planning combined with knowledge of the logistics can optimize support costs. A depot stocking plan can additionally vary because of many factors, including error coverage, system maturity, deferred repair, and maintenance familiarity. A dynamic (continuously updated) depot stocking plan would be cost effective. A dynamic depot model using Monte Carlo methods (ref. 9-11) includes unit delivery schedules, item usage per month, support personnel efficiency, and depot and base repair cycle times.

TABLE 9-6.—MAINTENANCE ACTION RECOMMENDATIONS

	Before busy hour	Busy hour	After busy hour	Off-shift time
Repair	Yes	Yes	Yes	Yes
Defer repair for (days)	0	0	1	1
Is second failure affecting service?	No	Yes	No	No
Probability of no similar second failure	0.95	0.90	0.82	0.60
Site failures last month	Low	High	Normal	Low
Site failures last year	Low	Low	Normal	Low
Transient error rate	Low	High	Low	Low

Reliability Management Activities

Performance Requirements

It is often difficult to translate customer performance requirements into design requirements, especially in the area of quality and reliability. Reliability encompasses both quantitative and qualitative measures. New terms in the computer industry, such as "robustness," are not formally metricized. However, we can adapt concepts for the overall performance process (ref. 9-12) to apply to reliability as presented in figure 9-3.

If a business's matrix of reliability requirements is reduced to one or more models, subjective and qualitative customer-oriented reliability measures can be translated into quantitative system-oriented reliability criteria. Figure 9-3 identifies both the top-down and bottom-up approaches to reliability validation, which includes (1) translation, (2) allocation, (3) requirements, and (4) planning.

With the identification of the agreed-to system-oriented reliability criteria, designer-oriented subsystem or module reliability parameters can be allocated as shown in figure 9-3, generally by a system reliability team. The team evaluates simple versus redundant configurations, levels of fault detection and correction implementations, software considerations, etc. System or module reliability modeling may specify reliability requirements for specific components. An example of such modeling is a failure modes and effects analysis (FMAEA) performed on a product to predict the probability of network failures due to a single failure or due to a failure after an accumulation of undetected failures.

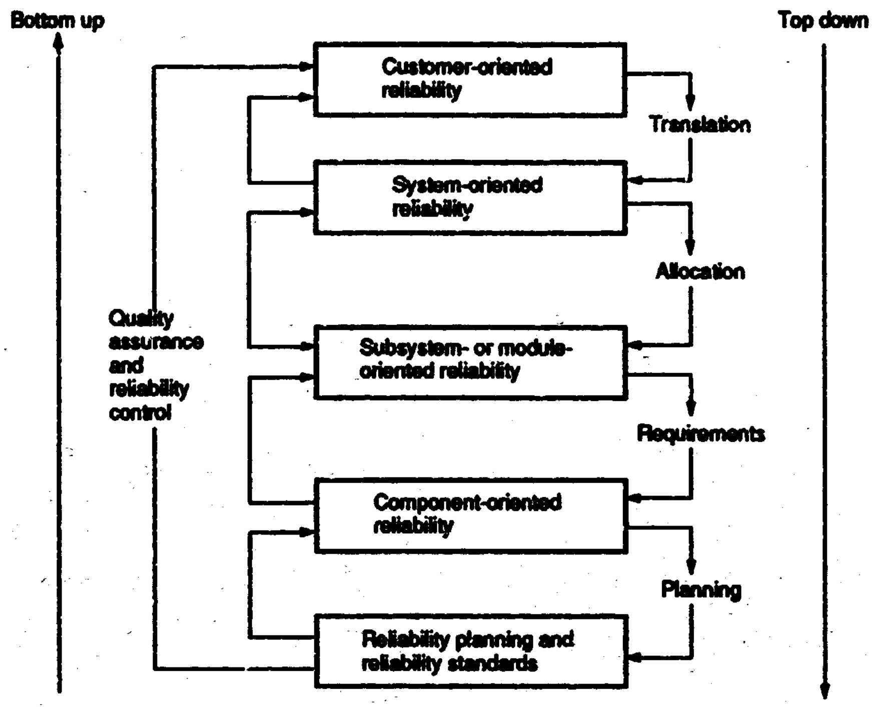

Figure 9-3.—Overall reliability process.

For example, a replacement product was to use a very large-scale integration (VLSI) implementation, and the protection against network failures needed to be assessed. An investigation found no apparent standard industry FMAEA method for VLSI components. Because future VLSI products may show an increasing need for FMAEA, it is important that an industry standard be generated. In the network examples discussed, a single fault could directly cause a customer-oriented problem.

The bottom-up approach to reliability validation ensures customer satisfaction. The appropriate certification, process metrics, and statistical in-process tests must be designed from the customer viewpoint. A step-by-step upward certification and design review using process metrics can be designed to ensure customer-oriented reliability. In addition, we can see the need for the independent upward path from reliability planning and standards to customer-oriented reliability in figure 9-3. This is the key to success, since reliability control cannot be bypassed or eliminated from design- or performance-related issues.

Specification Targets

A system can have a detailed performance or reliability specification that is based on customer requirements. The survivability of a telecommunications network is defined as the ability of the network to perform under stress caused by cable cuts or sudden and lengthy traffic overloads and after failures including equipment breakdowns. Thus, performance and availability have been combined into a unified metric. One area of telecommunications where these principles have been applied is the design and implementation of fiber-based networks. Reference 9-13 states that "the statistical observation that on the average 56 percent of the pairs in a copper cable are cut when the cable is dug up, makes the copper network 'structurally survivable.'" On the other hand, a fiber network can be assumed to be an all or nothing situation with 100 percent of the circuits being affected by a cable cut, failure, etc. In this case study, according to reference 9-13, "cross connects and allocatable capacity are utilized by the intelligent network operation system to dynamically reconfigure the network in the case of failures." Figure 9-4 (from ref. 9-14) presents a concept for specification targets.

Field Studies

The customer may observe specific results of availability. For instance, figure 9-5 has been the basis for the proposal of an IEC technology trend document (ref. 9-15).

System reliability testing is performed today to benchmark the reliability, availability, and dependability metrics of complex new hardware and software programs. Figure 9-6 (taken from ref. 9-1) presents the traditional viewpoint of the design, development, and production community on cumulative reliability growth. It is possible that the same data generated both curves in figure 9-6. When we measure the cumulative reliability growth, the decline of production coupled with a decline of reliability is masked. If we track the product on a quarterly basis, often the product shows a relaxation of process control, incorporation of old, marginal components into the last year's product manufacture, failure to incorporate the latest changes into service manuals, knowledgeable personnel

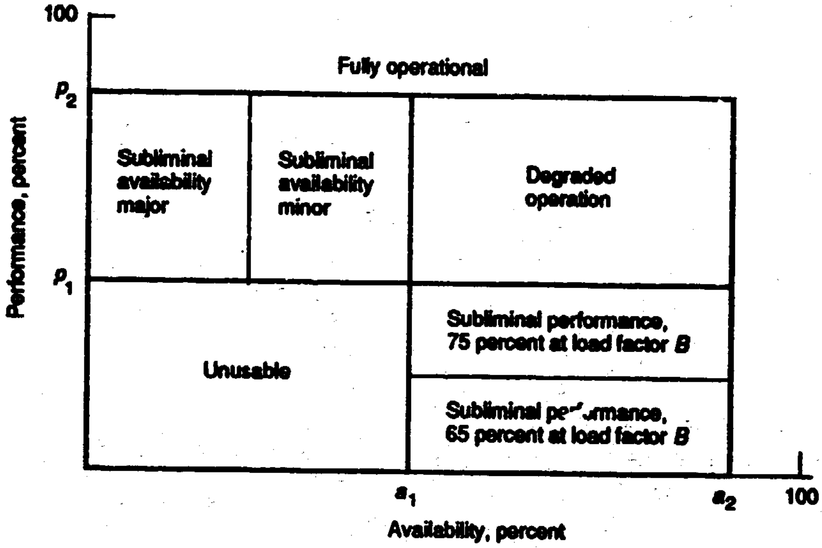

Figure 9-4.—Specification targets.

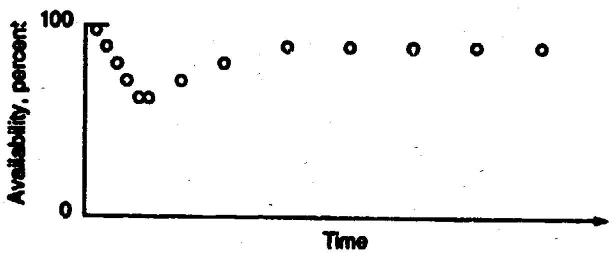

Figure 9-5.—Software availability.

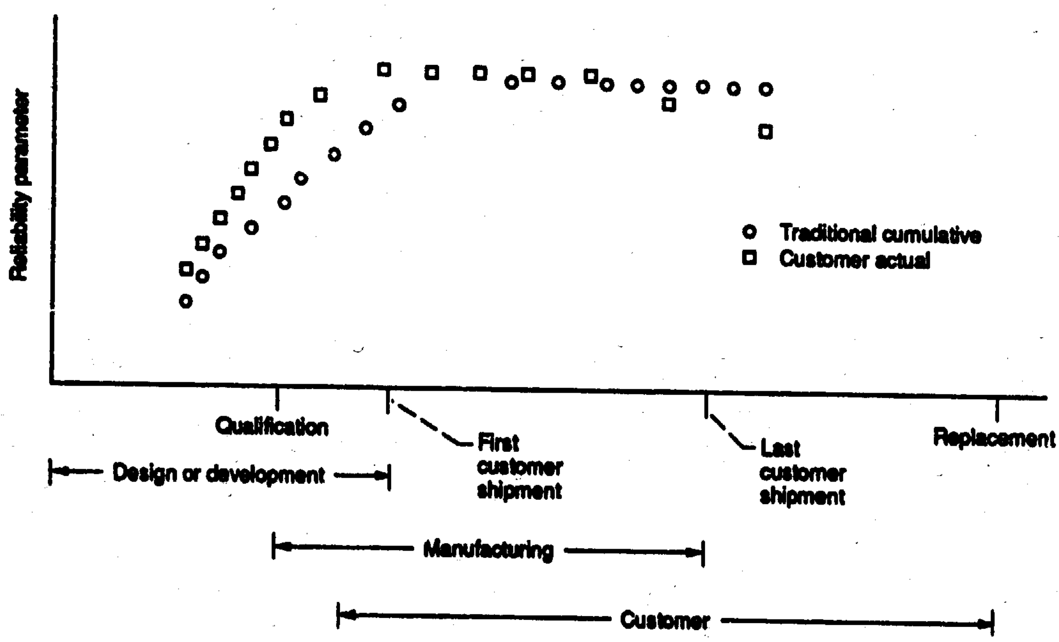

Figure 9-6.—Traditional viewpoint of reliability growth.

transferred to other products, etc. Thus, there is a need to track specific products on a quarterly basis (ref. 9-1).

Human Reliability

Analysis Methods

The major objectives of reliability management are to ensure that a selected reliability level for a product can be achieved on schedule in a cost-effective manner and that the customer perceives the selected reliability level. The current emphasis in reliability management is on meeting or exceeding customer expectations. We can view this as a challenge, but it should be viewed as the bridge between the user and the producer or provider. This bridge can be titled "human reliability." In the past, the producer was concerned with the process and the product and found reliability measurements that addressed both. Often there was no correlation between field data, the customer's perception of reliability, and the producer's reliability metrics. Surveys then began to indicate that the customer or user distinguished between reliability performance, response to order placement, technical support, service quality, etc.

Human Errors

Human reliability is defined (ref. 9-16) as "the probability of accomplishing a job or task successfully by humans at any required stage in system operations within a specified minimum time limit (if the time requirement is specified)." Although customers generally are not yet requiring human reliability models in addition to the requested hardware and software reliability models, the science of human reliability is well established.

Example

Presently, the focus in design is shifting from hardware and software reliability to human reliability. A recent 2½-year study by Bell Communication Research (ref. 9-17) indicated that reliability in planning, design, and field maintenance procedures must be focused on procedural errors, inadequate emergency actions, recovery and diagnostic programs, the design of preventive measures to reduce the likelihood of procedural errors, and the improvement of the human factors in the design and subsequent documentation. The study revealed the following results for outages or crashes as shown in figure 9-7. Approximately 40 percent of outage events and downtime is due to procedural problems (human error). In fact, if software recovery problems are included with procedural problems, 62 percent of the events and 68 percent of the downtime are due to human error. Therefore, human reliability planning, modeling, design, and implementation must be focused on in order to achieve customer satisfaction.

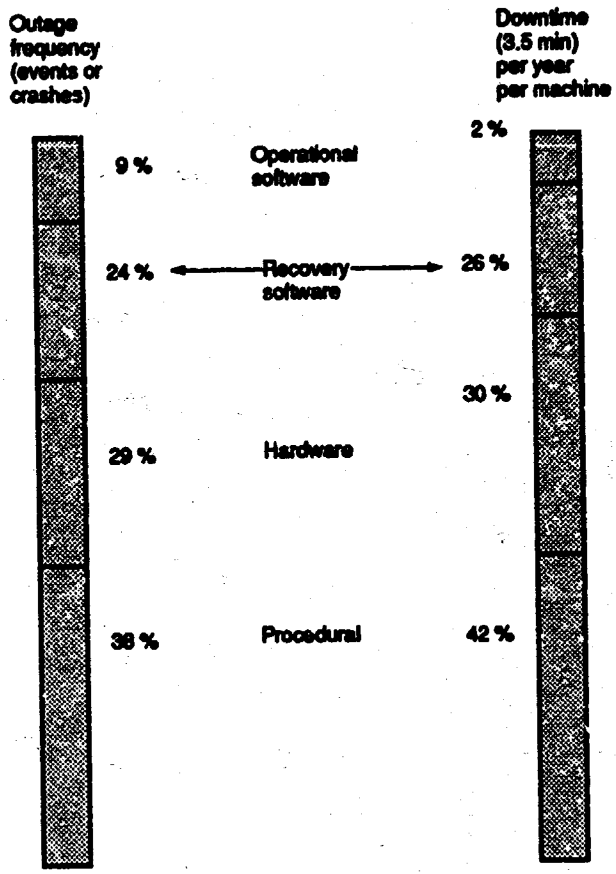

Figure 9-7.—Reliability characteristics.

Presentation of Reliability

Reliability testing usually occurs during product development and ends with the first product shipment. However, product reliability testing can be cost effectively run through the manufacturing life of the product to achieve both continued customer satisfaction and the inherent reliability of the product.

A major concern in planning reliability testing is the maturity of the specific manufacturing facility. For instance, a new plant may initially need three to five failures per week of tested product under controlled test environments in order to shape the manufacturing process and the product specifics. Therefore, detailed failure analysis will be conducted on 150 to 250 failed items per year. Once plant personnel begin to feel comfortable as a team and several of the plant's processes, products, or both are certified, the goal of one failure per week can be instituted in a medium-mature plant. The team in a mature plant with few failures can observe leading indicators that forewarn of possible problems and can prevent them from entering into the shipped product. Thus, in a mature plant the goal of one failure per 2 weeks can suffice as a benchmark for quality operations to achieve product reliability.

Engineering and Manufacturing

Measuring reliability in a practical way is a challenge. Reliability grows with product, process, and customer use maturity. We could measure, for example, the reliability at first customer shipment and the reliability during a 5-year production life. An effective start may be to establish a three- to five-level reliability tier concept (ref. 9–18). For example, table 9–7 presents a five-tier reliability concept. With this concept products can achieve first customer shipment at a mean time between failures (MTBF) of $\bar{T}$(min). Manufacturing and service will accept risks until $\bar{T}$(spec) is reached. Manufacturing has a commitment to drive the MTBF of the product up to $\bar{T}$(spec), and engineering has a commitment to provide resources for solving design problems until $\bar{T}$(spec) is reached. The qualification team working with this process is now involved throughout the design qualification process through field feedback. Ideally, the MTBF's of tiers 2 to 5 would be equal; however, the calibration of reliability modeling tools and the accuracy of field MTBF measurements are challenges yet to be met in some corporations and industries. Thus, a three- to five-tier approach is a practical and effective solution for developing reliability measurements.

Although the MTBF is between $\bar{T}$(min) and $\bar{T}$(spec), progress is tracked toward $\bar{T}$(spec) as a goal. The point is to find and fix the problems and thus improve the reliability of the product. Teamwork and commonality of purpose with manufacturing and engineering are necessary in order to deal with real problems and not symptoms. After $\bar{T}$(spec) has been achieved, an "insurance policy" is necessary to determine if anything has gone radically wrong. This can be a gross evaluation based on limited data as the "premiums" for a perfect "insurance policy" are too high. Once $\bar{T}$(spec) has been demonstrated, a trigger can be set at the 50-percent lower MTBF limit for control purposes. Improvement plans at this level should be based on the return on investment. At maturity, $\bar{T}$(intrinsic), dependence on reliability testing can be reduced. A few suggestions for reductions are testing fewer samples, shortening tests, and skipping testing for 1 or 2 months when the personnel feel comfortable with the product or process. With a reduced dependence on reliability testing, other manufacturing process data can be used for full control.

TABLE 9-7.—FIVE-TIER RELIABILITY CONCEPT

Tier	Mean time between failures	Description
1	$\bar{T}$(min)	Minimum demonstrated MTBF before shipping (statistical test)
2	$\bar{T}$(spec)	Specified MTBF that meets market needs and supports service pricing
3	$\bar{T}$(design)	Design goal MTBF (calculation)
4	$\bar{T}$(intrinsic)	Intrinsic MTBF (plant measurement)
5	$\bar{T}$(field)	Field MTBF measurement

User or Customer

Reliability growth has been studied, modeled, and analyzed—usually from the design and development viewpoint. Seldom is the process or product studied from the customer's or user's perspective. Furthermore, the reliability that the first customer observes with the first customer shipment can be quite different from the reliability that a customer will observe with a unit or system produced 5 years later, or last customer shipment. Because the customer's experience can vary with the maturity of a system, reliability growth is an important concept to customers and should be considered in the customer's purchasing decision.

The key to reliability growth is the ability to define the goals for the product or service from the customer's perspective while reflecting the actual situation in which the customer obtains the product or service. For large telecommunications switching systems there has been a rule of thumb for determining reliability growth. Often systems have been allowed to operate at a lower availability than the specified availability goal for the first 6 months to 1 year of operation (ref. 9–19). In addition, component part replacement rates have often been allowed to be 50 percent higher than specified for the first 6 months of operation. These allowances accommodated craftspersons learning patterns, software patches, design errors, etc.

The key to reliability growth is to have the growth measurement encompass the entire life cycle of the product. The concept is not new, only here the emphasis is placed on the customer's perspective. Reference 9–20 presents the goals of software reliability growth (table 9–8).

Table 9–8 covers a large complex system with built-in fault tolerance. Reference 9–21 regarded this system as not "technically or economically feasible to detect and fix all software problems in a system as large as No. 4 ESS [electronic switching system]. Consequently, a strong emphasis has been placed on making it sufficiently tolerant of software errors to provide successful operation and fault recovery in an environment containing software problems."

Reliability growth can be specified from "day 1" on a product development and can be measured or controlled on a product with a 10-year life until "day 5000." We can apply the philosophy of reliability knowledge generation principles, which is to generate reliability knowledge at the earliest possible time in the planning process and to add to this base for the duration of the product's useful life. To accurately measure and control reliability growth, we must examine the entire manufacturing life cycle. One method is the construction of a production life-cycle reliability growth chart.

Table 9–9 presents a chart for setting goals for small (e.g., a 60-line PABX or a personal computer), medium, and large systems. Small systems must achieve manufacturing, shipping, and installation maturity in 3 months in order to gain and keep a market share for present and future products. This is an achievable but difficult goal to reach. The difference in

TABLE 9-8.—1980 GENERIC QUALITY METRICS

[From reference 9-20.]

	Implementation phase				
	Require-ments	Design	Laboratory system test	Field test	Field performance
Open questions	0	------	------	------	------
Problems fixed, per words	---	------	1/500	1/1000	1/1000
Problems open, per words	---	1/5000	1/5000	1/2000	1/2000
Interrupts, per day	---	------	<20	<20	<25
Audits, per day	---	0	<10	<10	<25
Service affective incidents, per office month	---	------	0	0	1.8
Reinitializations, per month	---	------			1
Cutoff calls, per 10 000	---	------			<0.2
Denied calls, per 10 000	---	------			<0.7
Trunk out of service, min/yr	---	------			20

TABLE 9-9.—PRODUCTION LIFE-CYCLE RELIABILITY GROWTH CHART

	Year								
	1987				1988		. . .	1994	
	Quarter								
	Q1	Q2	Q3	Q4	Q1	Q2	. . .	Q3	Q4
Small system:									
Reliability growth, percent	5	0	0	0	0	0	. . .	0	0
Time to steady state, months	3	0	0	0	0	0	. . .	0	0
Medium system:									
Reliability growth, percent	100	50	25	10	10	10	. . .	10	10
Time to steady state, months	6	3	2	1	1	1	. . .	1	1
Large system:									
Reliability growth, percent	200	100	50	50	33	33	. . .	20	20
Time to steady state, months	12	9	6	3	3	3	. . .	3	3

reliability growth characterization between small systems and larger systems is that the software-hardware-firmware interaction, coupled with the human factors of production, installation, and usage, limits the reliability growth over the production life cycle for most large, complex systems.

In certain large telecommunications systems the long installation time allows the electronic part reliability to grow so that the customer observes the design growth and the production growth. Large, complex systems often offer a unique environment to each product installation, which dictates that a significant reliability growth will occur. Yet, with the difference that size and complexity impose on resultant product reliability growth, corporations with a wide scope of product lines should not present overall reliability growth curves on a corporate basis but must present individual product-line reliability growth pictures to achieve total customer satisfaction.

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Reliability Training[1]

1. Reliability management is concerned with what phases of the life cycle?

   A. Design and development B. Manufacturing C. Customer D. All of the above

2. Name a new style of organizing reliability activities.

   A. Functional B. Team C. Matrix D. Council

3. What are the functions of the diagnostic team or person?

   A. Review the internal reliability status
   B. Review reliability as perceived by the customer
   C. Recommend tasks to the reliability council
   D. Diagnose problems
   E. Design experiments
   F. Collect and analyze data
   G. All of the above

4. Name a goal category for a telephone instrument.

   A. Loss of service
   B. Mean time between failures
   C. Mishandled calls
   D. All of the above

5. A PABX with 800 lines has a service level reliability specification for the mean time between major losses of service (MTBF) of

   A. 150 days B. 1 hour C. 0.1 percent D. All of the above

6. A Petri net is composed of which of the following parts?

   A. A set of places
   B. A set of transitions
   C. An input function
   D. An output function
   E. All of the above

7. For a telecommunications system, what is the spares adequacy level for a network subsystem with spares depots?

   A. 0.999 B. 0.995 C. 0.95

8. Turnaround time depends on

   A. Replaceable unit failure rate
   B. Repair location
   C. Repair cost
   D. All of the above

9. Spares adequacy is the probability of having spares available.

   A. True B. False C. Do not know

[1] Answers are given at the end of this manual.

10. What is the normal maintenance action recommendation for the site to defer repair for (days) during off-shift time?

A. 0 B. 2 C. 1

11. The bottom-up approach to reliability makes use of planning, requirements, allocations, and customer orientation.

A. True B. False C. Do not know

12. Specification targets can be used to define what performance and availability requirements?

A. Fully operational
B. Subliminal availability
C. Degraded operation
D. Unusable
E. Subliminal performance
F. All of the above

13. Tracking a product on a quarterly basis often shows

A. A relaxation of process control
B. Incorporation of old marginal components
C. Failure to incorporate the latest changes into service manuals
D. Knowledgeable personnel transferred to other products
E. All of the above

14. If we consider recovery software and procedural problems as human error, human error can account for what percentage of outage and downtime problems?

a. Outage frequency, percent of events/crashes: A. 38 B. 55 C. 62
b. Downtime (3.5 min), percent per year per machine: A. 42 B. 51 C. 68

15. As a benchmark for quality operations to achieve product reliability, what is a reasonable goal (failures per week) for a mature plant?

A. 3.0 B. 1.0 C. 0.5

16. While the MTBF is between T(min) and T(spec), progress is tracked toward what goal?

A. T(design) B. T(spec) C. T(intrinsic)

17. The key to reliability growth is to have the growth measurement encompass

A. The design phase
B. The manufacturing phase
C. The testing phase
D. The user phase
E. The entire life cycle of the product

18. For a No. 4 ESS system in the field-test phase the number of interrupts per day can be

A. <20 B. >20 C. 40

19. An electronic system must achieve manufacturing, shipping, and installation maturity in what period of time (months) to gain and keep market share?

a. Small system: A. 1 B. 2 C. 3
b. Medium system: A. 4 B. 6 C. 12
c. Large system: A. 12 B. 8 C. 16

Appendix A

Reliabil:ty Information

The figures and tables in this appendix provide reference data to support chapters 2 to 6. For the most part these data are self-explanatory.

Figure A-1 contains operating failure rates for military standard parts. They relate to electronic, electromechanical, and some mechanical parts and are useful in making approximate reliability predictions as discussed in chapter 3. Their use, limitations, and validity are explained in chapter 4.

Figure A-2 provides failure rate information for making approximate reliability predictions for systems that use established-reliability parts, such as air- and ground-launched vehicles, airborne and critical ground support equipment, piloted aircraft, and orbiting satellites. The use of this figure is discussed in chapter 4.

Figure A-3 shows the relationship of operating application factor to nonoperating application factor. These data can be used to adjust failure rates for the mission condition. The use of this figure is also discussed in chapter 4.

Figure A-4 contains reliability curves for interpreting the results of attribute tests. They provide seven confidence levels, from 50 percent to 99 percent; and six test failure levels, from 0 to 5 failures. The use of these figures is discussed in chapter 5.

Table A-1 contains values of the negative exponential function e^{-x}, where $-x$ varies from 0 to -0.1999. The tabulated data make it easy to look up the reliability, where the product of failure rate λ (or 1/MTBF) and operating time t are substituted for $-x$. Use of this table is discussed in chapter 3 and frequently referred to in chapters 4 to 6.

Table A-2 contains tolerance factors for calculating the results of mean-time-between-failure tests. It provides seven confidence levels, from 50 to 99 percent for 0 to 15 observed failures. The use of this table is explained in the table. Examples are discussed in chapter 6.

Tables A-3 to A-5 contain tabulated data for safety margins, probability, sample size, and test-demonstrated safety margins for tests to failure. They provide three confidence levels, from 90 to 99 percent, and sample sizes from 5 to 100. Values similar to these are presented on the safety margin side of the reliability slide rule; the slide rule provides six confidence levels and sample sizes from 5 to 80. The use of these tables and the slide rule is discussed in chapter 6.

More information on this subject can be found in references A-1 and A-2.

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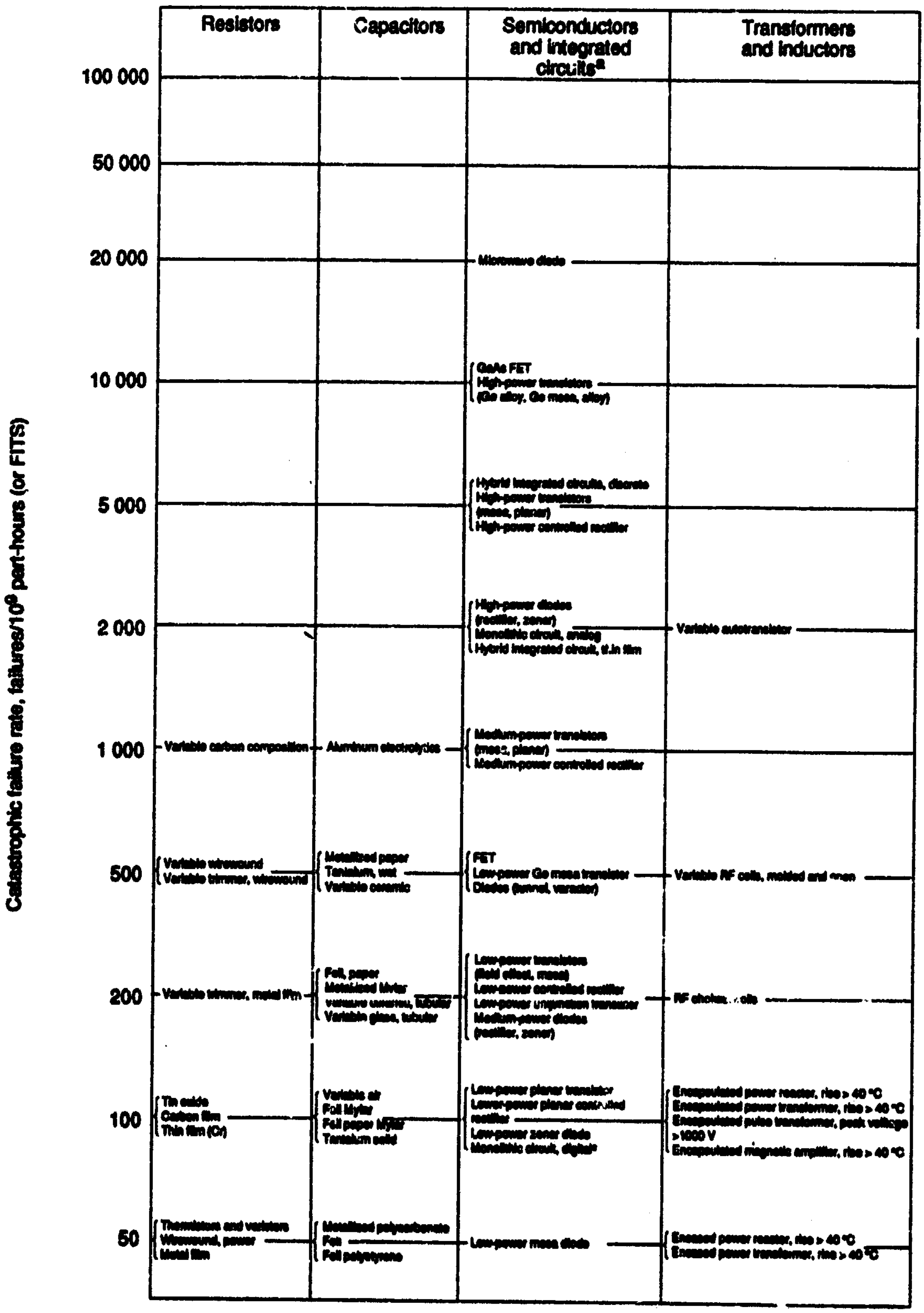
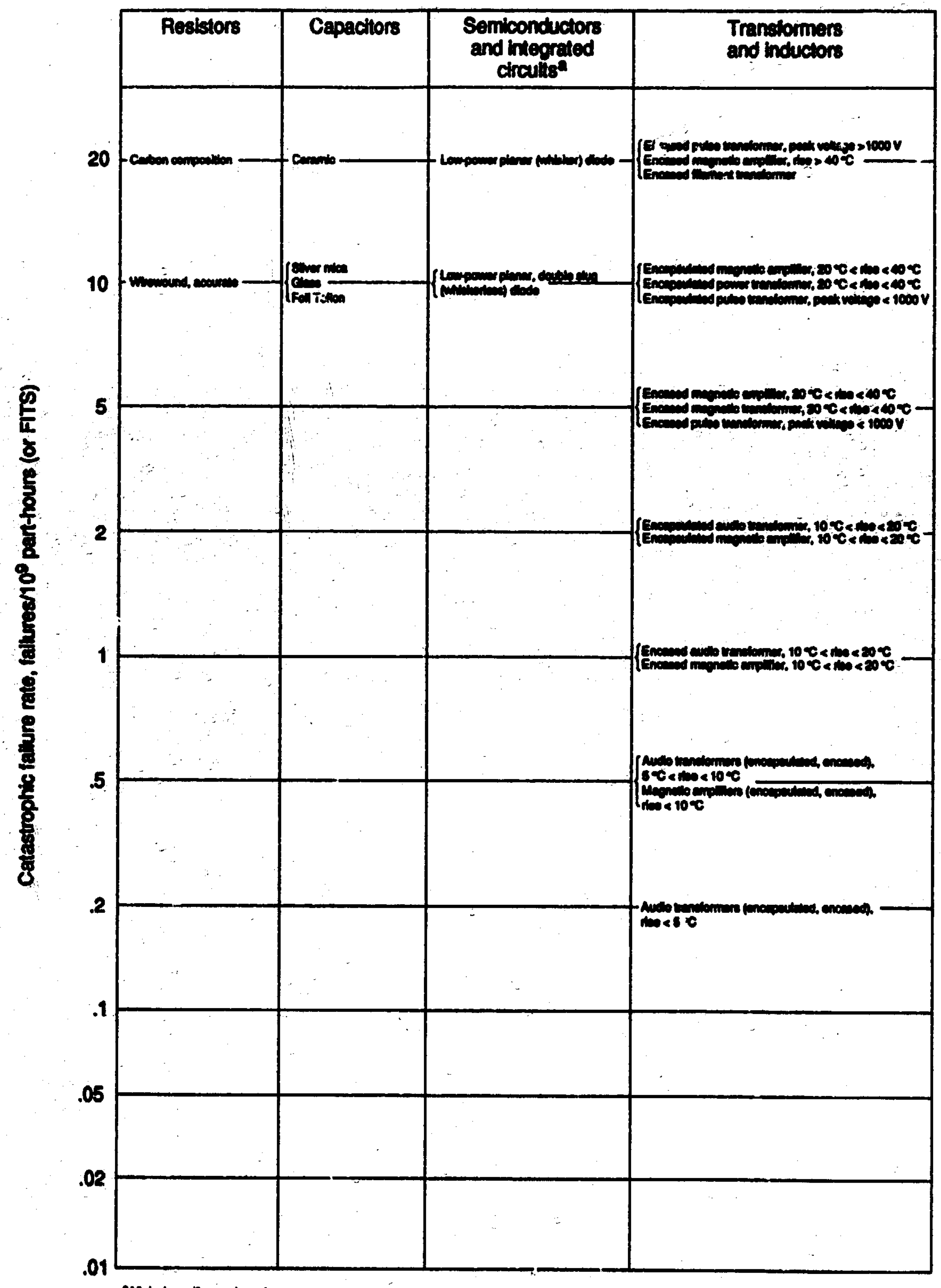


[a]All devices silicon unless shown.
[b]FITS per male-female pin connection.
[c]One circuit contains three transistors and four resistors.

Figure A-1.—Military standard catastrophic failure rates—operating mode. by dividing these values by the application factor shown in figure A-3.

Electromechanical rotating devices	Switches and relays	Connectors	Hydraulics	Hardware
	Acceleration timer switch			
	Thermal timer			
Electromechanical timers Motors (ac, dc), panel, D'Arsonval Synchronous devices, brush type	Thermal switch Stepper, Ledex, industrial			
dc torquers ac motor tachometers ac servomechanisms	Stepper, telephone type Electronic timers	Coaxial quick, float, solder		
	Dry circuit relay Sensitive relay, opening <100 mW	Coaxial bayonet, float, solder Coaxial quick, float, crimp Tape		
dc motors, power dc generators	Circuit breaker Contactor, load >10A Pushbutton switch One-sixth-size crystal can relay	Coaxial threaded, float, solder Coaxial bayonet, float, crimp	Dynamic seals	
ac motors, induction, power ac synchronous motors, power ac generators	Precision switch, limit Rotary switch Toggle switch Half-size crystal can relay	Coaxial quick, captive, solder Coaxial threaded, float, crimp Signal rectifier, solder, high density	Pumps/motors Electrohydraulic transducers Electrohydraulic servomechanisms Accumulators Actuators	
	Reed relay	Coaxial quick, captive, crimp Coaxial bayonet, captive, solder Signal circular, solder, high density		Bearings Isolators, rubber
	Microminiature crystal can relay, opening 250 mW	Coaxial bayonet, captive, crimp Coaxial threaded, captive, solder Signal rectifier, crimp, high density Signal rectifier, solder, miniature Signal edge, crimp Power rectifier, solder, blind mate		Clutches
	Squib switch	Coaxial threaded, captive, crimp Signal circular, solder, miniature Signal circular, crimp, high density Signal pin socket, solder Power rectifier, crimp, blind mate Power rectifier, solder, screw lock	Regulators Solenoid-operated valves	Explosive nuts and bolts

The failure rate for these parts in the nonoperating mode can be estimated
(From ref. A-1.)

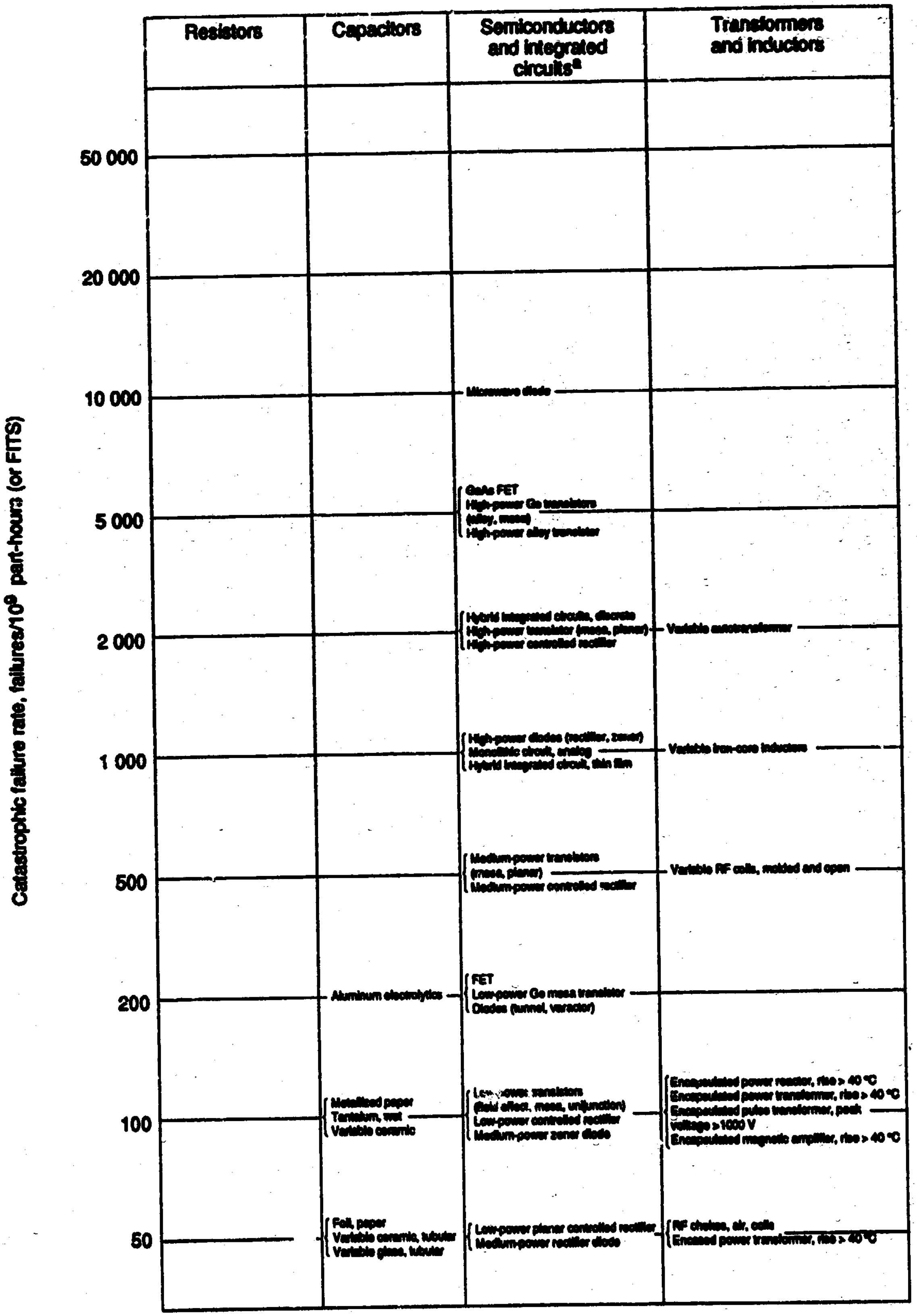


[a]All devices silicon unless shown.
[b]FITS per male-female pin connection.

Figure A-1.—

Electromechanical rotating devices	Switches and relays	Connectors[b]	Hydraulics	Hardware
	Mercury-wetted relay	Signal rectifier, crimp, miniature Signal pin socket, crimp Power, bayonet, solder Power, threaded, solder	Self-operating valves	Gears
	Mercury switch	Signal circular, crimp, miniature Signal edge, solder Power rectifier, crimp, screw lock	Manual valves Fittings, tubing (flexible)	Strap assemblies Slings
		Power, bayonet, crimp Power, threaded, crimp	Check valves Relief valves Static seals	Containers, sealed
				O-ring pins O-ring fasteners Blind rivets Inserts Permanent fasteners
			Reservoirs Gages Fluids, lubricants Fittings, tubing (rigid) Filters	
		Connection, one: solder, weld, or wirewrap		Bolts and screws, structural Nuts Clamps and couplings
				Bolts and screws, nonstructural
				Solid rivets

Concluded.



[a]All devices silicon unless shown.
[b]FITS per male-female pin connection.

Figure A-2.—High-reliability catastrophic failure rates—operating mode. The less than the values shown. (From ref. A-1.)

Electromechanical rotating devices	Switches and relays	Connectors[b]	Hydraulics	Hardware
	Acceleration timer switch			
	Thermal timer			
Electromechanical timers Motors (ac, dc), panel, D'Arsonval Synchronous devices, brush type	Thermal switch Stepper, Ledex, industrial			
dc torquers ac motor tachometers ac servomechanisms	Stepper, telephone type Electronic timers			
	Dry circuit relay Sensitive relay, opening < 100 mW	Coaxial quick, float, solder		
dc motors, power dc generators	Circuit breaker Contactor, load >10 A Push-button switch One-sixth-size crystal can relay	Coaxial bayonet, float, solder Coaxial quick, float, crimp Tape	Dynamic seals	
ac motors, induction, power type ac synchronous motors, power type ac generators	Precision switch, limit Rotary switch Toggle switch Half-size crystal can relay	Coaxial threaded, float, solder Coaxial bayonet, float, crimp	Pumps/motors Electrohydraulic transducers Electrohydraulic servomechanisms Accumulators Actuators	
	Reed relay	Coaxial quick, captive, solder Coaxial threaded, float, crimp Signal rectifier, solder, high density		Isolators, rubber Bearings
	Microminiature crystal can relay, opening <250 mW	Coaxial quick, captive, crimp Coaxial bayonet, captive, solder Signal circular, solder, high density		Clutches
	Squib switch	Coaxial bayonet, captive, crimp Coaxial threaded, captive, solder Signal rectifier, crimp, high density Signal rectifier, solder, miniature Signal edge, crimp Power rectifier, solder, blind mate	Regulators Solenoid-operated valves	Explosive nuts and bolts

failure rate for these parts in the nonoperating mode is about a factor of 10

Catastrophic failure rate, failures/10^9 part-hours (or FITS)

Failure rate	Resistors	Capacitors	Semiconductors and integrated circuits[a]	Transformers and inductors
20	Variable carbon composition	Variable air Foil paper Mylar Metallized Mylar	Low-power diodes (mesa, zener) Monolithic circuit, digital[c]	Encased pulse transformer, peak voltage >1000 V Encased magnetic amplifier, rise > 40 °C Encased power reactor, rise > 40 °C
10	Variable wirewound Variable trimmer, wirewound	Tantalum solid Metallized polycarbonate Foil Foil polystyrene Foil Mylar	Low-power planar transformer	Encapsulated magnetic amplifier, 20 °C < rise < 40 °C Encapsulated power transformer, 20 °C < rise < 40 °C Encapsulated pulse transformer, peak voltage < 1000 V Encapsulated filament transformer, peak voltage < 1000 V
5	Variable trimmer, metal film Wirewound, power and accurate	Ceramic Silver mica Glass Foil Teflon	Low-power planar diodes (whisker and double-slug whiskerless)	Encased magnetic amplifier, 20 °C < rise < 40 °C Encased power transformer, 20 °C < rise < 40 °C Encased pulse transformer, peak voltage < 1000 V
2	Tin oxide Carbon film Metal film			Encapsulated audio transformer, 10 °C < rise < 20 °C Encapsulated magnetic amplifier, 10 °C < rise < 20 °C
1	Thermistors and varistors, Carbon composition			Encased audio transformer, 10 °C < rise < 20 °C Encased magnetic amplifier, 10 °C < rise < 20 °C
.5				Audio transformers (encapsulated, encased), 5 °C < rise < 10 °C Magnetic amplifiers (encapsulated, encased), rise < 10 °C
.2				Audiotransformers (encapsulated, encased), rise < 5 °C
.1				
.05				
.02				

[a]All devices silicon unless shown.
[b]FITS per male-female pin connection.
[c]One circuit contains three resistors and four transistors.

Figure A-2.—

Electromechanical rotating devices	Switches and relays	Connectors[b]	Hydraulics	Hardware
	Mercury-wetted relay	Coaxial threaded, captive, crimp Signal circular, solder, miniature Signal circular, crimp, high density Signal, pin socket, solder Power rectifier, crimp, blind mate Power rectifier, solder, screw lock	Self-operating valves	Gears
	Mercury switch	Signal rectifier, crimp, miniature Signal, pin socket, crimp Power, bayonet, solder Power, threaded, solder	Manual valves Fittings; tubing (flexible)	Strap assemblies Slings
		Signal edge, solder Power rectifier, crimp, screw lock	Check valves Relief valves Static seals	Containers, sealed
		Power, bayonet, crimp Power, threaded, crimp Signal circular, crimp, miniature		O-ring pins O-ring fasteners Blind rivets Inserts Permanent fasteners
			Reservoir Gages Fluids, lubricants Fittings; tubing (rigid) Filters	
		Connection, one: solder, weld, or wirewrap		Bolts and screws, structural Nuts Clamps and couplings
				Bolts and screws, nonstructural
				Solid rivets

Concluded.

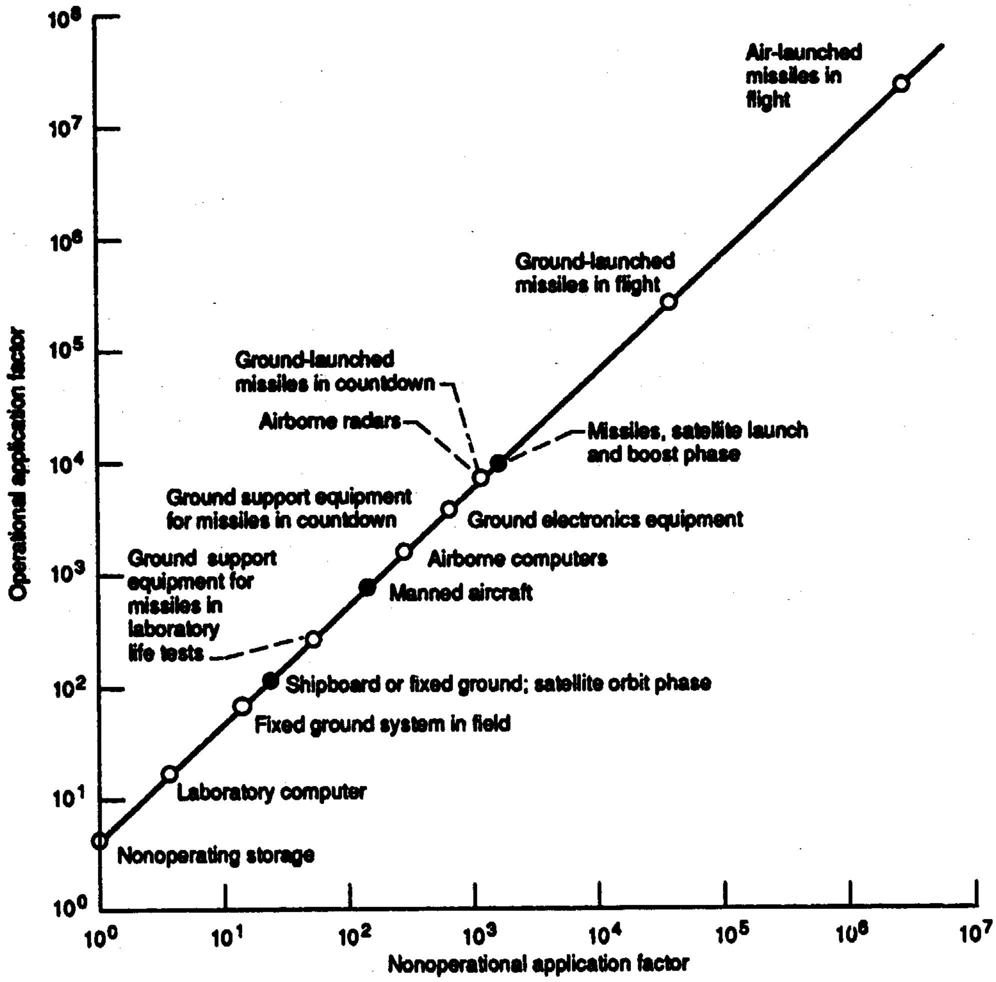
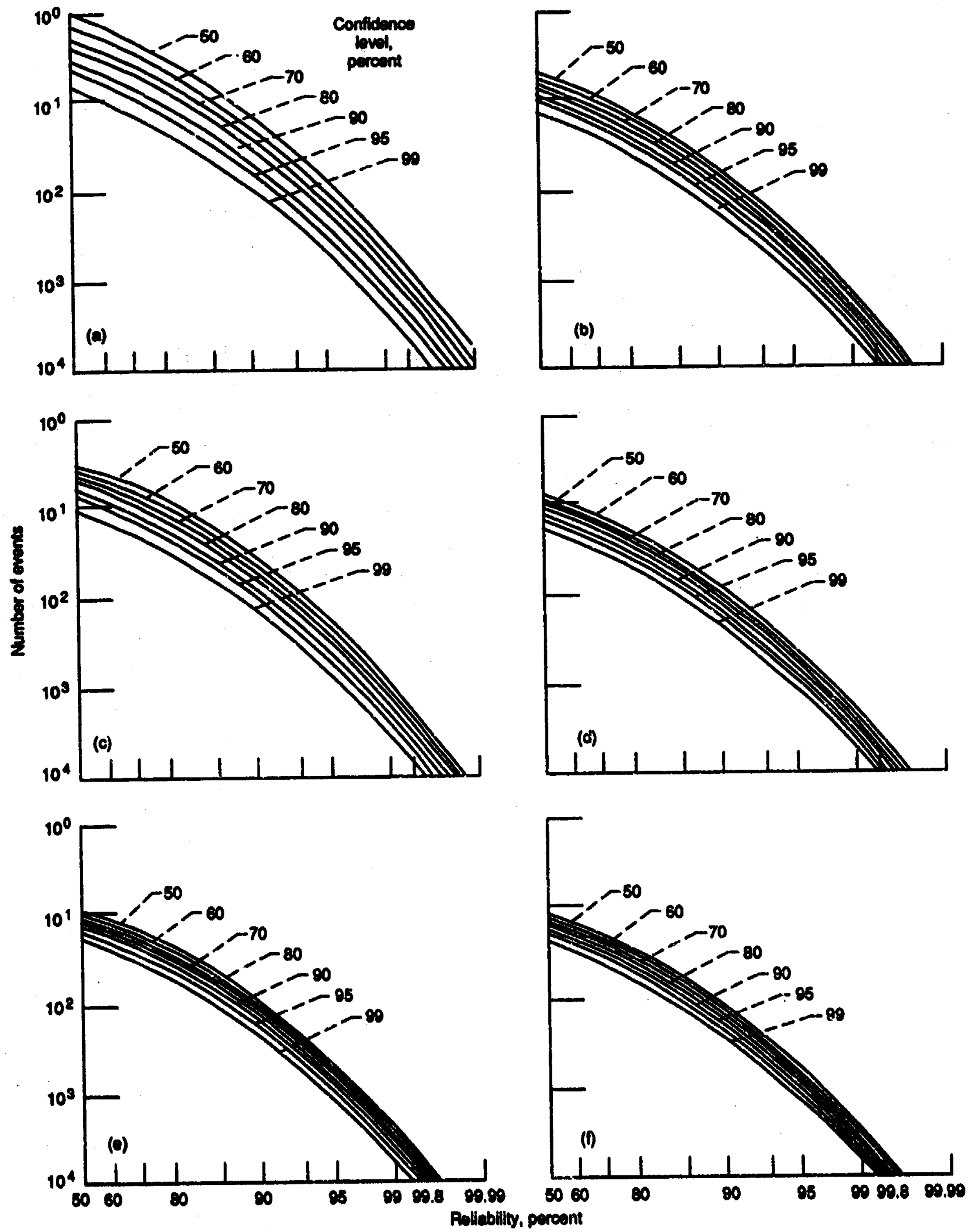


Figure A-3.—Application factor comparison for nonoperating storage of military standard electronic parts. MIL-STD-756 points (solid symbols) are given for comparison. (From ref. A-2.)



(a) When no failures are observed.
(b) When one failure is observed.
(c) When two failures are observed.
(d) When three failures are observed.
(e) When four failures are observed.
(f) When five failures are observed.

Figure A-4.—Confidence curves for attribute testing. (From ref. A-2.)

TABLE A-1.—VALUES OF NEGATIVE EXPONENTIAL FUNCTION e^{-x}

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.0000	1.00000	0.0050	0.99501	0.0100	0.99005	0.0150	0.98511	0.0200	0.98020	0.0250	0.97531
.0001	.99990	.0051	.99491	.0101	.98995	.0151	.98501	.0201	.98010	.0251	.97521
.0002	.99980	.0052	.99481	.0102	.98985	.0152	.98491	.0202	.98000	.0252	.97511
.0003	.99970	.0053	.99471	.0103	.98975	.0153	.98482	.0203	.97990	.0253	.97502
.0004	.99960	.0054	.99461	.0104	.98965	.0154	.98472	.0204	.97981	.0254	.97492
0.0005	0.99950	0.0055	0.99452	0.0105	0.98955	0.0155	0.98462	0.0205	0.97971	0.0255	0.97482
.0006	.99940	.0056	.99442	.0106	.98946	.0156	.98452	.0206	.97961	.0256	.97472
.0007	.99930	.0057	.99432	.0107	.98936	.0157	.98442	.0207	.97951	.0257	.97463
.0008	.99920	.0058	.99422	.0108	.98926	.0158	.98432	.0208	.97941	.0258	.97453
.0009	.99910	.0059	.99412	.0109	.98916	.0159	.98423	.0209	.97932	.0259	.97443
0.0010	0.99900	0.0060	0.99402	0.0110	0.98906	0.0160	0.98413	0.0210	0.97922	0.0260	0.97434
.0011	.99890	.0061	.99392	.0111	.98896	.0161	.98403	.0211	.97912	.0261	.97424
.0012	.99880	.0062	.99382	.0112	.98886	.0162	.98393	.0212	.97902	.0262	.97414
.0013	.99870	.0063	.99372	.0113	.98876	.0163	.98383	.0213	.97893	.0263	.97404
.0014	.99860	.0064	.99362	.0114	.98866	.0164	.98373	.0214	.97883	.0264	.97395
0.0015	0.99850	0.0065	0.99352	0.0115	0.98857	0.0165	0.98364	0.0215	0.97873	0.0265	0.97385
.0016	.99840	.0066	.99342	.0116	.98847	.0166	.98354	.0216	.97863	.0266	.97375
.0017	.99830	.0067	.99332	.0117	.98837	.0167	.98344	.0217	.97853	.0267	.97365
.0018	.99820	.0068	.99322	.0118	.98827	.0168	.98334	.0218	.97844	.0268	.97356
.0019	.99810	.0069	.99312	.0119	.98817	.0169	.98324	.0219	.97834	.0269	.97346
0.0020	0.99800	0.0070	0.99302	0.0120	0.98807	0.0170	0.98314	0.0220	0.97824	0.0270	0.97336
.0021	.99790	.0071	.99293	.0121	.98797	.0171	.98305	.0221	.97814	.0271	.97326
.0022	.99780	.0072	.99283	.0122	.98787	.0172	.98295	.0222	.97804	.0272	.97317
.0023	.99770	.0073	.99273	.0123	.98777	.0173	.98285	.0223	.97795	.0273	.97307
.0024	.99760	.0074	.99263	.0124	.98767	.0174	.98275	.0224	.97785	.0274	.97297
0.0025	0.99750	0.0075	0.99253	0.0125	0.98757	0.0175	0.98265	0.0225	0.97775	0.0275	0.97287
.0026	.99740	.0076	.99243	.0126	.98747	.0176	.98255	.0226	.97765	.0276	.97278
.0027	.99730	.0077	.99233	.0127	.98738	.0177	.98246	.0227	.97756	.0277	.97268
.0028	.99720	.0078	.99223	.0128	.98728	.0178	.98236	.0228	.97746	.0278	.97258
.0029	.99710	.0079	.99213	.0129	.98718	.0179	.98226	.0229	.97736	.0279	.97249
0.0030	0.99700	0.0080	0.99203	0.0130	0.98708	0.0180	0.98216	0.0230	0.97726	0.0280	0.97239
.0031	.99690	.0081	.99193	.0131	.98699	.0181	.98206	.0231	.97716	.0281	.97229
.0032	.99681	.0082	.99183	.0132	.98689	.0182	.98196	.0232	.97707	.0282	.97219
.0033	.99671	.0083	.99173	.0133	.98679	.0183	.98187	.0233	.97697	.0283	.97210
.0034	.99661	.0084	.99164	.0134	.98669	.0184	.98177	.0234	.97687	.0284	.97200
0.0035	0.99651	0.0085	0.99154	0.0135	0.98659	0.0185	0.98167	0.0235	0.97677	0.0285	0.97190
.0036	.99641	.0086	.99144	.0136	.98649	.0186	.98157	.0236	.97668	.0286	.97181
.0037	.99631	.0087	.99134	.0137	.98639	.0187	.98147	.0237	.97658	.0287	.97171
.0038	.99621	.0088	.99124	.0138	.98629	.0188	.98138	.0238	.97648	.0288	.97161
.0039	.99611	.0089	.99114	.0139	.98620	.0189	.98128	.0239	.97638	.0289	.97151
0.0040	0.99601	0.0090	0.99104	0.0140	0.98610	0.0190	0.98118	0.0240	0.97629	0.0290	0.97142
.0041	.99591	.0091	.99094	.0141	.98600	.0191	.98108	.0241	.97619	.0291	.97132
.0042	.99581	.0092	.99084	.0142	.98590	.0192	.98098	.0242	.97609	.0292	.97122
.0043	.99571	.0093	.99074	.0143	.98580	.0193	.98089	.0243	.97599	.0293	.97113
.0044	.99561	.0094	.99064	.0144	.98570	.0194	.98079	.0244	.97590	.0294	.97103
0.0045	0.99551	0.0095	0.99054	0.0145	0.98560	0.0195	0.98069	0.0245	0.97580	0.0295	0.97093
.0046	.99541	.0096	.99045	.0146	.98551	.0196	.98059	.0246	.97570	.0296	.97083
.0047	.99531	.0097	.99035	.0147	.98541	.0197	.98049	.0247	.97560	.0297	.97074
.0048	.99521	.0098	.99025	.0148	.98531	.0198	.98039	.0248	.97550	.0298	.97064
.0049	.99511	.0099	.99015	.0149	.98521	.0199	.98030	.0249	.97541	.0299	.97054

TABLE A-1.—Continued.

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.0300	0.97045	0.0350	0.96561	0.0400	0.96079	0.0450	0.95600	0.0500	0.95123	0.0550	0.94649
.0301	.97035	.0351	.96551	.0401	.96069	.0451	.95590	.0501	.95113	.0551	.94639
.0302	.97025	.0352	.96541	.0402	.96060	.0452	.95581	.0502	.95104	.0552	.94630
.0303	.97015	.0353	.96531	.0403	.96050	.0453	.95571	.0503	.95094	.0553	.94620
.0304	.97006	.0354	.96522	.0404	.96041	.0454	.95562	.0504	.95085	.0554	.94611
0.0305	0.96996	0.0355	0.96512	0.0405	0.96031	0.0455	0.95552	0.505	0.95075	0.0555	0.94601
.0306	.96986	.0356	.96503	.0406	.96021	.0456	.95542	.0506	.95066	.0556	.94592
.0307	.96977	.0357	.96493	.0407	.96012	.0457	.95533	.0507	.95056	.0557	.94582
.0308	.96967	.0358	.96483	.0408	.96002	.0458	.95523	.0508	.95047	.0558	.94573
.0309	.96957	.0359	.96474	.0409	.95993	.0459	.95514	.0509	.95037	.0559	.94563
0.0310	0.96948	0.0360	0.96464	0.0410	0.95983	0.0460	0.95504	0.0510	0.95028	0.0560	0.94554
.0311	.96938	.0361	.96454	.0411	.95973	.0461	.95495	.0511	.95018	.0561	.94544
.0312	.96928	.0362	.96445	.0412	.95964	.0462	.95485	.0512	.95009	.0562	.94535
.0313	.96918	.0363	.96435	.0413	.95954	.0463	.95476	.0513	.94999	.0563	.94526
.0314	.96909	.0364	.96425	.0414	.95945	.0464	.95466	.0514	.94990	.0564	.94516
0.0315	0.96899	0.0365	0.96416	0.0415	0.95935	0.0465	0.95456	0.0515	0.94980	0.0565	0.94507
.0316	.96889	.0366	.96406	.0416	.95925	.0466	.95447	.0516	.94971	.0566	.94488
.0317	.96879	.0367	.96397	.0417	.94916	.0467	.95437	.0517	.94961	.0567	.94488
.0318	.96870	.0368	.96387	.0418	.95906	.0468	.95428	.0518	.94952	.0568	.94478
.0319	.96860	.0369	.96377	.0419	.95897	.0469	.95418	.0519	.94942	.0569	.94469
0.0320	0.96851	0.0370	0.96368	0.0420	0.95887	0.0470	0.95409	0.0520	0.94933	0.0570	0.94450
.0321	.96841	.0371	.96358	.0421	.95877	.0471	.95399	.0521	.94923	.0571	.94450
.0322	.96831	.0372	.96348	.0422	.95868	.0472	.95390	.0522	.94914	.0572	.94441
.0323	.96822	.0373	.96339	.0423	.95858	.0473	.95380	.0523	.94904	.0573	.94431
.0324	.96812	.0374	.96329	.0424	.95849	.0474	.95371	.0524	.94895	.0574	.94422
0.0325	0.96802	0.0375	0.96319	0.0425	0.95839	0.0475	0.95361	0.0525	0.94885	0.0575	0.94412
.0326	.96793	.0376	.96310	.0426	.95829	.0476	.95352	.0526	.94876	.0576	.94403
.0327	.96783	.0377	.96300	.0427	.95820	.0477	.95342	.0527	.94866	.0577	.94393
.0328	.96773	.0378	.96291	.0428	.95810	.0478	.95332	.0528	.94857	.0578	.94384
.0329	.96764	.0379	.96281	.0429	.95801	.0479	.95323	.0529	.94847	.0579	.94374
0.0330	0.96754	0.0380	0.96271	0.0430	0.95791	0.0480	0.95313	0.0530	0.94838	0.0580	0.94365
.0331	.96744	.0381	.96262	.0431	.94782	.0481	.95304	.0531	.94829	.0581	.94356
.0332	.96735	.0382	.96252	.0432	.95772	.0482	.95294	.0532	.94819	.0582	.94346
.0333	.96725	.0383	.96242	.0433	.95762	.0483	.95285	.0533	.94810	.0583	.94337
.0334	.96715	.0384	.96233	.0434	.95753	.0484	.95275	.0534	.94800	.0584	.94327
0.0335	0.96705	0.0385	0.96223	0.0435	0.95743	0.0485	0.95266	0.0535	0.94791	0.0585	0.94318
.0336	.96696	.0386	.96214	.0436	.95734	.0486	.95256	.0536	.94781	.0586	.94308
.0337	.96686	.0387	.96204	.0437	.95724	.0487	.95247	.0537	.94772	.0587	.94299
.0338	.96676	.0388	.96194	.0438	.95715	.0488	.95237	.0538	.94762	.0588	.94289
.0339	.96667	.0389	.96185	.0439	.95705	.0489	.95228	.0539	.94753	.0589	.94280
0.0340	0.96657	0.0390	0.96175	0.0440	0.95695	0.0490	0.95218	0.0540	0.94743	0.0590	0.94271
.0341	.96647	.0391	.96165	.0441	.95686	.0491	.95209	.0541	.94734	.0591	.94261
.0342	.96638	.0392	.96156	.0442	.95676	.0492	.95199	.0542	.94724	.0592	.94252
.0343	.96628	.0393	.96146	.0443	.95667	.0493	.95190	.0543	.94715	.0593	.94242
.0344	.96618	.0394	.96137	.0444	.95657	.0494	.95180	.0544	.94705	.0594	.94233
0.0345	0.96609	0.0395	0.96127	0.0445	0.95648	0.0495	0.95171	0.0545	0.94696	0.0595	0.94224
.0346	.96599	.0396	.96117	.0446	.95638	.0496	.95161	.0546	.94686	.0596	.94214
.0347	.96590	.0397	.96108	.0447	.95628	.0497	.95151	.0547	.94677	.0597	.94205
.0348	.96580	.0398	.96098	.0448	.95619	.0498	.95142	.0548	.94667	.0598	.94195
.0349	.96570	.0399	.96089	.0449	.95609	.0499	.95132	.0549	.94658	.0599	.94186

TABLE A-1.—Continued.

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.0600	0.94176	0.0650	0.93707	0.0700	0.93239	0.0750	0.92774	0.0800	0.92312	0.0850	0.91851
.0601	.94167	.0651	.93697	.0701	.93230	.0751	.92765	.0801	.92302	.0851	.91842
.0602	.94158	.0652	.93688	.0702	.93221	.0752	.92756	.0802	.92293	.0852	.91833
.0603	.94148	.0563	.93679	.0703	.93211	.0753	.92747	.0803	.92284	.0853	.91824
.0604	.94139	.0654	.93669	.0704	.93202	.0754	.92737	.0804	.92275	.0854	.91814
0.0605	0.94129	0.0655	0.93660	0.0705	0.93193	0.0755	0.92728	0.0805	0.92265	0.0855	0.91805
.0606	.94120	.0656	.93651	.0706	.93183	.0756	.92719	.0806	.92256	.0856	.91796
.0607	.94111	.0657	.93641	.0707	.93174	.0757	.92709	.0807	.92247	.0857	.91787
.0608	.94101	.0658	.93632	.0708	.93165	.0758	.92700	0808	.92238	.0858	.91778
.0609	.94092	.0659	.93622	.0709	.93156	.0759	.92691	.0809	.92229	.0859	.91769
0.0610	0.94082	0.0660	0.93613	0.0710	0.93146	0.0760	0.92682	0.0810	0.92219	0.0860	0.91759
.0611	.94073	.0661	.93604	.0711	.93137	.0761	.92672	.0811	.92210	.0861	.91750
.0612	.94064	.0662	.93594	.0712	.93128	.0762	.92663	.0812	.92201	.0862	.91741
.0613	.94054	.0663	.93585	.0713	.93118	.0763	.92654	.0813	.92191	.0863	.91732
.0614	.94045	.0664	.93576	.0714	.93109	.0764	.92645	.0814	.92182	.0864	.91723
0.0615	0.94035	0.0665	0.93566	0.0715	0.93100	0.0765	0.92635	0.0815	0.92173	0.0865	0.91714
.0616	.94026	.0666	.93557	.0716	.93090	.0766	.92626	.0816	.92164	.0866	.91704
.0617	.94016	.0667	.93548	.0717	.93081	.0767	.92617	.0817	.92155	.0867	.91695
.0618	.94007	.0668	.93538	.0718	.93072	.0768	.92608	.0818	.92146	.0868	.91686
.0619	.93998	.0669	.93529	.0719	.93062	.0769	.92598	.0819	.92136	.0869	.91677
0.0620	0.93988	0.0670	0.93520	0.0720	0.93053	0.0770	0.92589	0.0820	0.92127	0.0870	0.91668
.0621	.93979	.0671	.93510	.0721	.93044	.0771	.92580	.0821	.92118	.0871	.91659
.0622	.93969	.0672	.93501	.0722	.93034	.0772	.92570	.0822	.92109	.0872	.91649
.0623	.93960	.0673	.93491	.0723	.93025	.0773	.92561	.0823	.92100	.0873	.91640
.0624	.93951	.0674	.93482	.0724	.93016	.0774	.92552	.0824	.92090	.0874	.91631
0.0625	0.93941	0.0675	0.93473	0.0725	0.93007	0.0775	0.92543	0.0825	0.92081	0.0875	0.91622
.0626	.93932	.0676	.93463	.0726	.92997	.0776	.92533	.0826	.92072	.0876	.91613
.0627	.93923	.0677	.93454	.0727	.92988	.0777	.92524	.0827	.92063	.0877	.91604
.0628	.93913	.0678	.93445	.0728	.92979	.0778	.92515	.0828	.92054	.0878	.91594
.0629	.93904	.0679	.93435	.0729	.92969	.0779	.92506	.0829	.92044	.0879	.91585
0.0630	0.93894	0.0680	0.93425	0.0730	0.92960	0.0780	0.92496	0.0830	0.92035	0.0880	0.91576
.0631	.93885	.0681	.93417	.0731	.92951	.0781	.92487	.0831	.92026	.0881	.91567
.0632	.93876	.0682	.93407	.0732	.92941	.0782	.92478	.0832	.92019	.0882	.91558
.0633	.93866	.0683	.93398	.0733	.92932	.0783	.92469	.0833	.92008	.0883	.91549
.0634	.93857	.0684	.93389	.0734	.92923	.0784	.92459	.0834	.91998	.0884	.91539
0.0635	0.93847	0.0685	0.93379	0.0735	0.92914	0.0785	0.92450	0.0835	0.91989	0.0885	0.91530
.0636	.93838	.0686	.93370	.0736	.92904	.0786	.92441	.0836	.91980	.0886	.91521
.0637	.93829	.0687	.93361	.0737	.92895	.0787	.92432	.0837	.91971	.0887	.91512
.0638	.93819	.0688	.93351	.0738	.92886	.0788	.92422	.0838	.91962	.0888	.91503
.0639	.93810	.0689	.93342	.0739	.92876	.0789	.92413	.0839	.91952	.0889	.91494
0.0640	0.93800	0.0690	0.93333	0.0740	0.92867	0.0790	0.92404	0.0840	0.91943	0.0890	0.91485
.0641	.93791	.0691	.93323	.0741	.92858	.0791	.92395	.0841	.91934	.0891	.91475
.0642	.93782	.0692	.93314	.0742	.92849	.0792	.92386	.0842	.91925	.0892	.91466
.0643	.93772	.0693	.93305	.0743	.92839	.0793	.92376	.0843	.91916	.0893	.91457
.0644	.93763	.0694	.93295	.0744	.92830	.0794	.92367	.0844	.91906	.0894	.91448
0.0645	0.93754	0.0695	0.93286	0.0745	0.92921	0.0795	0.92358	0.0845	0.91897	0.0895	0.91439
.0646	.93744	.0696	.93277	.0746	.92811	.0796	.92349	.0846	.91888	.0896	.91430
.0647	.93735	.0697	.93267	.0747	.92802	.0797	.92339	.0847	.91879	.0897	.91421
.0648	.93725	.0698	.93258	.0748	.92793	.0798	.92330	.0848	.91870	.0898	.91411
.0649	.93716	.0699	.93249	.0749	.92784	.0799	.92321	.0849	.91860	.0899	.91402

TABLE A-1.—Continued.

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.0900	0.91393	0.0950	0.90937	0.1000	0.90484	0.1050	0.90032	0.1100	0.89583	0.1150	0.89137
.0901	.91384	.0951	.90928	.1001	.90475	.1051	.90023	.1101	.89574	.1151	.89128
.0902	.91375	.0952	.90919	.1002	.90466	.1052	.90014	.1102	.89565	.1152	.89119
.0903	.91366	.0953	.90910	.1003	.90457	.1053	.90005	.1103	.89557	.1153	.89110
.0904	.91357	.0954	.90901	.1004	.90448	.1054	.89996	.1004	.89548	.1154	.89101
0.0905	0.91347	0.0955	0.90892	0.1005	0.90439	0.1055	0.89987	0.1105	0.89539	0.1155	0.89092
.0906	.91338	.0956	.90883	.1006	.90429	.1056	.89978	.1106	.89530	.1156	.89083
.0907	.91329	.0957	.90874	.1007	.90420	.1057	.89969	.1107	.89521	.1157	.89074
.0908	.91320	.0958	.90865	.1008	.90411	.1058	.89960	.1108	.89512	.1158	.89065
.0909	.91311	.0959	.90855	.1009	.90402	.1059	.89951	.1109	.89503	.1159	.89056
0.0910	0.91302	0.0960	0.90846	0.1010	0.90393	0.1060	0.89942	0.1110	0.89494	0.1160	0.89048
.0911	.91293	.0961	.90837	.1011	.90384	.1061	.89933	.1111	.89485	.1161	.89039
.0912	.91284	.0962	.90828	.1012	.90375	.1062	.89924	.1112	.89476	.1162	.89030
.0913	.91274	.0963	.90819	.1013	.90366	.1063	.89915	.1113	.89467	.1163	.89021
.0914	.91265	.0964	.90810	.1014	.90357	.1064	.89906	.1114	.89458	.1164	.89012
0.0915	0.91256	0.0965	0.90801	0.1015	0.90348	0.1065	0.89898	0.1115	0.89449	0.1165	0.89003
.0916	.91247	.0966	.90792	.1016	.90339	.1066	.89889	.1116	.89440	.1166	.88994
.0917	.91238	.0967	.90783	.1017	.90330	.1067	.89880	.1117	.89431	.1167	.88985
.0918	.91229	.0968	.90774	.1018	.90321	.1068	.89871	.1118	.89422	.1168	.88976
.0919	.9122[illegible]	.0969	.90765	.1019	.90312	.1069	.89862	.1119	.89413	.1169	.88967
0.0920	0.92111	0.0970	0.90756	0.1020	0.90303	0.1070	0.89853	0.1120	0.89404	0.1170	0.88959
.0921	.91201	.0971	.90747	.1021	.90294	.1071	.89844	.1121	.89395	.1171	.88950
.0922	.91192	.0972	.90737	.1022	.90285	.1072	.89835	.1122	.89387	.1172	.88941
.0923	.91183	.0973	.90728	.1023	.90276	.1073	.89826	.1123	.89378	.1173	.88932
.0924	.91174	.0974	.90719	.1024	.90267	.1074	.89817	.1124	.89369	.1174	.88923
0.0925	0.91165	0.0975	0.90710	0.1025	0.90258	0.1075	0.89808	0.1125	0.89360	0.1175	0.88914
.0926	.91156	.0976	.90701	.1026	.90249	.1076	.89799	.1126	.89351	.1176	.88905
.0927	.91147	.0977	.90692	.1027	.90240	.1077	.89790	.1127	.89342	.1177	.88896
.0928	.91138	.0978	.90683	.1028	.90231	.1078	.89781	.1128	.89333	.1178	.88887
.0929	.91128	.0979	.90674	.1029	.90222	.1079	.89772	.1129	.89324	.1179	.88878
0.0930	0.91119	0.0980	0.90665	0.1030	0.90213	0.1080	0.89763	0.1130	0.89315	0.1180	0.88870
.0931	.91110	.0981	.90656	.1031	.90204	.1081	.89754	.1131	.89306	.1181	.88861
.0932	.91101	.0982	.90647	.1032	.90195	.1082	.89745	.1132	.89297	.1182	.88852
.0933	.9109[illegible]	.0983	.90638	.1033	.90186	.1083	.89736	.1133	.89288	.1183	.88843
.0934	.91083	.0984	.90629	.1034	.90177	.1084	.89727	.1134	.89279	.1184	.88834
0.0935	0.91074	0.0985	0.90620	0.1035	0.90168	0.1085	0.89718	0.1135	0.89270	0.1185	0.88825
.0936	.91065	.0986	.90611	.1036	.90159	.1086	.89709	.1136	.89261	.1186	.88816
.0937	.91056	.0987	.90601	.1037	.90150	.1087	.89700	.1137	.89253	.1187	.88807
.0938	.91046	.0988	.90592	.1038	.90141	.1088	.89691	.1138	.89244	.1188	.88799
.0939	.91037	.0989	.90583	.1039	.90132	.1089	.89682	.1139	.89235	.1189	.88790
0.0940	0.91028	0.0990	0.90574	0.1040	0.90123	0.1090	0.89673	0.1140	0.89226	0.1190	0.98781
.0941	.91019	.0991	.90565	.1041	.90114	.1091	.89664	.1141	.89217	.1191	.88772
.0942	.91010	.0992	.90556	.1042	.90105	.1092	.89655	.1142	.89208	.1192	.88763
.0943	.91001	.0993	.90547	.1043	.90095	.1093	.89646	.1143	.89199	.1193	.88754
.0944	.90992	.0994	.90538	.1044	.90086	.1094	.89637	.1144	.89190	.1194	.88745
0.0945	0.90983	0.0995	0.90529	0.1045	0.90077	0.1095	0.89628	0.1145	0.89181	0.1195	0.88736
.0946	.90974	.0996	.90520	.1046	.90068	.1096	.89619	.1146	.89172	.1196	.88728
.0947	.90965	.0997	.90501	.1047	.90059	.1097	.89610	.1147	.89163	.1197	.88719
.0948	.90955	.0998	.90502	.1048	.90050	.1098	.89601	.1148	.89154	.1198	.88710
.0949	.90946	.0999	.90493	.1049	.90041	.1099	.89592	.1149	.89146	.1199	.88701

TABLE A-1.—Continued.

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.1200	0.88692	0.1250	0.88250	0.1300	0.87810	0.1350	0.87372	0.1400	0.86936	0.1450	0.86502
.1201	.88683	.1251	.88241	.1301	.87801	.1351	.87363	.1401	.86927	.1451	.86494
.1202	.88674	.1252	.88232	.1302	.87792	.1352	.87354	.1402	.86918	.1452	.86485
.1203	.88665	.1253	.88223	.1303	.87783	.1353	.87345	.1403	.86910	.1453	.86476
.1204	.88657	.1254	.88214	.1304	.87774	.1354	.87337	.1404	.86901	.1454	.86468
0.1205	0.88648	0.1255	0.88206	0.1305	0.87766	0.1355	0 87328	0.1405	0.86892	0.1455	0.86459
.1206	.88639	.1256	.88197	.1306	.87757	.1356	.87319	.1406	.86884	.1456	.86450
.1207	.88630	.1257	.88188	.1307	.87748	.1357	.87310	.1407	.86875	.1457	.86442
.1208	.88621	.1258	.88179	.1308	.87739	.1358	.87302	.1408	.86866	.1458	.86433
.1209	.88612	.1259	.88170	.1309	.87731	.1359	.87283	.1409	.86858	.1459	.86424
0.1210	0.88603	0.1260	0.88161	0.1310	0.87722	0.1360	0.87284	0.1410	0.86849	0.1460	0.86416
.1211	.88595	.1261	.88153	.1311	.87713	.1361	.87276	.1411	.86840	.1461	.86407
.1212	.88586	.1262	.88144	.1312	.87704	.1362	.87267	.1412	.86832	.1462	.86398
.1213	.88577	.1263	.88135	.1313	.87695	.1363	.87258	.1413	.86823	.1463	.86390
.1214	.88568	.1264	.88126	.1314	.87687	.1364	.87249	.1414	.86814	.1464	.86381
0.1215	0.88559	0.1265	0.88117	0.1315	0.87678	0.1365	0.87241	0.1415	0.86806	0.1465	0.86373
.1216	.88550	.1266	.88109	.1316	.87669	.1366	.87232	.1416	.86797	.1466	.86364
.1217	.88541	.1267	.88100	.1317	.87660	.1367	.87223	.1417	.86788	.1467	.86355
.1218	.88533	.1268	.88091	.1318	.87652	.1368	.87214	.1418	.86779	.1468	.86347
.1219	.88524	.1269	.88082	.1319	.87643	.1369	.87206	.1419	.86771	.1469	.86338
0.1220	0.88515	0.1270	0.88065	0.1320	0.87634	0.1370	0.87197	0.1420	0.86762	0.1470	0.86329
.1221	.88506	.1271		.1321	.87625	.1371	.87188	.1421	.86753	.1471	.86321
.1222	.88497	.1272	.88056	.1322	.87617	.1372	.87180	.1422	.86745	.1472	.86312
.1223	.88488	.1273	.88047	.1323	.87608	.1373	.87171	.1423	.86736	.1473	.86304
.1224	.88479	.1274	.88038	.1324	.87599	.1374	.87162	.1424	.86727	.1474	.86295
0.1225	0.88471	0.1275	0.88029	0.1325	0.87590	0.1375	0.87153	0.1425	0.86719	0.1475	0.86286
.1226	.88462	.1276	.88021	.1326	.87582	.1376	.87145	.1426	.86710	.1476	.86278
.1227	.88453	.1277	.88012	.1327	.87573	.1377	.87136	.1427	.86701	.1477	.86269
.1228	.88444	.1728	.88003	.1328	.87564	.1378	.87127	.1428	.86693	.1478	.86260
.1229	.88435	.1279	.87994	.1329	.87555	.1379	.87119	.1429	.86684	.1479	.86252
0.1230	0.88426	0.1280	0.87985	0.1330	0.87547	0.1380	0.87110	0.1430	0.86675	0.1480	0.86243
.1231	.88418	.1281	.87977	.1331	.87538	.1381	.87101	.1431	.86667	.1481	.86234
.1232	.88409	.1282	.87968	.1332	.87529	.1382	.87092	.1432	.86658	.1482	.86226
.1233	.88400	.1283	.87959	.1333	.87520	.1383	.87084	.1433	.86649	.1483	.86217
.1234	.88391	.1284	.87950	.1334	.87511	.1384	.87075	.1434	.86641	.1484	.86209
0.1235	0.88382	0.1285	0.87941	0.1335	0.87503	0.1385	0.87066	0.1435	0.86632	0.1485	0.86200
.1236	.88373	.1286	.87933	.1336	.87494	.1386	.87058	.1436	.86623	.1486	.86191
.1237	.88364	.1287	.87924	.1337	.87485	.1387	.87049	.1437	.86615	.1487	.86183
.1238	.88356	.1288	.87915	.1338	.87477	.1388	.87040	.1438	.86606	.1488	.86174
.1239	.88347	.1289	87906	.1339	.87468	.1389	.87031	.1439	.86597	.1489	.86166
0.1240	0.88338	0.1290	0.87897	0.1340	0.87459	0.1390	0.87023	0.1440	0.86589	0.1490	0.86157
.1241	.88329	.1291	.87889	.1341	.87450	.1391	.87014	.1441	.86580	.1491	.86148
.1242	.88320	.1292	.87880	.1342	.87442	.1392	.87005	.1442	.86571	.1492	.86140
.1243	.88311	.1293	.87871	.1343	.87433	.1393	.86997	.1443	.86563	.1493	.86131
.1244	.88303	.1294	.87862	.1344	.87424	.1394	.86988	.1444	.86554	.1494	.86122
0.1245	0.88294	0.1295	0.87853	0.1345	0.87415	0.1395	0.86979	0.1445	0.86545	0.1495	0.86114
.1246	.88285	.1296	.87845	.1346	.87407	.1396	.86971	.1446	.86537	.1496	.86105
.1247	.88276	.1297	.87836	.1347	.87398	.1397	.86962	.1447	.86528	.1497	.86097
.1248	.88267	.1298	.87827	.1348	.87389	.1398	.86953	.1448	.86520	.1498	.86088
.1249	.88256	.1299	.87818	.1349	.87380	.1399	.86945	.1449	.86511	.1499	.86079

TABLE A-1.—Continued.

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.1500	0.86071	0.1550	0.85642	0.1600	0.85214	0.1650	0.84789	0.1700	0.84366	0.1750	0.83946
.1501	.86062	.1551	.85633	.1601	.85206	.1651	.84781	.1701	.84358	.1751	.83937
.1502	.86054	.1552	.85624	.1602	.85197	.1652	.84772	.1702	.84350	.1752	.83929
.1503	.86045	.1553	.85616	.1603	.85189	.1653	.84764	.1703	.84341	.1753	.83921
.1504	.86036	.1554	.85607	.1604	.85180	.1654	.84755	.1704	.84333	.1754	.83912
0.1505	0.86028	0.1555	0.85599	0.1605	0.85172	0.1655	0.84747	0.1705	0.84324	0.1755	0.83904
.1506	.86019	.1556	.85590	.1606	.85163	.1656	.84739	.1706	.84316	.1756	.83895
.1507	.86010	.1557	.85582	.1607	.85155	.1657	.84730	.1707	.84307	.1757	.83887
.1508	.86002	.1558	.85573	.1608	.85146	.1658	.84722	.1708	.84299	.1758	.83879
.1509	.85993	.1559	.85564	.1609	.85138	.1659	.84713	.1709	.84296	.1759	.83870
0.1510	0.85985	0.1560	0.85556	0.1610	0.85129	0.1660	0.84705	0.1710	0.84282	0.1760	0.83862
.1511	.85976	.1561	.85547	.1611	.85121	.1661	.84696	.1711	.84274	.1761	.83853
.1512	.85968	.1562	.85539	.1612	.85112	.1662	.84688	.1712	.84265	.1762	.83845
.1513	.85959	.1563	.85530	.1613	.85104	.1663	.84679	.1713	.84257	.1763	.83837
.1514	.85950	.1564	.85522	.1614	.85095	.1664	.84671	.1714	.84248	.1764	.83828
0.1515	0.85942	0.1565	0.85513	0.1615	0.85087	0.1665	0.84662	0.1715	0.84240	0.1765	0.83820
.1516	.85933	.1566	.85505	.1616	.85078	.1666	.84654	.1716	.84231	.1766	.83811
.1517	.85925	.1567	.85496	.1617	.85070	.1667	.84645	.1717	.84223	.1767	.83803
.1518	.85916	.1568	.85488	.1618	.85061	.1668	.84637	.1718	.84215	.1768	.83795
.1519	.85907	.1569	.85479	.1619	.85053	.1669	.84628	.1719	.84206	.1769	.83786
0.1520	0.85899	0.1570	0.85470	0.1620	0.85044	0.1670	0.84620	0.1720	0.84198	0.1770	0.83778
.1521	.85890	.1571	.85462	.1621	.85036	.1671	.84611	.1721	.84189	.1771	.83770
.1522	.85882	.1572	.85453	.1622	.85027	.1672	.84603	.1722	.84181	.1772	.83761
.1523	.85873	.1573	.85445	.1623	.85019	.1673	.84595	.1723	.84173	.1773	.83753
.1524	.85864	.1574	.85436	.1624	.85010	.1674	.84586	.1724	.84164	.1774	.83744
0.1525	0.85856	0.1575	0.85428	0.1625	0.85002	0.1675	0.84578	0.1725	0.84156	0.1775	0.83736
.1526	.85847	.1576	.85412	.1626	.84993	.1676	.84569	.1726	.84147	.1776	.83728
.1527	.85839	.1577	.85411	.1627	.84985	.1677	.84561	.1727	.84139	.1777	.83719
.1528	.85830	.1578	.85402	.1628	.84976	.1678	84552	.1728	.84131	.1778	.83711
.1529	.85822	.1579	.85394	.1629	.84968	.1679	.84544	.1729	.84122	.1779	.83703
0.1530	0.85813	0.1580	0.85385	0.1630	0.84959	0.1680	0.84535	0.1730	0.84114	0.1780	0.83694
.1531	.85804	.1581	.85376	.1631	.84951	.1681	.84527	.1731	.84105	.1781	.83686
.1532	.85796	.1582	.85368	.1632	.84942	.1682	.84518	.1732	.84097	.1782	.83678
.1533	.85787	.1583	.85359	.1633	.84934	.1683	.84510	.1733	.84089	.1783	.83669
.1534	.85779	.1584	.85351	.1634	.84925	.1684	.84502	.1734	.84080	.1784	.83661
0.1535	0.85770	0.1585	0.85342	0.1635	0.84917	0.1685	0.84493	0.1735	0.84072	0.1785	0.83652
.1536	.85761	.1586	.85334	.1636	.84908	.1686	.84485	.1736	.84063	.1786	.83644
.1537	.85753	.1587	.85325	.1637	.84900	.1687	.84476	.1737	.84055	.1787	.83636
.1538	.85744	.1588	.85317	.1638	.84891	.1688	.84468	.1738	.84046	.1788	.83627
.1539	.85736	.1589	.85308	.1639	.84883	.1689	.84459	.1739	.84038	.1789	.83619
0.1540	0.85727	0.1590	0.85300	0.1640	0.84874	0.1690	0.84451	0.1740	0.84030	0.1790	0.83611
.1541	.85719	.1591	.85291	.1641	.84866	.1691	.84442	.1741	.84021	.1791	.83602
.1542	.85710	.1592	.85283	.1642	.84857	.1692	.84434	.1742	.84013	.1792	.83594
.1543	.85701	.1593	.85274	.1643	.84849	.1693	.84426	.1743	.84004	.1793	.83586
.1544	.85693	.1594	.85266	.1644	.84840	.1694	.84417	.1744	.83996	.1794	.83577
0.1545	0.85684	0.1595	0.85257	0.1645	0.84832	0.1695	0.84409	0.1745	0.83988	0.1795	0.83569
.1546	.85676	.1596	.85248	.1646	.84823	.1696	.84400	.1746	.83979	.1796	.83560
.1547	.85667	.1597	.85240	.1647	84815	.1697	.84392	.1747	.83971	.1797	.83552
.1548	.85659	.1598	.85231	.1648	.84806	.1698	.84383	.1748	.83962	.1798	.83544
.1549	.85650	.1599	.85223	.1649	.84798	.1699	.84375	.1749	.83954	.1799	.83535

TABLE A-1.—Concluded.

x	e^{-x}	x	e^{-x}	x	e^{-x}	x	e^{-x}
0.1800	0.83527	0.1850	0.83110	0.1900	0.82696	0.1950	0.82283
.1801	.83519	.1851	.83102	.1901	.82688	.1951	.82275
.1802	.83510	.1852	.83094	.1902	.82679	.1952	.82267
.1803	.83502	.1853	.83085	.1903	.82671	.1953	.82259
.1804	.83494	.1854	.83077	.1904	.82663	.1954	.82251
0.1805	0.83485	0.1855	0.83069	0.1905	0.82655	0.1955	0.82242
.1806	.83477	.1856	.83061	.1906	.82646	.1956	.82234
.1807	.83469	.1857	.83052	.1907	.82638	.1957	.82226
.1808	.83460	.1858	.83044	.1908	.82630	.1958	.82218
.1809	.83452	.1859	.83036	.1909	.82622	.1959	.82209
0.1810	0.83444	0.1860	0.83027	0.1910	0.82613	0.1960	0.82201
.1811	.83435	.1861	.83019	.1911	.82605	.1961	.82193
.1812	.83427	.1862	.83017	.1912	.82597	.1962	.82185
.1813	.83419	.1863	.83002	.1913	.82588	.1963	.82177
.1814	.83410	.1864	.82994	.1914	.82580	.1964	.82168
0.1815	0.83402	0.1865	0.82986	0.1915	0.82572	0.1965	0.82160
.1816	.83393	.1866	.82978	.1916	.82564	.1966	.82152
.1817	.83385	.1867	.82969	.1917	.82555	.1967	.82144
.1818	.83377	.1868	.82961	.1918	.82547	.1968	.82135
.1819	.83368	.1869	.82953	.1919	.82539	.1969	.82127
0.1820	0.83360	0.1870	0.82944	0.1920	0.82531	0.1970	0.82119
.1821	.83352	.1871	.82936	.1921	.82522	.1971	.82111
.1822	.83343	.1872	.82928	.1922	.82514	.1972	.82103
.1823	.83335	.1873	.82919	.1923	.82506	.1973	.82094
.1824	.83327	.1874	.82911	.1924	.82498	.1974	.82086
0.1825	0.83318	0.1875	0.82903	0.1925	0.82489	0.1975	0.82078
.1826	.83310	.1876	.82895	.1926	.82481	.1976	.82070
.1827	.83302	.1877	.82886	.1927	.82473	.1977	.82062
.1828	.83293	.1878	.82878	.1928	.82465	.1978	.82053
.1829	.83285	.1879	.82870	.1929	.82456	.1979	.82045
0.1830	0.83277	0.1880	0.82861	0.1930	0.82448	0.1980	0.82037
.1831	.83268	.1881	.82853	.1931	.82440	.1981	.82029
.1832	.83260	.1882	.82845	.1932	.82432	.1982	.82021
.1833	.83252	.1883	.82837	.1933	.82423	.1983	.82012
.1834	.83244	.1884	.82828	.1934	.82415	.1984	.82004
0.1835	0.83235	0.1885	0.82820	0.1935	0.82407	0.1985	0.81996
.1836	.83227	.1886	.82812	.1936	.82399	.1986	.81988
.1837	.83219	.1887	.82803	.1937	.83391	.1987	.81980
.1838	.83210	.1888	.82795	.1938	.82382	.1988	.81971
.1839	.83202	.1889	.82787	.1939	.82374	.1989	.81963
0.1840	0.83194	0.1890	0.82779	0.1940	0.82366	0.1990	0.81955
.1841	.83185	.1891	.82770	.1941	.82358	.1991	.81947
.1842	.83177	.1892	.82762	.1942	.82349	.1992	.81939
.1843	.83169	.1893	.82754	.1943	.82341	.1993	.81930
.1844	.83160	.1894	.82746	.1944	.82333	.1994	.81922
0.1845	0.83152	0.1895	0.82737	0.1945	0.82325	0.1995	0.81914
.1846	.83144	.1896	.82729	.1946	.82316	.1996	.81906
.1847	.83135	.1897	.82721	.1947	.82308	.1997	.81898
.1848	.83127	.1898	.82712	.1948	.82300	.1998	.81889
.1849	.83119	.1899	.82704	.1949	.82392	.1999	.81881

TABLE A-2.—TOLERANCE FACTORS FOR OBSERVED MTBF[a]

Confidence level, percent	Number of observed failures															
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
99	4.6	6.6	8.4	10.1	11.6	13.1	14.6	16.0	17.4	18.7	20.2	21.5	22.8	24.1	25.4	26.8
95	3.0	4.7	6.3	7.8	9.1	10.5	11.8	13.1	14.4	15.7	17.0	18.2	19.4	20.7	21.9	23.1
90	2.3	3.9	5.3	6.7	8.0	9.2	10.5	11.7	13.0	14.2	15.4	16.6	17.8	19.0	20.2	21.3
80	1.6	3.0	4.3	5.5	6.7	7.9	9.0	10.2	11.4	12.5	13.7	14.8	15.9	17.0	18.1	19.2
70	1.2	2.4	3.6	4.8	5.9	7.0	8.1	9.2	10.3	11.4	12.5	13.5	14.6	15.7	16.8	17.8
60	.9	2.0	3.1	4.2	5.2	6.3	7.4	8.4	9.4	10.5	11.5	12.5	13.6	14.7	15.7	16.7
50	.7	1.7	2.7	3.7	4.7	5.7	6.7	7.7	8.7	9.7	10.7	11.7	12.7	13.7	14.7	15.7

[a]To use this table

1. Calculate total test hours, $T = \sum_{i=1}^{n} N_i t_i$ where N is the i^{th} unit tested, t_i is the test time of N_i, and n is the total number of units tested.
2. Enter table under number of observed failures at desired confidence level to find tolerance factor.
3. Lower confidence limit of MTBF = T/(Tolerance factor).

TABLE A-3.—SAFETY MARGINS AT 99-PERCENT CONFIDENCE LEVEL

(a) Sample sizes 5 to 12

Safety margin, S_M	Probability, P_x	Sample size, N							
		5	6	7	8	9	10	11	12
−5.0	0	−2.6271	−2.7679	−2.8843	−2.9789	−3.0590	−3.1327	−3.1958	−3.2521
−4.0	0	−2.0487	−2.1655	−2.2612	−2.3404	−2.4052	−2.4667	−2.5188	−2.5652
−3.0	.0013	−1.4523	−1.5466	−1.6226	−1.6880	−1.7376	−1.7878	−1.8294	−1.8664
−2.0	.0227	.8028	.8810	.9415	.9923	−1.0351	−1.0740	−1.1071	−1.1364
−1.0	.1586	.0434	.0500	.1235	.1762	.2227	.2579	.2893	.3168
−0	.5000	1.6808	1.3681	1.1900	1.0602	.9617	.8914	.8320	.7833
.1	.5398	1.9138	1.5664	1.3628	1.2168	1.1126	1.0351	.9703	.9175
.2	.5792	2.1557	1.7665	1.5439	1.3850	1.2706	1.1844	1.1137	1.0563
.3	.6179	2.4041	1.9747	1.7328	1.5608	1.4352	1.3389	1.2617	1.1994
.4	.6554	2.6582	2.1986	1.9285	1.7380	1.6061	1.4975	1.4138	1.3463
.5	.6914	2.9406	2.4294	2.1304	1.9206	1.7775	1.6602	1.5697	1.4970
.6	.7257	3.2293	2.6662	2.3378	2.1082	1.9522	1.8270	1.7295	1.6512
.7	.7580	3.5232	2.9083	2.5500	2.3002	2.1309	1.9977	1.8927	1.8085
.8	.7881	3.8217	3.1551	2.7665	2.4961	2.3133	2.1719	2.0591	1.9689
.9	.8159	4.1244	3.4059	2.9869	2.6956	2.4989	2.3493	2.2285	2.1320
1.0	.8413	4.4425	3.6604	3.2107	2.8988	2.6875	2.5295	2.4005	2.2975
1.1	.8643	4.7756	3.9183	3.4375	3.1115	2.8846	2.7118	2.5745	2.4650
1.2	.8849	5.1124	4.1791	3.6672	3.3269	3.0842	2.8962	2.7506	2.6344
1.3	.9031	5.4524	4.4467	3.9006	3.5445	3.2860	3.0827	2.9285	2.8056
1.4	.9192	5.7952	4.7243	4.1431	3.7582	3.4851	3.2713	3.1083	2.9785
1.5	.9331	6.1405	5.0042	4.3877	3.9736	3.6855	3.4616	3.2897	3.1528
1.6	.9452	6.4881	5.2861	4.6340	4.1908	3.8874	3.6533	3.4724	3.3284
1.7	.9554	6.8377	5.5698	4.8820	4.4094	4.0907	3.8463	3.6563	3.5051
1.8	.9640	7.1891	5.8550	5.1279	4.6311	4.2953	4.0404	3.8412	3.6828
1.9	.9712	7.5422	6.1417	5.3723	4.8570	4.5010	4.2353	4.0269	3.8613
2.0	.9772	7.8966	6.4295	5.6180	5.0840	4.7077	4.4310	4.2135	4.0405
2.1	.9821	8.2524	6.7186	5.8647	5.3119	4.9153	4.6277	4.4008	4.2205
2.2	.9860	8.6094	7.0086	6.1125	5.5406	5.1238	4.8251	4.5889	4.4012
2.3	.9892	8.9675	7.2996	6.3612	5.7701	5.3330	5.0232	4.7776	4.5825
2.4	.9918	9.3265	7.5914	6.6107	6.0003	5.5429	5.2219	4.9670	4.7644
2.5	.9937	9.6865	7.8840	6.8609	6.2311	5.7534	5.4212	5.1568	4.9468
2.6	.9953	1.0472	8.1772	7.1119	6.4624	5.9646	5.6210	5.3472	5.1296
2.7	.9965	1.4011	8.4694	7.3635	6.6943	6.1762	5.8213	5.5380	5.3129
2.8	.9974	1.7549	8.7588	7.6191	6.9248	6.3894	6.0221	5.7292	5.4965
2.9	.9981	11.1094	9.0488	7.8753	7.1549	6.6040	6.2232	5.9207	5.6804
3.0	.9986	11.4647	9.3395	8.1319	7.3855	6.8191	6.4247	6.1126	5.8647
3.1	.9990	11.8207	9.6307	8.3889	7.6165	7.0345	6.6266	6.3047	6.0492
3.2	.9993	12.1773	9.9223	8.6463	7.8479	7.2502	6.8287	6.4972	6.2340
3.3	.9995	12.5345	1.2145	8.9040	8.0796	7.4662	7.0312	6.6900	6.4191
3.4	.9996	12.8922	1.5070	9.1620	8.3117	7.6825	7.2339	6.8830	6.6044
3.5	.9997	13.2505	1.8000	9.4203	8.5440	7.8990	7.4368	7.0762	6.7899
3.6	.9998	13.6092	11.0933	9.6739	8.7767	8.1157	7.6400	7.2697	6.9756
3.7	.9998	13.9684	11.3870	9.9377	9.0096	8.3326	7.8435	7.4633	7.1616

TABLE A-3.—Continued.

(a) Concluded.

Safety margin, S_M	Probability P_x	Sample size, N							
		5	6	7	8	9	10	11	12
3.8	0.9999	14.3280	11.6809	10.1968	9.2427	8.5497	8.0471	7.6572	7.3477
3.9		14.6880	11.9752	10.4560	9.4761	8.7671	8.2500	7.8512	7.5340
4.0		15.0488	12.2698	10.7155	9.7097	8.9845	8.4548	8.0454	7.7204
4.1		15.4090	12.5646	10.9751	9.9435	9.2022	8.6590	8.2397	7.9069
4.2		15.7700	12.8597	11.2350	10.1775	9.4200	8.8632	8.4342	8.0937
4.3		16.1313	13.1550	11.4950	10.4116	9.6379	9.0677	8.6288	8.2805
4.4		16.4929	13.4505	11.7551	10.6459	9.8559	9.2722	8.8235	8.4674
4.5		16.8547	13.7463	12.0154	10.8804	10.0741	9.4769	9.0184	8.6545
4.6		17.2168	14.0422	12.2758	11.1150	10.2924	9.6817	9.2134	8.8417
4.7		17.5792	14.3383	12.5364	11.3497	10.5108	9.8866	9.4084	9.0289
4.8		17.9417	14.6346	12.7970	11.5846	10.7293	10.0917	9.6036	9.2163
4.9	1.0000	18.3045	14.9310	13.0578	11.8196	10.9479	10.2968	9.7989	9.4038
5.0		18.6674	15.2276	13.3187	12.0547	11.1666	10.5020	9.9942	9.5913
5.1		19.0306	15.5243	13.5797	12.2900	11.3854	10.7073	10.1896	9.7789
5.2		19.3939	15.8212	13.8408	12.5253	11.6043	10.9127	10.3851	9.9666
5.3		19.7574	16.1182	14.1020	12.7607	11.8232	11.1181	10.5807	10.1543
5.4		20.1210	16.4153	14.3632	12.9962	12.0422	11.3237	10.7764	10.3422
5.5		20.4848	16.7125	14.6246	13.2318	12.2613	11.5293	10.9721	10.5300
5.6		20.8488	17.0099	14.8860	13.4675	12.4804	11.7350	11.1679	10.7180
5.7		21.2129	17.3073	15.1475	13.7033	12.6996	11.9407	11.3638	10.9060
5.8		21.5771	17.6049	15.4091	13.9391	12.9189	12.1465	11.5597	11.0941
5.9		21.9414	17.9025	15.6707	14.1751	13.1382	12.3524	11.7556	11.2822
6.0		22.3059	18.2003	15.9324	14.4110	13.3576	12.5583	11.9516	11.4703
6.1		22.6705	18.4981	16.1941	14.6471	13.5770	12.7643	12.1477	11.6585
6.2		23.0351	18.7960	16.4560	14.8832	13.7965	12.9703	12.3438	11.8468
6.3		23.3999	19.0940	16.7178	15.1194	14.0160	13.1763	12.5400	12.0351
6.4		23.7648	19.3921	16.9797	15.3556	14.2356	13.3825	12.7362	12.2234
6.5		24.1298	19.6902	17.2417	15.5919	14.4552	13.5886	12.9324	12.4118
6.6		24.4948	19.9884	17.5037	15.8282	14.6748	13.7948	13.1287	12.6002
6.7		24.8600	20.2867	17.7658	16.0646	14.8945	14.0011	13.3250	12.7887
6.8		25.2252	20.5850	18.0279	16.3011	15.1143	14.2074	13.5214	12.9771
6.9		25.5905	20.8834	18.2901	16.5375	15.3340	14.4137	13.7177	13.1657
7.0		25.9559	21.1819	18.5523	16.7741	15.5538	14.6200	13.9142	13.3542
7.1		26.3214	21.4804	18.8145	17.0106	15.7736	14.8264	14.1106	13.5428
7.2		26.6869	21.7789	19.0768	17.2472	15.9935	15.0328	14.3071	13.7314
7.3		27.0525	22.0776	19.3391	17.4839	16.2134	15.2393	14.5036	13.9200
7.4		27.4182	22.3762	19.6014	17.7206	16.4333	15.4458	14.7002	14.1087
7.5		27.7839	22.6749	19.8638	17.9573	16.6533	15.6523	14.8967	14.2974
7.6		28.1497	22.9737	20.1262	18.1940	16.8732	15.8588	15.0933	14.4861
7.7		28.5155	23.2725	20.3886	18.4308	17.0932	16.0654	15.2900	14.6748
7.8		28.8814	23.5714	20.6511	18.6676	17.3133	16.2720	15.4866	14.8636
7.9		29.2474	23.8702	20.9136	18.9045	17.5333	16.4786	15.6833	15.0524
8.0		29.6134	24.1692	21.1761	19.1414	17.7534	16.6852	15.8800	15.2412

TABLE A-3.—Continued.

(b) Sample sizes 13 to 20

Safety margin, S_M	Probability, P_x	Sample size, N							
		13	14	15	16	17	18	19	20
−5.0	0	−3.3027	−3.3485	−3.3903	−3.4287	−3.4642	−3.4972	−3.5280	−3.5567
−4.0	0	−2.6069	−2.6447	−2.6792	−2.7109	−2.7401	−2.7673	−2.7927	−2.8163
−3.0	.0013	−1.8997	−1.9299	−1.9573	−1.9826	−2.005[illegible]	−2.0275	−2.0477	−2.0666
−2.0	.0227	−1.1628	−1.1866	−1.2083	−1.2281	−1.246[illegible]	−1.2633	−1.2791	−1.2937
−1.0	.1586	−.3411	−.3628	−.3823	−.4000	−.4162	−.4309	−.4445	−.4571
0	.5000	.7424	.7074	.6770	.6503	.6265	.6049	.5854	.5677
.1	.5398	.8733	.8357	.8031	7745	.7491	.7260	.7052	.6863
.2	.5792	1.0085	.9679	.9328	.9022	.8751	.8503	.8281	.8079
.3	.6179	1.1477	1.1039	1.0662	1.0332	1.0042	.9777	.9540	.9325
.4	.6554	1.2905	1.2433	1.2028	1.1675	1.1364	1.1081	1.0827	1.0598
.5	.6914	1.4369	1.3862	1.3428	1.3049	1.2717	1.2414	1.2143	1.1898
.6	.7257	1.5867	1.5324	1.4859	1.4454	1.4099	1.3775	1.3485	1.3224
.7	.7580	1.7393	1.6810	1.6312	1.5880	1.5500	1.5155	1.4846	1.4569
.8	.7881	1.8947	1.8324	1.7792	1.7331	1.6926	1.6558	1.6230	1.5935
.9	.8159	2.0527	1.9862	1.9294	1.8803	1.8372	1.7981	1.7632	1.7319
1.0	.8413	2.2130	2.1422	2.0818	2.0295	1.9838	1.9423	1.9053	1.8721
1.1	.8643	2.3752	2.3000	2.2359	2.1805	2.1320	2.0880	2.0489	2.0138
1.2	.8849	2.5393	2.4596	2.3917	2.3330	2.2817	2.2352	2.1939	2.1568
1.3	.9031	2.7050	2.6208	2.5491	2.4871	2.4329	2.3839	2.3403	2.3012
1.4	.9192	2.8722	2.7833	2.7077	2.6424	2.5853	2.5337	2.4877	2.4466
1.5	.9331	3.0408	2.9472	2.8675	2.7988	2.7388	2.6845	2.6362	2.5930
1.6	.9452	3.2106	3.1122	3.0285	2.9563	2.8932	2.8363	2.7856	2.7403
1.7	.9554	3.3815	3.2782	3.1905	3.1147	3.0486	2.9890	2.9360	2.8885
1.8	.9640	3.5534	3.4452	3.3533	3.2740	3.2049	3.1425	3.0870	3.0374
1.9	.9712	3.7259	3.6129	3.5168	3.4340	3.3617	3.2966	3.2387	3.1870
2.0	.9772	3.8992	3.7812	3.6810	3.5946	3.5193	3.4513	3.3910	3.3370
2.1	.9821	4.0732	3.9503	3.8459	3.7559	3.6774	3.6066	3.5438	3.4876
2.2	[illegible]	4.2479	4.1200	4.0113	3.9177	3.8360	3.7625	3.6972	3.6388
2.3	.9892	4.4232	4.2902	4.1773	4.0800	3.9952	3.9188	3.8510	3.7904
2.4	.9918	4.5990	4.4610	4.3438	4.2429	4.1548	4.0756	4.0052	3.9424
2.5	.9937	4.7753	4.6322	4.5107	4.4061	4.3149	4.2327	4.1599	4.0947
2.6	.9953	4.9520	4.8038	4.6781	4.5697	4.4753	4.3903	4.3149	4.2474
2.7	.9965	5.1291	4.9759	4.8458	4.7337	4.6361	4.5482	4.4702	4.4005
2.8	.9974	5.3066	5.1482	5.0138	4.8980	4.7971	4.7063	4.6258	4.5538
2.9	.9981	5.4843	5.3208	5.1820	5.0625	4.9584	4.8647	4.7816	4.7073
3.0	.9986	5.6624	5.4937	5.3505	5.2273	5.1198	5.0232	4.9376	4.8610
3.1	.9990	5.8407	5.6668	5.5193	5.3922	5.2816	5.1820	5.0938	5.0149
3.2	.9993	6.0192	5.8402	5.6882	5.5575	5.4435	5.3411	5.2502	5.1690
3.3	.9995	6.1981	6.0138	5.8575	5.7229	5.6056	5.5003	5.4069	5.3234
3.4	.9996	6.3771	6.1876	6.0269	5.8885	5.7680	5.6597	5.5637	5.4779
3.5	.9997	6.5564	6.3617	6.1965	6.0544	5.9305	5.8193	5.7207	5.6325
3.6	.9998	6.7358	6.5359	6.3663	6.2204	6.0932	5.9790	5.8778	5.7873
3.7	.9998	6.9154	6.7103	6.5363	6.3865	6.2560	6.1389	6.0351	5.9423

TABLE A-3.—Continued.

(b) Concluded.

Safety margin, S_M	Probability, P_x	Sample size, N							
		13	14	15	16	17	18	19	20
3.8	0.9999	7.0952	6.8849	6.7064	6.5528	6.4190	6.2990	6.1925	6.0974
3.9		7.2752	7.0596	6.8767	6.7193	6.5822	6.4591	6.3501	6.2526
4.0		7.4553	7.2344	7.0471	6.8859	6.7454	6.6194	6.5077	6.4079
4.1		7.6356	7.4094	7.2176	7.0526	6.9088	6.7798	6.6655	6.5633
4.2		7.8159	7.5845	7.3883	7.2194	7.0723	6.9403	6.8234	6.7189
4.3		7.9964	7.7597	7.5590	7.3863	7.2359	7.1010	6.9814	6.8745
4.4		8.1771	7.9351	7.7299	7.5533	7.3995	7.2617	7.1394	7.0302
4.5		8.3578	8.1105	7.9009	7.7204	7.5633	7.4225	7.2976	7.1860
4.6		8.5386	8.2861	8.0719	7.8877	7.7272	7.5833	7.4558	7.3419
4.7		8.7195	8.4617	8.2431	8.0550	7.8911	7.7443	7.6141	7.4978
4.8		8.9005	8.6374	8.4143	8.2223	8.0551	7.9053	7.7725	7.6538
4.9	1.0000	9.0816	8.8132	8.5856	8.3898	8.2192	8.0664	7.9310	7.8099
5.0		9.2628	8.9890	8.7570	8.5573	8.3834	8.2276	8.0895	7.9660
5.1		9.4440	9.1650	8.9284	8.7249	8.5476	8.3888	8.2480	8.1222
5.2		9.6253	9.3410	9.0999	8.8925	8.7119	8.5501	8.4067	8.2785
5.3		9.8067	9.5170	9.2715	9.0602	8.8763	8.7114	8.5654	8.4348
5.4		9.9881	9.6932	9.4431	9.2280	9.0406	8.8728	8.7241	8.5912
5.5		10.1696	9.8694	9.6148	9.3958	9.2051	9.0843	8.8829	8.7476
5.6		10.3512	10.0456	9.7865	9.5637	9.3696	9.1958	9.0417	8.9040
5.7		10.5328	10.2219	9.9583	9.7316	9.5341	9.3573	9.2006	9.0605
5.8		10.7145	10.3983	10.1302	9.8996	9.6987	9.5189	9.3595	9.2171
5.9		10.8962	10.5746	10.3020	10.0676	9.8634	9.6805	9.5184	9.3736
6.0		11.0779	10.7511	10.4740	10.2356	10.0280	9.8422	9.6774	9.5302
6.1		11.2598	10.9276	10.6459	10.4037	10.1928	10.0039	9.8365	9.6869
6.2		11.4416	11.1041	10.8179	10.5718	10.3575	10.1656	9.9955	9.8435
6.3		11.6235	11.2806	10.9900	10.7400	10.5223	10.3274	10.1546	10.0003
6.4		11.8054	11.4572	11.1621	10.9082	10.6871	10.4892	10.3138	10.1570
6.5		11.9874	11.6339	11.3342	11.0764	10.8520	10.6510	10.4729	10.3138
6.6		12.1694	11.8105	11.5063	11.2447	11.0168	10.8129	10.6321	10.4706
6.7		12.3514	11.9873	11.6785	11.4130	11.1817	10.9748	10.7913	10.6274
6.8		12.5335	12.1640	11.8507	11.5813	11.3467	11.1367	10.9505	10.7842
6.9		12.7156	12.3407	12.0230	11.7496	11.5116	11.2986	11.1098	10.9411
7.0		12.8978	12.5175	12.1952	11.9180	11.6766	11.4606	11.2691	11.0980
7.1		12.0799	12.6944	12.3675	12.0864	11.8416	11.6226	11.4284	11.2549
7.2		13.2621	12.8712	12.5398	12.2548	12.0067	11.7846	11.5877	11.4119
7.3		13.4443	13.0481	12.7122	12.4233	12.1717	11.9466	11.7471	11.5688
7.4		13.6266	13.2250	12.8846	12.5918	12.3368	12.1087	11.9065	11.7253
7.5		13.8088	13.4019	13.0569	12.7603	12.5019	12.2708	12.0659	11.8823
7.6		13.9911	13.5788	13.2294	12.9288	12.6671	12.4329	12.2253	12.0398
7.7		14.1734	13.7558	13.4018	13.0973	12.8322	12.5950	12.3847	12.1969
7.8		14.3558	13.9328	13.5742	13.2659	12.9974	12.7571	12.5442	12.3539
7.9		14.5381	14.1098	13.7467	13.4344	13.1626	12.9193	12.7036	12.5110
8.0		14.7205	14.2868	13.9192	13.6030	13.3278	13.0814	12.8631	12.6681

TABLE A-3.—Continued.

(c) Sample sizes 21 to 28

Safety margin, S_M	Probability, P_x	Sample size, N							
		21	22	23	24	25	26	27	28
−5.0	0	−3.5836	−3.6090	−3.6328	−3.6554	−3.6767	−3.6970	−3.7162	−3.7346
−4.0	0	−2.8385	−2.8594	−2.8790	−2.8976	−2.9152	−2.9318	−2.9477	−2.9628
−3.0	.0013	−2.0842	−2.1008	−2.1164	−2.1312	−2.1451	−2.1584	−2.1710	−2.1830
−2.0	.0227	−1.3075	−1.3204	−1.3325	−1.3439	−1.3548	−1.3650	−1.3748	−1.3840
−1.0	.1586	−.4688	−.4797	−.4900	−.4996	−.5087	−.5173	−.5254	−.5331
0	.5000	.5514	.5366	.5228	.5101	.4982	.4872	.4769	.4671
.1	.5398	.6691	.6533	.6387	.6253	.6128	.6011	.5902	.5800
.2	.5792	.7896	.7728	.7574	.7432	.7299	.7176	.7061	.6953
.3	.6179	.9130	.8952	.8788	.8637	.8496	.8366	.8244	.8130
.4	.6554	1.0390	1.0201	1.0026	.9866	.9717	.9579	.9450	.9330
.5	.6914	1.1677	1.1475	1.1289	1.1119	1.0961	1.0814	1.0678	1.0550
.6	.7257	1.2988	1.2773	1.2576	1.2395	1.2227	1.2071	1.1926	1.1791
.7	.7580	1.4318	1.4089	1.3880	1.3687	1.3510	1.3345	1.3191	1.3048
.8	.7881	1.5669	1.5426	1.5204	1.5000	1.4811	1.4637	1.4474	1.4323
.9	.8159	1.7036	1.6779	1.6544	1.6327	1.6128	1.5943	1.5771	1.5611
1.0	.8413	1.8421	1.8149	1.7900	1.7671	1.7460	1.7265	1.7084	1.6914
1.1	.8643	1.9821	1.9533	1.9270	1.9028	1.8806	1.8600	1.8408	1.8230
1.2	.8849	2.1234	2.0930	2.0652	2.0397	2.0163	1.9945	1.9744	1.9556
1.3	.9031	2.2659	2.2339	2.2046	2.1778	2.1531	2.1302	2.1090	2.0892
1.4	.9192	2.4095	2.3758	2.3451	2.3169	2.2909	2.2669	2.2446	2.2238
1.5	.9331	2.5540	2.5187	2.4864	2.4568	2.4296	2.4044	2.3810	2.3592
1.6	.9452	2.6995	2.6624	2.6286	2.5976	2.5690	2.5426	2.5182	2.4954
1.7	.9554	2.8457	2.8069	2.7716	2.7391	2.7093	2.6817	2.6561	2.6322
1.8	.9640	2.9927	2.9522	2.9152	2.8813	2.8501	2.8213	2.7946	2.7697
1.9	.9712	3.1403	3.0980	3.0594	3.0241	2.9915	2.9615	2.9336	2.9077
2.0	.9772	3.2884	3.2443	3.2041	3.1673	3.1334	3.1021	3.0731	3.0461
2.1	.9821	3.4370	3.3912	3.3493	3.3110	3.2758	3.2432	3.2130	3.1849
2.2	.9860	3.5862	3.5385	3.4950	3.4552	3.4186	3.3847	3.3534	3.3242
2.3	.9892	3.7357	3.6862	3.6411	3.5998	3.5618	3.5267	3.4941	3.4638
2.4	.9918	3.8857	3.8344	3.7876	3.7448	3.7053	3.6689	3.6352	3.6038
2.5	.9937	4.0360	3.9829	3.9344	3.8901	3.8492	3.8115	3.7766	3.7441
2.6	.9953	4.1867	4.1317	4.0816	4.0357	3.9934	3.9544	3.9183	3.8847
2.7	.9965	4.3377	4.2808	4.2290	4.1816	4.1379	4.0976	4.0603	4.0255
2.8	.9974	4.4890	4.4302	4.3767	4.3277	4.2826	4.2410	4.2025	4.1666
2.9	.9981	4.6404	4.5798	4.5246	4.4741	4.4276	4.3847	4.3449	4.3079
3.0	.9986	4.7920	4.7296	4.6727	4.6206	4.5727	4.5284	4.4874	4.4493
3.1	.9990	4.9439	4.8796	4.8210	4.7673	4.7180	4.6724	4.6302	4.5909
3.2	.9993	5.0959	5.0297	4.9694	4.9142	4.8634	4.8166	4.7731	4.7327
3.3	.9995	5.2482	5.1801	5.1181	5.0613	5.0091	4.9609	4.9162	4.8747
3.4	.9996	5.4006	5.3306	5.2669	5.2085	5.1549	5.1053	5.0594	5.0168
3.5	.9997	5.5532	5.4813	5.4158	5.3559	5.3008	5.2500	5.2028	5.1590
3.6	.9998	5.7059	5.6321	5.5649	5.5034	5.4469	5.3947	5.3463	5.3014
3.7	.9998	5.8587	5.7831	5.7142	5.6511	5.5931	5.5396	5.4899	5.4438

TABLE A-3.—Continued.

(c) Concluded.

Safety margin, S_M	Probability P_x	Sample size, N							
		21	22	23	24	25	26	27	28
3.8	0.9999	6.0117	5.9342	5.8635	5.7989	5.7394	5.6845	5.6337	5.5864
3.9		6.1648	6.0854	6.0130	5.9467	5.8858	5.8296	5.7775	5.7291
4.0		6.3180	6.2367	6.1626	6.0947	6.0324	5.9748	5.9215	5.8719
4.1		6.4714	6.3881	6.3122	6.2428	6.1790	6.1201	6.0655	6.0147
4.2		6.6248	6.5396	6.4620	6.3910	6.3257	6.2654	6.2096	6.1577
4.3		6.7788	6.6912	6.6118	6.5392	6.4725	6.4109	6.3538	6.3007
4.4		6.9319	6.8428	6.7618	6.6876	6.6193	6.5564	6.4980	6.4438
4.5		7.0856	6.9946	6.9118	6.8360	6.7663	6.7020	6.6424	6.5870
4.6		7.2393	7.1464	7.0619	6.9844	6.9133	6.8476	6.7868	6.7302
4.7		7.3931	7.2983	7.2120	7.1330	7.0604	6.9933	6.9312	6.8735
4.8		7.5470	7.4503	7.3622	7.2816	7.2075	7.1391	7.0757	7.0168
4.9	1.0000	7.7010	7.6023	7.5125	7.4303	7.3547	7.2849	7.2203	7.1602
5.0		7.8550	7.7544	7.6628	7.5790	7.5019	7.4308	7.3649	7.3037
5.1		8.0090	7.9065	7.8132	7.7278	7.6492	7.5768	7.5096	7.4472
5.2		8.1632	8.0587	7.9636	7.8766	7.7966	7.7227	7.6543	7.5907
5.3		8.3173	8.2110	8.1141	8.0255	7.9440	7.8688	7.7991	7.7343
5.4		8.4716	8.3633	8.2646	8.1744	8.0914	8.0148	7.9439	7.8780
5.5		8.6258	8.5156	8.4152	8.3233	8.2389	8.1609	8.0887	8.0215
5.6		8.7801	8.6680	8.5658	8.4723	8.3864	8.3071	8.2336	8.1653
5.7		8.9345	8.8204	8.7165	8.6214	8.5340	8.4533	8.3785	8.3091
5.8		9.0889	8.9728	8.8671	8.7704	8.6815	8.5995	8.5235	8.4529
5.9		9.2433	9.1253	9.0179	8.9195	8.8292	8.7458	8.6685	8.5967
6.0		9.3978	9.2778	9.1686	9.0687	8.9768	8.8920	8.8135	8.7405
6.1		9.5523	9.4304	9.3194	8.2179	9.1245	8.0384	8.9585	8.8844
6.2		9.7068	9.5830	9.4702	9.3671	9.2722	9.1847	9.1036	9.0283
6.3		9.8614	9.7356	9.6211	9.5163	9.4200	9.3311	9.2487	9.1722
6.4		10.0160	9.8882	9.7720	9.6655	9.5677	9.4775	9.3938	9.3161
6.5		10.1706	10.0409	9.9229	9.8148	9.7155	9.6239	9.5390	9.4601
6.6		10.3252	10.1936	10.0738	9.9641	8.8634	9.7703	9.6842	9.6041
6.7		10.4799	10.3463	10.2247	10.1135	10.0112	9.9168	9.8294	9.7481
6.8		10.6346	10.4991	10.3757	10.2628	10.1591	10.0633	9.9746	9.8921
6.9		10.7893	10.6519	10.5267	10.4122	10.3070	10.2098	10.1198	10.0362
7.0		10.9441	10.8047	10.6777	10.5616	10.4549	10.3563	10.2651	10.1803
7.1		11.0988	10.9575	10.8288	10.7110	10.6028	10.5029	10.4104	10.3244
7.2		11.2536	11.1103	10.9798	10.8605	10.7507	10.6495	10.5557	10.4685
7.3		11.4084	11.2632	11.1309	11.0099	10.8987	10.7961	10.7010	10.6126
7.4		11.5632	11.4160	11.2820	11.1594	11.0467	10.9427	10.8463	10.7567
7.5		11.7181	11.5689	11.4331	11.3089	11.1947	11.0893	10.9916	10.9009
7.6		11.8729	11.7218	11.5843	11.4584	11.3427	11.2359	11.1370	11.0451
7.7		12.0278	11.8748	11.7354	11.6079	11.4907	11.3826	11.2824	11.1893
7.8		12.1827	12.0277	11.8866	11.7575	11.6388	11.5292	11.4278	11.3335
7.9		12.3376	12.1807	12.0378	11.9070	11.7868	11.6759	11.5732	11.4777
8.0		12.4926	12.3337	12.1890	12.0566	11.9349	11.8226	11.7186	11.6219

TABLE A-3.—Continued.

(d) Sample sizes 30 to 100

Safety margin, S_M	Probability, P_x	Sample size, N							
		30	40	50	60	70	80	90	100
−5.0	0	−3.7688	−3.9040	−4.0005	−4.0741	−4.1328	−4.1810	−4.2216	−4.2565
−4.0	0	−2.9910	−3.1021	−3.1815	−3.2420	−3.2901	−3.3297	−3.3630	−3.3916
−3.0	.0013	−2.2053	−2.2935	−2.2564	−2.4042	−2.4422	−2.4735	−2.4998	−2.5224
−2.0	.0227	−1.4013	−1.4691	−1.5172	−1.5536	−1.5825	−1.6063	−1.6262	−1.6432
−1.0	.1586	−.5473	−.6024	−.6406	−.6692	−.6918	−.7101	−.7254	−.7385
0	.5000	.4494	.3835	.3401	.3087	.2846	.2655	.2497	.2364
.1	.5398	.5613	.4924	.4472	.4146	.3897	.3700	.3537	.3401
.2	.5792	.6756	.6033	.5560	.5221	.4963	.4758	.4590	.4449
.3	.6179	.7923	.7162	.6665	.6311	.6042	.5828	.5654	.5508
.4	.6554	.9110	.8308	.7786	.7415	.7134	.6912	.6730	.6579
.5	.6914	1.0318	.9471	.8922	.8533	.8239	.8007	.7818	.7659
.6	.7257	1.1545	1.0650	1.0073	.9664	.9356	.9113	.8915	.8750
.7	.7580	1.2788	1.1844	1.1236	1.0806	1.0483	1.0229	1.0022	.9850
.8	.7881	1.4048	1.3051	1.2411	1.1960	1.1621	1.1355	1.1138	1.0958
.9	.8159	1.5321	1.4270	1.3596	1.3122	1.2767	1.2488	1.2261	1.2073
1.0	.8413	1.6608	1.5500	1.4792	1.4294	1.3922	1.3629	1.3392	1.3195
1.1	.8643	1.7906	1.6740	1.5996	1.5474	1.5084	1.4778	1.4530	1.4324
1.2	.8849	1.9215	1.7989	1.7208	1.6662	1.6253	1.5933	1.5673	1.5458
1.3	.9031	2.0535	1.9247	1.8429	1.7856	1.7429	1.7094	1.6823	1.6598
1.4	.9192	2.1863	2.0513	1.9656	1.9057	1.8610	1.8260	1.7977	1.7742
1.5	.9331	2.3198	2.1784	2.0888	2.0262	1.9795	1.9430	1.9135	1.8890
1.6	.9452	2.4542	2.3062	2.2126	2.1473	2.0986	2.0605	2.0297	2.0042
1.7	.9554	2.5891	2.4346	2.3369	2.2688	2.2180	2.1784	2.1463	2.1198
1.8	.9640	2.7247	2.5635	2.4617	2.3908	2.3379	2.2966	2.2632	2.2356
1.9	.9712	2.8608	2.6928	2.5868	2.5130	2.4580	2.4151	2.3804	2.3517
2.0	.9772	2.9972	2.8225	2.7123	2.6356	2.5784	2.5339	2.4979	2.4681
2.1	.98[illegible]	3.1341	2.9525	2.8380	2.7584	2.6991	2.6529	2.6155	2.5846
2.2	[illegible]	3.2715	3.0829	2.9641	2.8815	2.8201	2.7721	2.7334	2.7014
2.3	.9892	3.4091	3.2136	3.0905	3.0049	2.9412	2.8916	2.8515	2.8184
2.4	.9918	3.5471	3.3445	3.2171	3.1285	3.0626	3.0112	2.9698	2.9355
2.5	.9937	3.6854	3.4757	3.3439	3.2523	3.1842	3.1311	3.0882	3.0528
2.6	.9953	3.8240	3.6072	3.4709	3.3763	3.3059	3.2511	3.2068	3.1702
2.7	.9965	3.9628	3.7389	3.5982	3.5005	3.4278	3.3712	3.3256	3.2878
2.8	.9974	4.1019	3.8707	3.7256	3.6248	3.5499	3.4915	3.4444	3.4055
2.9	.9981	4.2411	4.0028	3.8531	3.7493	3.6720	3.6119	3.5634	3.5233
3.0	.9986	4.3805	4.1349	3.9808	3.8738	3.7943	3.7324	3.6825	3.6412
3.1	.9990	4.5201	4.2673	4.1086	3.9985	3.9167	3.8530	3.8017	3.7592
3.2	.9993	4.6598	4.3997	4.2366	4.1234	4.0393	3.9737	3.9209	3.8773
3.3	.9995	4.7997	4.5323	4.3646	4.2483	4.1619	4.0945	4.0403	3.9954
3.4	.9996	4.9397	4.6650	4.4928	4.3733	4.2845	4.2154	4.1597	4.1137
3.5	.9997	4.0799	4.7979	4.6211	4.4984	4.4073	4.3364	4.2792	4.2320
3.6	.9998	5.2202	4.9308	4.7494	4.6236	4.5302	4.4574	4.3988	4.3503
3.7	.9998	5.3606	5.0638	4.8778	4.7489	4.6531	4.5785	4.5184	4.4688

TABLE A-3.—Concluded.

(d) Concluded.

Safety margin, S_M	Probability P_x	Sample size, N							
		30	40	50	60	70	80	90	100
3.8	0.9999	5.5011	5.1969	5.0064	4.8742	4.7761	4.6997	4.6381	4.5872
3.9	↓	5.6417	5.3301	5.1350	4.9996	4.8991	4.8209	4.7579	4.7058
4.0		5.7824	5.4634	5.2636	5.1251	5.0223	4.9422	4.8777	4.8243
4.1		5.9232	5.5968	5.3923	5.2506	5.1454	5.0635	4.9975	4.9430
4.2		6.0640	5.7302	5.5211	5.3762	5.2686	5.1849	5.1174	5.0615
4.3		6.2049	5.8637	5.6500	5.5019	5.3919	5.3063	5.2373	5.1803
4.4		6.3459	5.9972	5.7789	5.6275	5.5152	5.4277	5.3573	5.2991
4.5		6.4870	6.1308	5.9078	5.7533	5.6385	5.5492	5.4773	5.4178
4.6		6.6281	6.2645	6.0368	5.8791	5.7619	5.6708	5.5974	5.5367
4.7		6.7693	6.3982	6.1659	6.0049	5.8853	5.7923	5.7174	5.6555
4.8		6.9106	6.5319	6.2949	6.1307	6.0088	5.9139	5.8375	5.7744
4.9	1.0000	7.0518	6.6657	6.4241	6.2566	6.1323	6.0356	5.9577	5.8933
5.0	↓	7.1932	6.7996	6.5532	6.3825	6.2558	6.1572	6.0778	6.0122
5.1		7.3346	6.9334	6.6824	6.5085	6.3794	6.2789	6.1980	6.1311
5.2		7.4760	7.0673	6.8116	6.6345	6.5029	6.4006	6.3182	6.2501
5.3		7.6175	7.2013	6.9409	6.7605	6.6266	6.5224	6.4384	6.3691
5.4		7.7590	7.3353	7.0702	6.8865	6.7502	6.6441	6.5587	6.4881
5.5		7.9005	7.4693	7.1995	7.0126	6.8738	6.7659	6.6790	6.6071
5.6		8.0421	7.6033	7.3288	7.1386	6.9975	6.8877	6.7993	6.7262
5.7		8.1837	7.7374	7.4582	7.2648	7.1212	7.0095	6.9196	6.8452
5.8		8.3254	7.8715	7.5876	7.3909	7.2449	7.1314	7.0399	6.9643
5.9		8.4671	8.0056	7.7170	7.5170	7.3687	7.2532	7.1603	7.0834
6.0		8.6088	8.1398	7.84[illegible]4	7.6432	7.4924	7.3751	7.2806	7.2025
6.1		8.7505	8.2740	7.9759	7.7694	7.6162	7.4970	7.4010	7.3217
6.2		8.8923	8.4082	8.1054	7.8956	7.7400	7.6189	7.5214	7.4408
6.3		9.0341	8.5424	8.2348	8.0218	7.8638	7.7409	7.6418	7.5600
6.4		9.1759	8.6766	8.3644	8.1481	7.9876	7.8627	7.7622	7.6791
6.5		9.3177	8.8109	8.4939	8.2744	8.1114	7.9847	7.8827	7.7983
6.6		9.4596	8.9452	8.6234	8.4006	8.2353	8.1067	8.0031	7.9175
6.7		9.6015	9.0794	8.7530	8.5269	8.3592	8.2286	8.1236	8.0367
6.8		9.7434	9.2138	8.8826	8.6532	8.4830	8.3506	8.2440	8.1559
6.9		9.8853	9.3481	9.0122	8.7795	8.6069	8.4726	8.3645	8.2751
7.0		10.0272	9.4824	9.1418	8.9059	8.7308	8.5946	8.4850	8.3944
7.1		10.1692	9.6168	9.2714	9.0322	8.8547	8.7167	8.6055	8.5136
7.2		10.3112	9.7512	9.4010	9.1586	8.9786	8.8387	8.7260	8.6329
7.3		10.4532	9.8856	9.5307	9.2849	9.1026	8.9607	8.8465	8.7521
7.4		10.5952	10.0200	9.6603	9.4113	9.2265	8.0828	8.9670	8.8714
7.5		10.7372	10.1544	9.7900	9.5377	9.3505	9.2048	9.0876	8.9907
7.6		10.8792	10.2888	9.9197	9.6641	9.4744	9.3269	9.2081	9.1100
7.7		11.0213	10.4233	10.0494	9.7905	9.5984	9.4490	9.3287	9.2293
7.8		11.1633	10.5577	10.1791	9.9169	9.7224	9.5711	9.4492	9.3486
7.9		11.3054	10.6922	10.3088	10.0433	9.8464	9.6932	9.5698	9.4679
8.0	↓	11.4475	10.8266	10.4385	10.1698	9.9704	9.8153	9.6904	9.5872

TABLE A-4.—SAFETY MARGINS AT 95-PERCENT CONFIDENCE LEVEL

(a) Sample sizes 5 to 12

Safety margin, S_M	Probability, P_x	Sample size, N							
		5	6	7	8	9	10	11	12
−5.0	0	−3.1600	−3.2797	−3.3759	−3.4551	−3.5230	−3.5814	−3.6328	−3.6783
−4.0	0	−2.4898	−2.5882	−2.6674	−2.7326	−2.7884	−2.8364	−2.8787	−2.9161
−3.0	.0013	−1.8066	−1.8847	−1.9477	−1.9995	−2.0438	−2.0819	−2.1155	−2.1453
−2.0	.0227	−1.0897	−1.1507	−1.1999	−1.2402	−1.2748	−1.3044	−1.3304	−1.3535
−1.0	.1586	−.2651	−.3241	−.3691	−.4062	−.4363	−.4625	−.4847	−.5043
0	.5000	.9538	.8223	.7340	.6697	.6197	.5796	.5464	.5184
.1	.5398	1.1123	.9664	.8692	.7989	.7449	.7017	.6661	.6361
.2	.5792	1.2779	1.1159	1.0094	.9328	.8741	.8273	.7889	.7567
.3	.6179	1.4509	1.2710	1.1543	1.0708	1.0071	.9562	.9148	.8803
.4	.6554	1.6309	1.4315	1.3035	1.2118	1.1428	1.0882	1.0436	1.0065
.5	.6914	1.8155	1.5966	1.4564	1.3565	1.2822	1.2231	1.1751	1.1353
.6	.7257	2.0061	1.7662	1.6130	1.5047	1.4247	1.3609	1.3093	1.2666
.7	.7580	2.2023	1.9395	1.7729	1.6562	1.5691	1.5010	1.4456	1.3999
.8	.7881	2.4022	2.1163	1.9362	1.8104	1.7162	1.6434	1.5841	1.5353
.9	.8159	2.6057	2.2960	2.1023	1.9666	1.8658	1.7878	1.7245	1.6724
1.0	.8413	2.8129	2.4788	2.2709	2.1252	2.0174	1.9343	1.8667	1.8112
1.1	.8643	3.0237	2.6647	2.4415	2.2858	2.1710	2.0822	2.0103	1.9514
1.2	.8849	3.2370	2.8528	2.6140	2.4483	2.3263	2.2317	2.1554	2.0930
1.3	.9031	3.4526	3.0427	2.7883	2.6123	2.4830	2.3826	2.3018	2.2358
1.4	.9192	3.6702	3.2340	2.9642	2.7779	2.6408	2.5345	2.4493	2.3796
1.5	.9331	3.8896	3.4268	3.1415	2.9447	2.7998	2.6876	2.5977	2.5243
1.6	.9452	4.1105	3.6211	3.3200	3.1126	2.9598	2.8416	2.7471	2.6699
1.7	.9554	4.3329	3.8165	3.4997	3.2815	3.1208	2.9965	2.8972	2.8163
1.8	.9640	4.5564	4.0134	3.6803	3.4513	3.2826	3.1522	3.0482	2.9633
1.9	.9712	4.7811	4.2113	3.8618	3.6216	3.4451	3.3085	3.1997	3.1110
2.0	.9772	5.0067	4.4101	4.0440	3.7927	3.6083	3.4654	3.3518	3.2592
2.1	.9821	5.2332	4.6097	4.2270	3.9645	3.7721	3.6229	3.5044	3.4079
2.2	.9860	5.4605	4.8099	4.4106	4.1368	3.9364	3.7810	3.6575	3.5570
2.3	.9892	5.6885	5.0108	4.5947	4.3096	4.1013	3.9394	3.8111	3.7066
2.4	.9918	5.9171	5.2122	4.7794	4.4829	4.2665	4.0983	3.9650	3.8565
2.5	.9937	6.1463	5.4142	4.9646	4.6567	4.4322	4.2576	4.1193	4.0068
2.6	.9953	6.3761	5.6166	5.1501	4.8308	4.5982	4.4172	4.2739	4.1574
2.7	.9965	6.6063	5.8193	5.3361	5.0053	4.7646	4.5771	4.4289	4.3083
2.8	.9974	6.8369	6.0223	5.5222	5.1801	4.9311	4.7373	4.5840	4.4594
2.9	.9981	7.0679	6.2256	5.7086	5.3552	5.0978	4.8977	4.7394	4.6107
3.0	.9986	7.2993	6.4292	5.8953	5.5306	5.2647	5.0584	4.8950	4.7622
3.1	.9990	7.5311	6.6332	6.0823	5.7062	5.4319	5.2193	5.0508	4.9139
3.2	.9993	7.7631	6.8374	6.2696	5.8821	5.5993	5.3803	5.2068	5.0658
3.3	.9995	7.9954	7.0418	6.4570	6.0582	5.7668	5.5416	5.3630	5.2179
3.4	.9996	8.2280	7.2465	6.6447	6.2344	5.9346	5.7030	5.5194	5.3701
3.5	.9997	8.4608	7.4514	6.8326	6.4109	6.1026	5.8646	5.6759	5.5225
3.6	.9998	8.6939	7.6565	7.0207	6.5875	6.2707	6.0264	5.8325	5.6750
3.7	.9998	8.9271	7.8618	7.2089	6.7643	6.4389	6.1882	5.9893	5.8277

TABLE A-4.—Continued.

(a) Concluded.

Safety margin, S_M	Probability P_x	Sample size, N							
		5	6	7	8	9	10	11	12
3.8	0.9999	9.1606	8.0673	7.3973	6.9412	6.6073	6.3502	6.1462	5.9804
3.9	↓	9.3942	8.2729	7.5858	7.1182	6.7758	6.5124	6.3032	6.1333
4.0		9.6280	8.4787	7.7745	7.2954	6.9444	6.6746	6.4603	6.2852
4.1		9.8619	8.6846	7.9633	7.4727	7.1132	6.8369	6.6175	6.4393
4.2		10.0960	8.8906	8.1522	7.6501	7.2820	6.9994	6.7748	6.5924
4.3		10.3302	9.0968	8.3413	7.8276	7.4510	7.1619	6.9322	6.7456
4.4		10.5645	9.3031	8.5304	8.0052	7.6200	7.3245	7.0896	6.8989
4.5		10.7990	9.5095	8.7196	8.1829	7.7891	7.4872	7.2472	7.0523
4.6		11.0336	9.7160	8.9090	8.3606	7.9583	7.6499	7.4048	7.2057
4.7		11.2683	9.9225	9.0984	8.5385	8.1276	7.8128	7.5625	7.3592
4.8	↓	11.5031	10.1292	8.2879	8.7164	8.2969	7.9757	7.7202	7.5128
4.9	1.0000	11.7379	10.3359	9.4775	8.8944	8.4664	8.1386	7.8780	7.6664
5.0		11.9729	10.5428	9.6671	9.0725	8.6358	8.3017	8.0358	7.8200
5.1		12.2080	10.7497	9.8568	9.2506	8.8054	8.4647	8.1938	7.9738
5.2		12.4431	10.9567	10.0466	9.4288	8.9750	8.6279	8.3517	8.1275
5.3		12.6783	11.1637	10.2364	9.6070	9.1446	8.7910	8.5097	8.2813
5.4		12.9136	11.3708	10.4263	9.7853	9.3143	8.9543	8.6678	8.4352
5.5		13.1489	11.5780	10.6163	9.9636	9.4841	9.1176	8.8259	8.5891
5.6		13.3843	11.7852	10.8063	10.1420	9.6539	9.2809	8.9840	8.7430
5.7		13.6198	11.9925	10.9963	10.3205	9.8237	9.4442	9.1422	8.8970
5.8		13.8553	12.1998	11.1865	10.4989	9.9936	9.6076	9.3004	9.0510
5.9		14.0909	12.4072	11.3766	10.6775	10.1635	9.7710	9.4586	9.2050
6.0		14.3265	12.6146	11.5668	10.8560	10.3335	9.9345	9.6169	9.3591
6.1		14.5622	12.8221	11.7570	11.0346	10.5034	10.0980	9.7752	9.5132
6.2		14.7979	13.0296	11.9473	11.2133	10.6735	10.2615	9.9336	9.6673
6.3		15.0337	13.2372	12.1376	11.3919	10.8435	10.4251	10.0919	9.8215
6.4		15.2695	13.4447	12.3279	11.5706	11.0136	10.5887	10.2503	9.9757
6.5		15.5054	13.6524	12.5183	11.7494	11.1837	10.7523	10.4087	10.1299
6.6		15.7413	13.8600	12.7087	11.9281	11.3539	10.9160	10.5672	10.2841
6.7		15.9772	14.0677	12.8992	12.1069	11.5241	11.0796	10.7257	10.4384
6.8		16.2132	14.2755	13.0897	12.2857	11.6943	11.2433	10.8842	10.5927
6.9		16.4492	14.4832	13.2802	12.4646	11.8645	11.4070	11.0427	10.7470
7.0		16.6852	14.6910	13.4707	12.6434	12.0347	11.5708	11.2012	10.9013
7.1		16.9213	14.8988	13.6612	12.8223	12.2050	11.7345	11.3598	11.0556
7.2		17.1574	15.1067	13.8518	13.0013	12.3753	11.8983	11.5183	11.2100
7.3		17.3935	15.3146	14.0424	13.1802	12.5456	12.0621	11.6769	11.3644
7.4		17.6297	15.5225	14.2330	13.3592	12.7160	12.2259	11.8356	11.5187
7.5		17.8659	15.7304	14.4237	13.5381	12.8863	12.3898	11.9942	11.6732
7.6		18.1021	15.9383	14.6144	13.7171	13.0567	12.5536	12.1528	11.8276
7.7		18.3383	16.1463	14.8050	13.8962	13.2271	12.7175	12.3115	11.9820
7.8		18.5746	16.3543	14.9958	14.0752	13.3975	12.8814	12.4702	12.1365
7.9		18.8109	16.5623	15.1865	14.2542	13.5679	13.0453	12.6289	12.2909
8.0	↓	19.0472	16.7703	15.3772	14.4333	13.7384	13.2092	12.7876	12.4454

TABLE A-4.—Continued.

(b) Sample sizes 13 to 20

Safety margin, S_M	Probability, P_x	Sample size, N							
		13	14	15	16	17	18	19	20
−5.0	0	−3.7191	−3.7560	−3.7895	−3.8202	−3.8485	−3.8747	−3.8990	−3.9217
−4.0	0	−2.9497	−2.9800	−3.0076	−3.0329	−3.0561	−3.0777	−3.0977	−3.1164
−3.0	.0013	−2.1719	−2.1960	−2.2179	−2.2380	−2.2564	−2.2735	−2.2894	−2.3042
−2.0	.0227	−1.3741	−1.3927	−1.4097	−1.4251	−1.4394	−1.4525	−1.4647	−1.4761
−1.0	.1586	−.5217	−.5373	−.5514	−.5642	−.5759	−.5866	−.5965	−.6057
−0	.5000	.4943	.4733	.4547	.4382	.4234	.4100	.3978	.3867
.1	.5398	.6104	.5881	.5684	.5510	.5353	.5212	.5084	.4967
.2	.5792	.7293	.7055	.6846	.6661	.6496	.6347	.6211	.6087
.3	.6179	.8509	.8256	.8033	.7837	.7661	.7503	.7359	.7228
.4	.6554	.9751	.9480	.9243	.9034	.8848	.8679	.8527	.8388
.5	.6914	1.1017	1.0727	1.0475	1.0252	1.0054	.9875	.9713	.9566
.6	.7257	1.2306	1.1996	1.1727	1.1490	1.1279	1.1089	1.0918	1.0762
.7	.7580	1.3614	1.3284	1.2997	1.2745	1.2521	1.2319	1.2137	1.1972
.8	.7881	1.4942	1.4591	1.4286	1.4018	1.3780	1.3566	1.3373	1.3197
.9	.8159	1.6286	1.5912	1.5588	1.5304	1.5052	1.4825	1.4620	1.4435
1.0	.8413	1.7647	1.7250	1.6906	1.6605	1.6338	1.6097	1.5880	1.5684
1.1	.8643	1.9021	1.8600	1.8236	1.7917	1.7635	1.7380	1.7151	1.6945
1.2	.8849	2.0407	1.9962	1.9577	1.9240	1.8942	1.8674	1.8432	1.8214
1.3	.9031	2.1806	2.1335	2.0929	2.0574	2.0260	1.9977	1.9723	1.9493
1.4	.9192	2.3213	2.2718	2.2290	2.1916	2.1585	2.1288	2.1020	2.0779
1.5	.9331	2.4630	2.4109	2.3659	2.3265	2.2918	2.2606	2.2325	2.2072
1.6	.9452	2.6055	2.5507	2.5035	2.4622	2.4258	2.3930	2.3636	2.3371
1.7	.9554	2.7487	2.6913	2.6418	2.5986	2.5605	2.5261	2.4954	2.4676
1.8	.9640	2.8926	2.8325	2.7807	2.7355	2.6957	2.6598	2.6277	2.5986
1.9	.9712	3.0370	2.9743	2.9202	2.8730	2.8313	2.7939	2.7604	2.7301
2.0	.9772	3.1820	3.1165	3.0601	3.0108	2.9674	2.9284	2.8935	2.8619
2.1	.9821	3.3274	3.2592	3.2004	3.1491	3.1040	3.0633	3.0270	2.9941
2.2	.9860	3.4733	3.4023	3.3411	3.2878	3.2409	3.1986	3.1608	3.1267
2.3	.9892	3.6196	3.5458	3.4823	3.4269	3.3781	3.3343	3.2950	3.2596
2.4	.9918	3.7662	3.6896	3.6237	3.5662	3.5156	3.4702	3.4295	3.3927
2.5	.9937	3.9131	3.8338	3.7654	3.7059	3.6535	3.6064	3.5642	3.5262
2.6	.9953	4.0604	3.9782	3.9075	3.8458	3.7915	3.7428	3.6991	3.6598
2.7	.9965	4.2079	4.1229	4.0497	3.9860	3.9298	3.8794	3.8343	3.7987
2.8	.9974	4.3557	4.2678	4.1922	4.1263	4.0684	4.0163	3.9697	3.9277
2.9	.9981	4.5036	4.4129	4.3349	4.2669	4.2071	4.1533	4.1053	4.0619
3.0	.9986	4.6518	4.5582	4.4777	4.4076	4.3459	4.2905	4.2409	4.1963
3.1	.9990	4.8001	4.7036	4.6207	4.5485	4.4849	4.4279	4.3768	4.3308
3.2	.9993	4.9486	4.8493	4.7639	4.6895	4.6241	4.5653	4.5128	4.4654
3.3	.9995	5.0973	4.9951	4.9072	4.8307	4.7634	4.7030	4.6489	4.6002
3.4	.9996	5.2461	5.1410	5.0507	4.9720	4.9028	4.8407	4.7851	4.7351
3.5	.9997	5.3950	5.2871	5.1943	5.1135	5.4024	4.9786	4.9215	4.8701
3.6	.9998	5.5441	5.4333	5.3380	5.2550	5.1820	5.1165	5.0580	5.0052
3.7	.9998	5.6933	5.5796	5.4818	5.3967	5.3218	5.2546	5.1945	5.1404

TABLE A-4.—Continued.

(b) Concluded.

Safety margin, S_M	Probability P_x	Sample size, N							
		13	14	15	16	17	18	19	20
3.8	0.9999	5.8426	5.7260	5.6257	5.5385	5.4617	5.3928	5.3312	5.2757
3.9		5.9920	5.8725	5.7697	5.6803	5.6016	5.5310	5.4679	5.4110
4.0		6.1416	6.0191	5.9139	5.8223	5.7417	5.6694	5.6047	5.5465
4.1		6.2912	6.1658	6.0580	5.9643	5.8818	5.8078	5.7416	5.6820
4.2		6.4408	6.3126	6.2023	6.1064	6.0220	5.9463	5.8785	5.8176
4.3		6.5906	6.4594	6.3467	6.2485	6.1622	6.0848	6.0156	5.9532
4.4		6.7404	6.6063	6.4911	6.3908	6.3025	6.2234	6.1526	6.0889
4.5		6.8903	6.7533	6.6355	6.5331	6.4429	6.3621	6.2989	6.2247
4.6		7.0403	6.9003	6.7801	6.6754	6.5834	6.5008	6.4270	6.3605
4.7		7.1903	7.0474	6.9247	6.8178	6.7239	6.6396	6.5642	6.4963
4.8		7.3404	7.1946	7.0693	6.9603	6.8644	6.7784	6.7015	6.6322
4.9	1.0000	7.4906	7.3418	7.2140	7.1028	7.0050	6.9173	6.8388	6.7682
5.0		7.6408	7.4891	7.3588	7.2454	7.1456	7.0562	6.9762	6.9042
5.1		7.7910	7.6364	7.5035	7.3879	7.2863	7.1951	7.1136	7.0402
5.2		7.9413	7.7837	7.6484	7.5306	7.4270	7.3341	7.2510	7.1763
5.3		8.0916	7.9311	7.7932	7.6733	7.5677	7.4731	7.3885	7.3124
5.4		8.2420	8.0786	7.9381	7.8160	7.7085	7.6122	7.5260	7.4485
5.5		8.3924	8.2260	8.0831	7.9587	7.8494	7.7513	7.6636	7.5846
5.6		8.5429	8.3735	8.2281	8.1015	7.9902	7.8904	7.8012	7.7208
5.7		8.6934	8.5211	8.3731	8.2443	8.1311	8.0296	7.9388	7.8571
5.8		8.8439	8.6687	8.5181	8.3872	8.2720	8.1687	8.0764	7.9933
5.9		8.9944	8.8163	8.6632	8.5300	8.4129	8.3080	8.2141	8.1296
6.0		9.1450	8.9639	8.8083	8.6729	8.5539	8.4472	8.3518	8.2659
6.1		9.2956	9.1115	8.9534	8.8159	8.6949	8.5864	8.4895	8.4022
6.2		9.4463	9.2592	9.0986	8.9588	8.8359	8.7257	8.6272	8.5385
6.3		9.5969	9.4069	9.2438	9.1018	8.9770	8.8650	8.7650	8.6749
6.4		9.7476	9.5547	9.3890	9.2448	9.1180	9.0044	8.9028	8.8113
6.5		9.8983	9.7024	9.5342	9.3878	9.2591	9.1437	9.0406	8.9477
6.6		10.0491	9.8502	9.6794	9.5309	9.40[illegible]	9.2831	9.1784	9.0841
6.7		10.1998	9.9980	9.8247	9.6739	9.5413	9.4225	9.3162	9.2205
6.8		10.3506	10.1458	9.9700	9.8170	9.6825	9.5619	9.4540	9.3570
6.9		10.5014	10.2937	10.1153	9.9601	9.8236	9.7013	9.5919	9.4935
7.0		10.6522	10.4416	10.2606	10.1032	9.9648	9.8407	9.7298	9.6299
7.1		10.8031	10.5894	10.4059	10.2463	10.1060	9.9802	9.8677	9.7664
7.2		10.9539	10.7373	10.5513	10.3895	10.2472	10.1196	10.0056	9.9030
7.3		11.1048	10.8852	10.6967	10.5326	10.3884	10.2591	10.1435	10.0395
7.4		11.2557	11.0332	10.8421	10.67[illegible]	10.5296	10.3986	10.2815	10.1760
7.5		11.4066	11.1811	10.9875	10.81[illegible]	10.6709	10.5381	10.4194	10.3126
7.6		11.5575	11.3291	11.1329	10.9622	10.8122	10.6776	10.5574	10.4491
7.7		11.7084	11.4770	11.2783	11.1054	10.9534	10.8172	10.6954	10.5837
7.8		11.85[illegible]4	11.6250	11.4237	11.2487	11.0947	10.9567	10.8334	10.7223
7.9		12.0[illegible]	11.7730	11.5692	11.3919	11.2360	11.0963	10.9714	10.8589
8.0		12.1613	11.9210	11.7147	11.5352	11.3773	11.2358	11.1094	10.9955

TABLE A-4.—Continued.

(c) Sample sizes 21 to 28

Safety margin, S_M	Probability, P_x	Sample size, N							
		21	22	23	24	25	26	27	28
−5.0	0	−3.9429	−3.9629	−3.9816	−3.9993	−4.0160	−4.0318	−4.0468	−4.0612
−4.0	0	−3.1338	−3.1502	−3.1656	−3.1801	−3.1938	−3.2068	−3.2192	−3.2310
−3.0	.0013	−2.3180	−2.3310	−2.3432	−2.3547	−2.3655	−2.3759	−2.3856	−2.3949
−2.0	.0227	−1.4867	−1.4966	−1.5060	−1.5148	−1.5231	−1.5310	−1.5385	−1.5456
−1.0	.1586	−.6142	−.6222	−.6297	−.6368	−.6434	−.6497	−.6556	−.6613
−0	.5000	.3764	.3669	.3581	.3499	.3422	.3350	.3283	.3219
.1	.5398	.4859	.4759	.4667	.4581	.4501	.4426	.4356	.4289
.2	.5792	.5974	.5869	.5772	.5682	.5598	.5519	.5446	.5376
.3	.6179	.7108	.6998	.6896	.6801	.6713	.6630	.6553	.6480
.4	.6554	.8261	.8145	.8037	.7937	.7844	.7757	.7676	.7599
.5	.6914	.9432	.9308	.9194	.9089	.8991	.8899	.8813	.8733
.6	.7257	1.0619	1.0489	1.0368	1.0256	1.0153	1.0056	.9966	.9881
.7	.7580	1.1821	1.1683	1.1555	1.1437	1.1328	1.1226	1.1130	1.1041
.8	.7881	1.3038	1.2891	1.2756	1.2632	1.2516	1.2408	1.2307	1.2213
.9	.8159	1.4266	1.4111	1.3968	1.3837	1.3715	1.3601	1.3495	1.3396
1.0	.8413	1.5506	1.5343	1.5192	1.5053	1.4925	1.4805	1.4693	1.4588
1.1	.8643	1.6756	1.6584	1.6425	1.6279	1.6144	1.6017	1.5900	1.5790
1.2	.8849	1.8016	1.7834	1.7667	1.7513	1.7371	1.7238	1.7114	1.6999
1.3	.9031	1.9284	1.9093	1.8918	1.8756	1.8606	1.8466	1.8337	1.8215
1.4	.9192	2.0560	2.0359	2.0175	2.0005	1.9848	1.9701	1.9565	1.9438
1.5	.9331	2.1842	2.1631	2.1438	2.1260	2.1095	2.0942	2.0799	2.0666
1.6	.9452	2.3130	2.2910	2.2707	2.2521	2.2349	2.2188	2.2039	2.1899
1.7	.9554	2.4424	2.4194	2.3982	2.3787	2.3607	2.3439	2.3283	2.3[illegible]8
1.8	.9640	2.5723	2.5482	2.5262	2.5058	2.4870	2.4695	2.4532	2.4360
1.9	.9712	2.7026	2.6775	2.6545	2.6333	2.6137	2.5955	2.5785	2.5626
2.0	.9772	2.8333	2.8072	2.7832	2.7611	2.7407	2.7217	2.7041	2.6876
2.1	.9821	2.9644	2.9372	2.9123	2.8893	2.8681	2.8483	2.8300	2.8128
2.2	.9860	3.0958	3.0675	3.0416	3.0178	2.9957	2.9753	2.9562	2.9384
2.3	.9892	3.2275	3.1982	3.1713	3.1465	3.1237	3.1024	3.0827	3.0642
2.4	.9918	3.3594	3.3291	3.3012	3.2756	3.2519	3.2298	3.2094	3.1902
2.5	.9937	3.4917	3.4602	3.4314	3.4048	3.3803	3.3575	3.3363	3.3165
2.6	.9953	3.6241	3.5916	3.5617	3.5343	3.5089	3.4853	3.4634	3.4429
2.7	.9965	3.7568	3.7231	3.6923	3.6639	3.6377	3.6134	3.5907	3.5695
2.8	.9974	3.8896	3.8549	3.8231	3.7938	3.7667	3.7416	3.7182	3.6963
2.9	.9981	4.0226	3.9868	3.9540	3.9237	3.8958	3.8699	3.8458	3.8233
3.0	.9986	4.1558	4.1188	4.0850	4.0538	4.0251	3.9984	3.9735	3.9503
3.1	.9990	4.2891	4.2510	4.2162	4.1841	4.1544	4.1269	4.1013	4.0775
3.2	.9993	4.4225	4.3833	4.3475	4.3145	4.2839	4.2557	4.2293	4.2047
3.3	.9995	4.5560	4.5158	4.4789	4.4449	4.4136	4.3845	4.3574	4.3321
3.4	.9996	4.6897	4.6483	4.6104	4.5755	4.5433	4.5134	4.4856	4.4596
3.5	.9997	4.8235	4.7810	4.7420	4.7062	4.6731	4.6424	4.6138	4.5872
3.6	.9998	4.9573	4.9137	4.8738	4.8370	4.8030	4.7715	4.7422	4.7148
3.7	.9998	5.0913	5.0466	5.0056	4.9679	4.9330	4.9007	4.8707	4.8426

TABLE A-4.—Continued.

(c) Concluded.

Safety margin, S_M	Probability P_x	Sample size, N							
		21	22	23	24	25	26	27	28
3.8	0.9999	5.2254	5.1795	5.1375	5.0988	5.0631	5.0300	4.9992	4.9704
3.9		5.3595	5.3125	5.2695	5.2298	5.1933	5.1593	5.1278	5.0983
4.0		5.4937	5.4456	5.4015	5.3609	5.3235	5.2887	5.2564	5.2262
4.1		5.6280	5.5787	5.5336	5.4921	5.4538	5.4182	5.3851	5.3542
4.2		5.7623	5.7119	5.6658	5.6233	5.5841	5.5477	5.5139	5.4823
4.3		5.8967	5.8452	5.7980	5.7546	5.7145	5.6773	5.6427	5.6104
4.4		6.0311	5.9785	5.9303	5.8859	5.8449	5.8069	5.7716	5.7386
4.5		6.1657	6.1119	6.0626	6.0173	5.9754	5.9366	5.9005	5.8668
4.6		6.3002	6.2453	6.1950	6.1487	6.1060	6.0663	6.0295	5.9951
4.7		6.4348	6.3788	6.3274	6.2802	6.2366	6.1961	6.1585	6.1234
4.8		6.5695	6.5123	6.4599	6.4117	6.3672	6.3259	6.2875	6.2517
4.9	1.0000	6.7042	6.6458	6.5924	6.5433	6.4979	6.4558	6.4166	6.3801
5.0		6.8389	6.7794	6.7250	6.6748	6.6286	6.5856	6.5457	6.5085
5.1		6.9737	6.9131	6.8575	6.8065	6.7593	6.7156	6.6749	6.6369
5.2		7.1085	7.0467	6.9902	6.9381	6.8901	6.8455	6.8041	6.7654
5.3		7.2433	7.1804	7.1228	7.0698	7.0209	6.9755	6.9333	6.8939
5.4		7.3782	7.3141	7.2555	7.2015	7.1517	7.1055	7.0625	7.0224
5.5		7.5131	7.4479	7.3882	7.3333	7.2826	7.2355	7.1918	7.1510
5.6		7.6480	7.5817	7.5209	7.4651	7.4134	7.3656	7.3211	7.2795
5.7		7.7830	7.7155	7.6537	7.5969	7.5443	7.4957	7.4504	7.4081
5.8		7.9180	7.8493	7.7865	7.7287	7.6753	7.6258	7.5797	7.5368
5.9		8.0530	7.9832	7.9193	7.8605	7.8062	7.7559	7.7091	7.6654
6.0		8.1880	8.1171	8.0521	7.9924	7.9372	7.8861	7.8385	7.7941
6.1		8.3231	8.2510	8.1850	8.1243	8.0682	8.0162	7.9679	7.9228
6.2		8.4582	8.3849	8.3179	8.2562	8.1992	8.1464	8.0973	8.0515
6.3		8.5933	8.5189	8.4508	8.3881	8.3303	8.2766	8.2267	8.1802
6.4		8.7284	8.6529	8.5837	8.5201	8.4613	8.4069	8.3562	8.3089
6.5		8.8635	8.7868	8.7166	8.6520	8.5924	8.5371	8.4857	8.4377
6.6		8.9987	8.9208	8.8496	8.7840	8.7235	8.6674	8.6152	8.5664
6.7		9.1338	9.0549	8.9825	8.9160	8.8546	8.7976	8.7447	8.6952
6.8		9.2690	9.1889	9.1155	9.0480	8.9857	8.9279	8.8742	8.8240
6.9		9.4042	9.3230	9.2485	9.1801	9.1168	9.0582	9.0037	8.9528
7.0		9.5395	9.4570	9.3815	9.3121	9.2480	9.1885	9.1333	9.0817
7.1		9.6747	9.5911	9.5146	9.4442	9.3791	9.3189	9.2628	9.2105
7.2		9.8099	9.7252	9.6476	9.5762	9.5103	9.4492	9.3924	9.3394
7.3		9.9452	9.8593	9.7807	9.7083	9.6415	9.5796	9.5220	9.4682
7.4		10.0805	9.9934	9.9137	9.8404	9.7727	9.7099	9.6516	9.5971
7.5		10.2158	10.1276	10.0468	9.9725	9.9039	9.8403	9.7812	9.7260
7.6		10.3511	10.2617	10.1799	10.1046	10.0351	9.9707	9.9108	9.8549
7.7		10.4864	10.3959	10.3130	10.2368	10.1664	10.1011	10.0404	9.9838
7.8		10.6217	10.5300	10.4461	10.3689	10.2976	10.2315	10.1700	10.1127
7.9		10.7570	10.6642	10.5792	10.5010	10.4288	10.3619	10.2997	10.2416
8.0		10.8924	10.7984	10.7123	10.6332	10.5601	10.4923	10.4293	10.3705

TABLE A-4.—Continued.

(d) Sample sizes 30 to 100

Safety margin, S_M	Probability, P_x	Sample size, N							
		30	40	50	60	70	80	90	100
−5.0	0	−4.0878	−4.1922	−4.2661	−4.3220	−4.3664	−4.4028	−4.4333	−4.4594
−4.0	0	−3.2528	−3.3386	−3.3992	−3.4452	−3.4815	−3.5113	−3.5364	−3.5578
−3.0	.0013	−2.4123	−2.4801	−2.5280	−2.5642	−2.5929	−2.6164	−2.6361	−2.6530
−2.0	.0227	−1.5589	−1.6105	−1.6469	−1.6743	−1.6960	−1.7138	−1.7286	−1.7413
−1.0	.1586	−.6717	−.7122	−.7402	−.7612	−.7777	−.7911	−.8023	−.8118
−0	.5000	.3102	.2664	.2371	.2158	.1993	.1861	.1752	.1660
.1	.5398	.4168	.3713	.3411	.3191	.3022	.2886	.2775	.2681
.2	.5792	.5249	.4776	.4462	.4235	.4060	.3921	.3806	.3710
.3	.6179	.6347	.5852	.5526	.5290	.5109	.4965	.4846	.4747
.4	.6554	.7459	.6941	.6600	.6354	.6166	.6017	.5894	.5791
.5	.6914	.8586	.8042	.7685	.7429	.7233	.7078	.6950	.6843
.6	.7257	.9726	.9154	.8781	.8512	.8308	.8146	.8014	.7903
.7	.7580	1.0877	1.0277	.9885	.9604	.9391	.9222	.9084	.8968
.8	.7881	1.2041	1.1409	1.0998	1.0704	1.0481	1.0304	1.0160	1.0039
.9	.8159	1.3214	1.2550	1.2118	1.1811	1.1577	1.1393	1.1242	1.1116
1.0	.8413	1.4398	1.3699	1.3246	1.2924	1.2680	1.2487	1.2329	1.2198
1.1	.8643	1.5589	1.4855	1.4381	1.4043	1.3787	1.3586	1.3421	1.3284
1.2	.8849	1.6788	1.6018	1.5520	1.5167	1.4900	1.4689	1.4518	1.4374
1.3	.9031	1.7994	1.7187	1.6666	1.6296	1.6017	1.5797	1.5618	1.5468
1.4	.9192	1.9206	1.8361	1.7816	1.7429	1.7138	1.6908	1.6721	1.6566
1.5	.9331	2.0423	1.9539	1.8970	1.8566	1.8262	1.8023	1.7828	1.7666
1.6	.9452	2.1645	2.0722	2.0127	1.9707	1.9390	1.9140	1.8938	1.8769
1.7	.9554	2.2873	2.1908	2.1289	2.0851	2.0521	2.0261	2.0050	1.9874
1.8	.9640	2.4104	2.3099	2.2453	2.1997	2.1654	2.1384	2.1164	2.0981
1.9	.9712	2.5338	2.4292	2.3620	2.3146	2.2789	2.2509	2.2281	2.2091
2.0	.9772	2.6576	2.5488	2.4790	2.4297	2.3927	2.3635	2.3399	2.3202
2.1	.9821	2.7817	2.6686	2.5962	2.5451	2.5066	2.4764	2.4519	2.4314
2.2	.9860	2.9061	2.7887	2.7136	2.6606	2.6207	2.5894	2.5640	2.5428
2.3	.9892	3.0307	2.9090	2.8312	2.7763	2.7350	2.7026	2.6763	2.6544
2.4	.9918	3.1555	3.0295	2.9489	2.8921	2.8494	2.8159	2.7887	2.7661
2.5	.9937	3.2905	3.1502	3.0669	3.0081	2.9640	2.9293	2.9012	2.8778
2.6	.9953	3.4058	3.2710	3.1849	3.1243	3.0787	3.0429	3.0139	2.9897
2.7	.9965	3.5311	3.3920	3.3031	3.2405	3.1935	3.1565	3.1266	3.1017
2.8	.9974	3.6567	3.5131	3.4214	3.3568	3.3083	3.2703	3.2394	3.2137
2.9	.9981	3.7824	3.6344	3.5398	3.4733	3.4233	3.3841	3.3523	3.3258
3.0	.9986	3.9082	3.7557	3.6583	3.5898	3.5384	3.4980	3.4653	3.4380
3.1	.9990	4.0341	3.8771	3.7769	3.7064	3.6535	3.6120	3.5783	3.5503
3.2	.9993	4.1601	3.9987	3.8956	3.8231	3.7687	3.7260	3.6914	3.6626
3.3	.9995	4.2863	4.1203	4.0144	3.9399	3.8840	3.8401	3.8045	3.7750
3.4	.9996	4.4125	4.2420	4.1332	4.0567	3.9993	3.9542	3.9177	3.8874
3.5	.9997	4.5388	4.3638	4.2521	4.1736	4.1147	4.0684	4.0310	3.9998
3.6	.9998	4.6652	4.4856	4.3711	4.2905	4.2301	4.1827	4.1443	4.1123
3.7	.9998	4.7917	4.6075	4.4901	4.4075	4.3456	4.2970	4.2576	4.2249

TABLE A-4.—Concluded.

(d) Concluded.

Safety margin, S_M	Probability P_x	Sample size, N							
		30	40	50	60	70	80	90	100
3.8	0.9999	4.9182	4.7295	4.6092	4.5246	4.4611	4.4114	4.3710	4.3375
3.9		5.0448	4.8515	4.7283	4.6417	4.5767	4.5257	4.4844	4.4501
4.0		5.1715	4.9736	4.8475	4.7588	4.6923	4.6402	4.5979	4.5628
4.1		5.2982	5.0958	4.9667	4.8760	4.8080	4.7546	4.7114	4.6755
4.2		5.4250	5.2179	5.0860	4.9932	4.9237	4.8691	4.8249	4.7882
4.3		5.5519	5.3402	5.2053	5.1105	5.0394	4.9836	4.9385	4.9009
4.4		5.6788	5.4624	5.3246	5.2278	5.1551	5.0982	5.0520	5.0137
4.5		5.8057	5.5847	5.4440	5.3451	5.2709	5.2128	5.1656	5.1265
4.6		5.9327	5.7071	5.5634	5.4624	5.3867	5.3274	5.2793	5.2393
4.7		6.0597	5.8294	5.6828	5.5798	5.5025	5.4420	5.3929	5.3521
4.8		6.1867	5.9519	5.8023	5.6972	5.6184	5.5566	5.5066	5.4650
4.9	1.0000	6.3138	6.0743	5.9218	5.8146	5.7343	5.6713	5.6203	5.5779
5.0		6.4409	6.1968	6.0413	5.9321	5.8502	5.7860	5.7340	5.6908
5.1		6.5681	6.3193	6.1608	6.0495	5.9661	5.9007	5.8477	5.8037
5.2		6.6952	6.4418	6.2804	6.1670	6.0820	6.0154	5.9614	5.9166
5.3		6.8224	6.5643	6.4000	6.2845	6.1980	6.1302	6.0752	6.0296
5.4		6.9497	6.6869	6.5196	6.4021	6.3140	6.2449	6.1890	6.1425
5.5		7.0769	6.8095	6.6392	6.5196	6.4300	6.3597	6.3028	6.2555
5.6		7.2042	6.9321	6.7588	6.6372	6.5460	6.4745	6.4166	6.3685
5.7		7.3315	7.0547	6.8785	6.7548	6.6620	6.5893	6.5304	6.4815
5.8		7.4588	7.1774	6.9982	6.8723	6.7780	6.7041	6.6442	6.5945
5.9		7.5862	7.3000	7.1179	6.9900	6.8941	6.8189	6.7581	6.7075
6.0		7.7136	7.4227	7.2376	7.1076	7.0101	6.9338	6.8719	6.8205
6.1		7.8409	7.5454	7.3573	7.2252	7.1262	7.0486	6.9858	6.9336
6.2		7.9683	7.6681	7.4770	7.3429	7.2423	7.1635	7.0996	7.0466
6.3		8.0958	7.7908	7.5968	7.4605	7.3584	7.2784	7.2135	7.1597
6.4		8.2232	7.9136	7.7165	7.5782	7.4745	7.3932	7.3274	7.2727
6.5		8.3506	8.0363	7.8363	7.6959	7.5906	7.5081	7.4413	7.3858
6.6		8.4781	8.1591	7.9561	7.8136	7.7068	7.6230	7.5552	7.4989
6.7		8.6056	8.2819	8.0759	7.9313	7.8229	7.7379	7.6691	7.6120
6.8		8.7331	8.4047	8.1957	8.0490	7.9390	7.8529	7.7831	7.7251
6.9		8.8606	8.5275	8.3155	8.1667	8.0552	7.9678	7.8970	7.8382
7.0		8.9881	8.6503	8.4354	8.2844	8.1714	8.0827	8.0109	7.9513
7.1		9.1157	8.7731	8.5552	8.4022	8.2875	8.1977	8.1249	8.0644
7.2		9.2432	8.8960	8.6750	8.5199	8.4037	8.3126	8.2388	8.1776
7.3		9.3708	9.0188	8.7949	8.6377	8.5199	8.4276	8.3528	8.2907
7.4		9.4983	9.1417	8.9147	8.7554	8.6361	8.5425	8.4668	8.4038
7.5		9.6259	9.2645	9.0346	8.8732	8.7523	8.6575	8.5807	8.5170
7.6		9.7535	9.3874	9.1545	8.9910	8.8685	8.7725	8.6947	8.6301
7.7		9.8811	9.5103	9.2744	9.1088	8.9847	8.8874	8.8087	8.7433
7.8		10.0087	9.6332	9.3943	9.2266	9.1009	9.0024	8.9227	8.8564
7.9		10.1363	9.7561	9.5142	9.3444	9.2171	9.1174	9.0367	8.9696
8.0		10.2639	9.8790	9.6341	9.4622	9.3334	9.2324	9.1507	9.0828

TABLE A-5.—SAFETY MARGINS AT 90-PERCENT CONFIDENCE LEVEL

(a) Sample sizes 5 to 12

Safety margin, S_M	Probability, P_x	Sample size, N							
		5	6	7	8	9	10	11	12
−5.0	0	−3.5162	−3.6140	−3.6932	−3.7586	−3.8146	−3.8623	−3.9045	−3.9418
−4.0	0	−2.7824	−2.8627	−2.9278	−2.9816	−3.0276	−3.0669	−3.1016	−3.1323
−3.0	.0013	−2.0381	−2.1018	−2.1535	−2.1962	−2.2327	−2.2640	−2.2915	−2.3159
−2.0	.0227	−1.2682	−1.3178	−1.3578	−1.3910	−1.4192	−1.4435	−1.4647	−1.4835
−1.0	.1586	−.4225	−.4673	−.5023	−.5312	−.5548	−.5752	−.5927	−.6082
−0	.5000	.6857	.6023	.5439	.5000	.4657	.4373	.4137	.3936
.1	.5398	.8218	.7303	.6665	.6190	.5822	.5518	.5267	.5053
.2	.5792	.9632	.8623	.7930	.7412	.7014	.6689	.6420	.6193
.3	.6179	1.1098	.9984	.9229	.8664	.8234	.7885	.7598	.7355
.4	.6554	1.2615	1.1385	1.0557	.9946	.9481	.9105	.8797	.8537
.5	.6914	1.4168	1.2817	1.1912	1.1256	1.0750	1.0347	1.0016	.9739
.6	.7257	1.5762	1.4282	1.3296	1.2591	1.2043	1.1610	1.1256	1.0960
.7	.7580	1.7392	1.5777	1.4709	1.3945	1.3358	1.2894	1.2514	1.2197
.8	.7881	1.9057	1.7301	1.6147	1.5321	1.4693	1.4196	1.3789	1.3451
.9	.8159	2.0753	1.8851	1.7608	1.6715	1.6044	1.5512	1.5078	1.4717
1.0	.8413	2.2475	2.0422	1.9087	1.8127	1.7412	1.6843	1.6381	1.5997
1.1	.8643	2.4221	2.2010	2.0578	1.9558	1.8792	1.8186	1.7695	1.7288
1.2	.8849	2.5988	2.3615	2.2085	2.1003	2.0184	1.9542	1.9021	1.8590
1.3	.9031	2.7769	2.5237	2.3605	2.2460	2.1589	2.0908	2.0357	1.9901
1.4	.9192	2.9564	2.6871	2.5138	2.3923	2.3003	2.2283	2.1701	2.1220
1.5	.9331	3.1372	2.8518	2.6681	2.5396	2.4426	2.3666	2.3052	2.2546
1.6	.9452	3.3192	3.0175	2.8234	2.6878	2.5857	2.5057	2.4411	2.3879
1.7	.9554	3.5024	3.1841	2.9795	2.8367	2.7296	2.6454	2.5776	2.5217
1.8	.9640	3.6868	3.3516	3.1353	2.9863	2.8740	2.7857	2.7147	2.6562
1.9	.9712	3.8720	3.5198	3.2938	3.1366	3.0189	2.9265	2.8522	2.7910
2.0	.9772	4.0580	3.6886	3.4519	3.2873	3.1643	3.0678	2.9902	2.9262
2.1	.9821	4.2446	3.8580	3.6105	3.4386	3.3101	3.2095	3.1285	3.0619
2.2	.9860	4.4318	4.0279	3.7696	3.5903	3.4563	3.3515	3.2672	3.1979
2.3	.9892	4.6195	4.1983	3.9292	3.7424	3.6029	3.4940	3.4063	3.3341
2.4	.9918	4.8076	4.3691	4.0891	3.8948	3.7499	3.6367	3.5456	3.4707
2.5	.9937	4.9962	4.5403	4.2493	4.0476	3.8971	3.7797	3.6852	3.6076
2.6	.9953	5.1851	4.7118	4.4099	4.2006	4.0446	3.9230	3.8251	3.7446
2.7	.9965	5.3742	4.8836	4.5707	4.3539	4.1924	4.0665	3.9652	3.8819
2.8	.9974	5.5636	5.0557	4.7318	4.5075	4.3404	4.2102	4.1054	4.0194
2.9	.9981	5.7532	5.2281	4.8931	4.6613	4.4886	4.3541	4.2459	4.1570
3.0	.9986	5.9431	5.4007	5.0547	4.8152	4.6870	4.4982	4.3865	4.2948
3.1	.9990	6.1332	5.5735	5.2164	4.9694	4.7855	4.6425	4.5273	4.4328
3.2	.9993	6.3236	5.7465	5.3784	5.1237	4.9342	4.7869	4.6683	4.5709
3.3	.9995	6.5142	5.9197	5.5405	5.2782	5.0831	4.9314	4.8093	4.7091
3.4	.9996	6.7049	6.0931	5.7027	5.4328	5.2321	5.0761	4.9505	4.8475
3.5	.9997	6.8958	6.2666	5.8651	5.5876	5.3812	5.2209	5.0918	4.9859
3.6	.9998	7.0869	6.4402	6.0276	5.7425	5.5305	5.3658	5.2332	5.1245
3.7	.9998	7.2781	6.6140	6.1903	5.8975	5.6798	5.5108	5.3747	5.2631

TABLE A-5.—Continued.

(a) Concluded.

Safety margin, S_M	Probability, P_x	Sample size, N							
		5	6	7	8	9	10	11	12
3.8	0.9999	7.4694	6.7879	6.3531	6.0526	5.8293	5.6559	5.5163	5.4018
3.9		7.6609	6.9619	6.5159	6.2078	5.9788	5.8011	5.6580	5.5406
4.0		7.8525	7.1360	6.6789	6.3631	6.1284	5.9464	5.7998	5.6795
4.1		8.0442	7.3102	6.8419	6.5185	6.2781	6.0917	5.9416	5.8185
4.2		8.2360	7.4845	7.0051	6.6740	6.4279	6.2371	6.0835	5.9575
4.3		8.4279	7.6589	7.1683	6.8295	6.5778	6.3826	6.2254	6.0965
4.4		8.6199	7.8334	7.3316	6.9851	6.7277	6.5282	6.3675	6.2357
4.5		8.8120	8.0079	7.4949	7.1408	6.8777	6.6738	6.5095	6.3749
4.6		9.0041	8.1826	7.6584	7.2965	7.0277	6.8194	6.6517	6.5141
4.7		9.1963	8.3572	7.8219	7.4523	7.1778	6.9651	6.7938	6.6534
4.8		9.3886	8.5320	7.9854	7.6082	7.3280	7.1109	6.9361	6.7927
4.9	1.0000	9.5809	8.7068	8.1490	7.7641	7.4782	7.2567	7.0783	6.9321
5.0		9.7734	8.8816	8.3127	7.9200	7.6284	7.4025	7.2206	7.0715
5.1		9.9658	9.0566	8.4764	8.0760	7.7787	7.5484	7.3630	7.2109
5.2		10.1584	9.2315	8.6401	8.2320	7.9290	7.6944	7.5054	7.3504
5.3		10.3509	9.4065	8.8039	8.3881	8.0794	7.8403	7.6478	7.4900
5.4		10.5436	9.5816	8.9677	8.5442	8.2298	7.9863	7.7902	7.6295
5.5		10.7362	9.7567	9.1316	8.7004	8.3802	8.1324	7.9327	7.7691
5.6		10.9289	9.9318	9.2955	8.8566	8.5307	8.2784	8.0752	7.9087
5.7		11.1217	10.1070	9.4595	9.0128	8.6812	8.4245	8.2178	8.0483
5.8		11.3145	10.2822	9.6235	9.1691	8.8318	8.5706	8.3603	8.1880
5.9		11.5073	10.4574	9.7875	9.3253	8.9823	8.7168	8.5029	8.3277
6.0		11.7002	10.6327	9.9515	9.4817	9.1329	8.8630	8.6456	8.4674
6.1		11.8931	10.8080	10.1156	9.6380	9.2835	9.0092	8.7882	8.6071
6.2		12.0860	10.9833	10.2797	9.7944	9.4342	9.1554	8.9309	8.7469
6.3		12.2790	11.1587	10.4438	9.9508	9.5848	9.3016	9.0736	8.8866
6.4		12.4720	11.3341	10 6079	10.1072	9.7355	9.4479	9.2163	9.0264
6.5		12.6650	11.5095	10.7721	10.2636	9 8862	9.5942	9.3590	9.1662
6.6		12.8581	11.6849	10.9363	10.4201	10.0869	9.7405	9.5017	9.3061
6.7		13.0512	11.8604	11.1005	10.5765	10.1877	9.8868	9.6445	9.4459
6.8		13.2443	12.0359	11.2648	10.7330	10.3385	10.0332	9.7873	9.5858
6.9		13.4374	12.2114	11.4290	10.8896	10.4892	10.1795	9.9301	9.7257
7.0		13.6305	12.3869	11.5933	11.0461	10.6400	10.3259	10.0729	9.8656
7.1		13.8237	12.5625	11.7576	11.2026	10.7909	10.4723	10.2157	10.0055
7.2		14.0169	12.7380	11.9219	11.3592	10.9417	10.6187	10.3585	10.1454
7.3		14.2101	12.9136	12.0863	11.5158	11.0925	10.7651	10.5014	10.2853
7.4		14.4033	13.0892	12.2506	11.6724	11.2434	10.9116	10.6443	10.4253
7.5		14.5966	13.2648	12.4150	11.8290	11.3943	11.0580	10.7872	10.5652
7.6		14.7899	13.4405	12.5794	11.9857	11.5452	11.2045	10.9300	10.7052
7.7		14.9831	13.6161	12.7437	12.1423	11.6961	11.3509	11.0729	10.8452
7.8		15.1764	13.7918	12.9082	12.2990	11.8470	11.4974	11.2159	10.9852
7.9		15.3698	13.9675	13.0726	12.4556	11.9979	11.6439	11.3588	11.1252
8.0		15.5631	14.1432	13.2370	12.6123	12.1488	11.7904	11.5017	11.2652

TABLE A-5.—Continued.

(b) Sample sizes 13 to 20

Safety margin, S_M	Probability, P_x	Sample size, N							
		13	14	15	16	17	18	19	20
−5.0	0	−3.9752	−4.0054	−4.0328	−4.0579	−4.0809	−4.1023	−4.1221	−4.1406
−4.0	0	−3.1598	−3.1846	−3.2072	−3.2278	−3.2468	−3.2643	−3.2806	−3.2958
−3.0	.0013	−2.3377	−2.3574	−2.3753	−2.3916	−2.4067	−2.4206	−2.4335	−2.4455
−2.0	.0227	−1.5003	−1.5155	−1.5293	−1.5419	−1.5534	−1.5641	−1.5739	−1.5831
−1.0	.1586	−.6220	−.6343	−.6455	−.6556	−.6649	−.6734	−.6813	−.6886
−0	.5000	.3762	.3609	.3473	.3351	.3242	.3143	.3052	.2969
.1	.5398	.4869	.4707	.4564	.4436	.4321	.4217	.4123	.4036
.2	.5792	.5997	.5826	.5675	.5540	.5419	.5310	.5210	.5119
.3	.6179	.7146	.6964	.6804	.6662	.6534	.6419	.6314	.6218
.4	.6554	.8315	.8122	.7952	.7801	.7666	.7544	.7433	.7332
.5	.6914	.9502	.9296	.9116	.8956	.8813	.8684	.8566	.8460
.6	.7257	1.0707	1.0488	1.0296	1.0126	.9975	.9838	.9714	.9601
.7	.7580	1.1927	1.1694	1.1490	1.1310	1.1149	1.1004	1.0873	1.0753
.8	.7881	1.3163	1.2915	1.2698	1.2507	1.2336	1.2182	1.2044	1.1917
.9	.8159	1.4411	1.4148	1.3918	1.3715	1.3534	1.3372	1.3225	1.3091
1.0	.8413	1.5673	1.5393	1.5149	1.4935	1.4744	1.4572	1.4416	1.4275
1.1	.8643	1.6944	1.6648	1.6390	1.6163	1.5962	1.5780	1.5616	1.5467
1.2	.8849	1.8225	1.7913	1.7640	1.7401	1.7188	1.6996	1.6824	1.6667
1.3	.9031	1.9516	1.9186	1.8899	1.8646	1.8422	1.8220	1.8038	1.7873
1.4	.9192	2.0814	2.0466	2.0164	1.9898	1.9662	1.9450	1.9259	1.9086
1.5	.9331	2.2119	2.1753	2.1435	2.1156	2.0908	2.0686	2.0485	2.0303
1.6	.9452	2.3430	2.3046	2.2712	2.2420	2.2160	2.1927	2.1716	2.1526
1.7	.9554	2.4747	2.4344	2.3995	2.3688	2.3416	2.3172	2.2952	2.2753
1.8	.9640	2.6069	2.5648	2.5282	2.4962	2.4677	2.4422	2.4192	2.3984
1.9	.9712	2.7395	2.6955	2.6573	2.6238	2.5942	2.5675	2.5435	2.5218
2.0	.9772	2.8725	2.8266	2.7868	2.7519	2.7210	2.6932	2.6682	2.6456
2.1	.9821	3.0059	2.9580	2.9166	2.8802	2.8480	2.8191	2.7931	2.7696
2.2	.9860	3.1396	3.0898	3.0467	3.0089	2.9754	2.9454	2.9183	2.8939
2.3	.9892	3.2736	3.2219	3.1771	3.1378	3.1031	3.0719	3.0438	3.0184
2.4	.9918	3.4079	3.3542	3.3077	3.2670	3.2309	3.1986	3.1695	3.1432
2.5	.9937	3.5424	3.4867	3.4385	3.3963	3.3590	3.3255	3.2954	3.2681
2.6	.9953	3.6771	3.6195	3.5696	3.5259	3.4873	3.4526	3.4214	3.3932
2.7	.9965	3.8121	3.7525	3.7009	3.6557	3.6157	3.5799	3.5476	3.5185
2.8	.9974	3.9472	3.8856	3.8323	3.7856	3.7444	3.7073	3.6740	3.6439
2.9	.9981	4.0825	4.0189	3.9639	3.9157	3.8731	3.8349	3.8005	3.7695
3.0	.9986	4.2179	4.1523	4.0956	4.0459	4.0020	3.9626	3.9272	3.8952
3.1	.9990	4.3535	4.2859	4.2275	4.1763	4.1310	4.0904	4.0539	4.0209
3.2	.9993	4.4893	4.4197	4.3594	4.3067	4.2602	4.2183	4.1808	4.1468
3.3	.9995	4.6251	4.5535	4.4915	4.4373	4.3894	4.3464	4.3078	4.2728
3.4	.9996	4.7611	4.6874	4.6237	4.5680	4.5187	4.4745	4.4348	4.3989
3.5	.9997	4.8971	4.8215	4.7560	4.6988	4.6481	4.6027	4.5619	4.5251
3.6	.9998	5.0333	4.9556	4.8884	4.8296	4.7777	4.7310	4.6892	4.6513
3.7	.9998	5.1695	5.0898	5.0209	4.9605	4.9072	4.8594	4.8165	4.7775

TABLE A-5.—Continued.

(b) Concluded.

Safety margin, S_M	Probability P_x	Sample size, N							
		13	14	15	16	17	18	19	20
3.8	0.9999	5.3059	5.2241	5.1534	5.0916	5.0369	4.9879	4.9438	4.9040
3.9		5.4423	5.3585	5.2860	5.2226	5.1666	5.1164	5.0712	5.0305
4.0		5.5788	5.4929	5.4187	5.3538	5.2964	5.2449	5.1987	5.1570
4.1		5.7153	5.6274	5.5514	5.4850	5.4263	5.3736	5.3263	5.2835
4.2		5.8519	5.7620	5.6842	5.6162	5.5562	5.5022	5.4539	5.4101
4.3		5.9886	5.8966	5.8171	5.7475	5.6861	5.6310	5.5815	5.5368
4.4		6.1253	6.0313	5.9500	5.8789	5.8161	5.7598	5.7092	5.6635
4.5		6.2621	6.1660	6.0829	6.0103	5.9461	5.8886	5.8369	5.7902
4.6		6.3989	6.3008	6.2159	6.1418	6.0762	6.0174	5.9647	5.9170
4.7		6.5358	6.4356	6.3490	6.2733	6.2064	6.1463	6.0925	6.0438
4.8	↓	6.6727	6.5704	6.4821	6.4048	6.3365	6.2743	6.2203	6.1707
4.9	1.0000	6.8096	6.7053	6.6152	6.5364	6.4667	6.4043	6.3482	6.2975
5.0		6.9466	6.8402	6.7483	6.6680	6.5970	6.5333	6.4761	6.4245
5.1		7.0836	6.9752	6.8815	6.7996	6.7272	6.6623	6.6040	6.5514
5.2		7.2207	7.1102	7.0147	6.9313	6.8575	6.7914	6.7320	6.6784
5.3		7.3578	7.2452	7.1480	7.0630	6.9878	6.9205	6.8600	6.8054
5.4		7.4949	7.3803	7.2813	7.1947	7.1182	7.0496	6.9880	6.9324
5.5		7.6321	7.5154	7.4146	7.3264	7.2486	7.1787	7.1160	7.0594
5.6		7.7693	7.6505	7.5479	7.4582	7.3790	7.3079	7.2441	7.1865
5.7		7.9065	7.7856	7.6813	7.5900	7.5094	7.4371	7.3722	7.3136
5.8		8.0437	7.9208	7.8146	7.7218	7.63 8	7.5663	7.5003	7.4407
5.9		8.1809	8.0560	7.9480	7.8537	7.7703	7.6955	7.6284	7.5678
6.0		8.3182	8.1912	8.0815	7.9855	7.9008	7.8248	7.7566	7.6960
6.1		8.4555	8.3264	8.2149	8.1174	8.0313	7.9540	7.8847	7.8221
6.2		8.5928	8.4617	8.3484	8.2493	8.1618	8.0833	8.0129	7.9493
6.3		8.7302	8.5969	8.4818	8.3812	8.2924	8.2126	8.1411	8.0765
6.4		8.8675	8.7322	8.6153	8.5131	8.4229	8.3420	8.2693	8.2037
6.5		9.0049	8.8675	8.7488	8.6451	8.5535	8.4713	8.3975	8.3309
6.6		9.1423	9.0028	8.8824	8.7771	8.6841	8.6006	8.5258	8.4582
6.7		9.2797	9.1382	9.0159	8.9090	8.8147	8.7300	8.6540	8.5854
6.8		9.4171	9.2735	9.1495	9.0410	8.9453	8.8594	8.7823	8.7127
6.9		9.5546	9.4089	9.2830	9.1730	9.0759	8.9888	8.9106	8.8400
7.0		9.6920	9.5443	9.4166	9.3051	9.2065	9.1182	9.0389	8.9672
7.1		9.8295	9.6797	9.5502	9.4371	9.3372	9.2476	9.1672	9.0945
7.2		9.9670	9.8151	9.6838	9.5691	9.4679	9.3770	9.2955	9.2219
7.3		10.1045	9.9505	9.8175	9.7012	9.5985	9.5064	9.4238	9.3492
7.4		10.2420	10.0859	9.9511	9.8333	9.7292	9.6359	9.5521	9.4765
7.5		10.3795	10.2213	10.0847	9.9653	9.8599	9.7653	9.6805	9.6038
7.6		10.5170	10.3568	10.2184	10.0974	9.9906	9.8948	9.8088	9.7312
7.7		10.6546	10.4923	10.3521	10.2295	10.1213	10.0243	8.8372	9.8585
7.8		10.7921	10.6277	10.4857	10.3616	10.2521	10.1538	10.0656	9.9859
7.9		10.9297	10.7632	10.6194	10.4938	10.3828	10.2833	10.1940	10.1133
8.0	↓	11.0672	10.8987	10.7531	10.6259	10.5135	10.4128	10.3223	10.2407

TABLE A-5.—Continued.

(c) Sample sizes 21 to 28

Safety margin S_M	Probability, P_x	Sample size, N							
		21	22	23	24	25	26	27	28
−5.0	0	−4.1579	−4.1740	−4.1893	−4.2036	−4.2172	−4.2300	−4.2422	−4.2538
−4.0	0	−3.3100	−3.3233	−3.3358	−3.3476	−3.3587	−3.3693	−3.3793	−3.3888
−3.0	.0013	−2.4568	−2.4673	−2.4772	−2.4865	−2.4954	−2.5037	−2.5116	−2.5192
−2.0	.0227	−1.5917	−1.5998	−1.6073	−1.6145	−1.6212	−1.6276	−1.6336	−1.6393
−1.0	.1586	−.6954	−.7017	−.7077	−.7133	−.7186	−.7235	−.7283	−.7327
0	.5000	.2893	.2821	.2755	.2694	.2636	.2582	.2531	.2483
.1	.5398	.3956	.3882	.3813	.3749	.3689	.3633	.3580	.3530
.2	.5792	.5035	.4958	.4886	.4819	.4757	.4698	.4643	.4591
.3	.6179	.6130	.6049	.5973	.5903	.5837	.5776	.5719	.5664
.4	.6554	.7239	.7153	.7074	.7000	.6931	.6867	.6807	.6750
.5	.6914	.8362	.8271	.8188	.8110	.8037	.7970	.7906	.7847
.6	.7257	.9497	.9402	.9314	.9232	.9155	.9084	.9017	.8955
.7	.7580	1.0644	1.0543	1.0450	1.0364	1.0283	1.0208	1.0138	1.0072
.8	.7881	1.1802	1.1695	1.1597	1.1506	1.1421	1.1342	1.1269	1.1199
.9	.8159	1.2969	1.2857	1.2754	1.2658	1.2568	1.2485	1.2407	1.2334
1.0	.8413	1.4146	1.4028	1.3919	1.3818	1.3724	1.3636	1.3554	1.3477
1.1	.8643	1.5331	1.5207	1.5092	1.4985	1.4886	1.4794	1.4708	1.4627
1.2	.8849	1.6524	1.6392	1.6271	1.6159	1.6055	1.5959	1.5868	1.5783
1.3	.9031	1.7723	1.7585	1.7457	1.7340	1.7231	1.7129	1.7034	1.6945
1.4	.9192	1.8927	1.8783	1.8649	1.8526	1.8411	1.8305	1.8205	1.8112
1.5	.9331	2.0137	1.9985	1.9846	1.9716	1.9596	1.9485	1.9380	1.9283
1.6	.9452	2.1352	2.1193	2.1047	2.0911	2.0786	2.0669	2.0560	2.0458
1.7	.9554	2.2571	2.2405	2.2252	2.2111	2.1979	2.1857	2.1743	2.1637
1.8	.9640	2.3794	2.3621	2.3461	2.3313	2.3176	2.3049	2.2930	2.2819
1.9	.9712	2.5021	2.4839	2.4673	2.4519	2.4376	2.4244	2.4120	2.4004
2.0	.9772	2.6250	2.6061	2.5888	2.5728	2.5579	2.5441	2.5312	2.5192
2.1	.9821	2.7482	2.7286	2.7105	2.6939	2.6784	2.6641	2.6507	2.6382
2.2	.9860	2.8716	2.8513	2.8325	2.8152	2.7992	2.7843	2.7704	2.7574
2.3	.9892	2.9953	2.9742	2.9547	2.9368	2.9202	2.9047	2.8903	2.8768
2.4	.9918	3.1192	3.0973	3.0772	3.0586	3.0413	3.0253	3.0104	2.9964
2.5	.9937	3.2433	3.2206	3.1998	3.1805	3.1627	3.1461	3.1306	3.1162
2.6	.9953	3.3676	3.3441	3.3225	3.3026	3.2842	3.2670	3.2510	3.2361
2.7	.9965	3.4920	3.4677	3.4454	3.4249	3.4058	3.3881	3.3716	3.3561
2.8	.9974	3.6165	3.5915	3.5685	3.5472	3.5276	3.5093	3.4922	3.4763
2.9	.9981	3.7412	3.7154	3.6916	3.6697	3.6495	3.6306	3.6130	3.5966
3.0	.9986	3.8660	3.8394	3.8149	3.7924	3.7714	3.7520	3.7339	3.7170
3.1	.9990	3.9909	3.9635	3.9383	3.9151	3.8935	3.8735	3.8549	3.8374
3.2	.9993	4.1160	4.0877	4.0618	4.0379	4.0157	3.9951	3.9759	3.9580
3.3	.9995	4.2411	4.2120	4.1854	4.1608	4.1380	4.1168	4.0971	4.0786
3.4	.9996	4.3663	4.3364	4.3090	4.2838	4.2603	4.2386	4.2183	4.1994
3.5	.9997	4.4916	4.4609	4.4328	4.4068	4.3828	4.3604	4.3396	4.3201
3.6	.9998	4.6169	4.5855	4.5566	4.5299	4.5053	4.4823	4.4610	4.4410
3.7	.9998	4.7423	4.7101	4.6805	4.6531	4.6278	5.6043	4.5824	4.5619

TABLE A-5.—Continued.

(c) Concluded.

Safety margin, S_M	Probability, P_x	Sample size, N							
		21	22	23	24	25	26	27	28
3.8	0.9999	4.8678	4.8348	4.8044	4.7764	4.7504	4.7263	4.7039	4.6829
3.9		4.9934	4.9595	4.9284	4.8997	4.8731	4.8484	4.8254	4.8039
4.0		5.1190	5.0843	5.0524	5.0231	4.9958	4.9706	4.9470	4.9250
4.1		5.2447	5.2091	5.1765	5.1465	5.1186	5.0927	5.0686	5.0461
4.2		5.3704	5.3340	5.3007	5.2699	5.2414	5.2150	5.1903	5.1673
4.3		5.4961	5.4590	5.4259	5.3934	5.3643	5.3372	5.3120	5.2885
4.4		5.6219	5.5840	5.5491	5.5170	5.4872	5.4595	5.4338	5.4097
4.5		5.7478	5.7090	5.6734	5.6405	5.6101	5.5819	5.5556	5.5310
4.6		5.8737	5.8340	5.7977	5.7641	5.7331	5.7043	5.6774	5.6523
4.7		5.9996	5.9591	5.9220	5.8878	5.8561	5.8267	5.7992	5.7736
4.8		6.1255	6.0843	6.0464	6.0115	5.9791	5.9491	5.9211	5.8950
4.9	1.0000	6.2515	6.2094	6.1708	6.1352	6.1022	6.0716	6.0430	6.0164
5.0		6.3775	6.3346	6.2952	6.2589	6.2253	6.1941	6.1650	6.1378
5.1		6.5035	6.4598	6.4197	6.3827	6.3484	6.3166	6.2869	6.2592
5.2		6.6296	6.5851	6.5442	6.5065	6.4716	6.4391	6.4089	6.3807
5.3		6.7557	6.7103	6.6687	6.6303	6.5947	6.5617	6.5309	6.5022
5.4		6.8818	6.8356	6.7932	6.7541	6.7179	6.6843	6.6530	6.6237
5.5		7.0080	6.9609	6.9178	6.8780	6.8411	6.8069	6.7750	6.7452
5.6		7.1341	7.0863	7.0423	7.0019	6.9644	6.9295	6.8971	6.8668
5.7		7.2603	7.2116	7.1669	7.1257	7.0876	7.0522	7.0192	6.9883
5.8		7.3865	7.3370	7.2916	7.2497	7.2109	7.1749	7.1413	7.1099
5.9		7.5127	7.4624	7.4162	7.3736	7.3342	7.2975	7.2634	7.2315
6.0		7.6390	7.5878	7.5408	7.4975	7.4575	7.4202	7.3856	7.3531
6.1		7.7652	7.7132	7.6655	7.6215	7.5808	7.5430	7.5077	7.4748
6.2		7.8915	7.8387	7.7902	7.7455	7.7041	7.6657	7.6299	7.5964
6.3		8.0178	7.9641	7.9149	7.8695	7.8275	7.7884	7.7521	7.7181
6.4		8.1441	8.0896	8.0396	7.9935	7.9508	7.9112	7.8743	7.8398
6.5		8.2704	8.2151	8.1643	8.1175	8.0742	8.0340	7.9965	7.9614
6.6		8.3967	8.3406	8.2891	8.2415	8.1976	8.1567	8.1187	8.0831
6.7		8.5231	8.4661	8.4138	8.3656	8.3210	8.2795	8.2409	8.2048
6.8		8.6494	8.5916	8.5386	8.4897	8.4444	8.4023	8.3632	8.3266
6.9		8.7758	8.7172	8.6633	8.6137	8.5678	8.5252	8.4854	8.4483
7.0		8.9022	8.8427	8.7881	8.7378	8.6912	8.6480	8.6077	8.5700
7.1		9.0285	8.9683	8.9129	8.8619	8.1847	8.7708	8.7300	8.6918
7.2		9.1549	9.0938	9.0377	8.9860	8.9381	8.8937	8.8522	8.8135
7.3		9.2814	9.2194	8.1625	8.1101	9.0616	9.0165	8.9745	8.9353
7.4		9.4078	9.3450	9.2873	9.2342	9.1850	9.1394	9.0968	9.0571
7.5		9.5342	9.4706	9.4122	9.3583	9.3085	8.2622	9.2191	9.1788
7.6		9.6606	9.5962	9.5370	9.4825	9.4320	9.3851	9.3414	9.3006
7.7		9.7871	9.7218	9.6619	9.6066	9.5555	9.5080	9.4638	9.4224
7.8		9.9135	9.8474	9.7867	9.7308	9.6790	9.6309	9.5861	9.5442
7.9		10.0400	9.9730	9.9116	9.8549	9.8025	9.7538	9.7084	9.6660
8.0		10.1665	10.0987	10.0365	9.9791	9.9260	9.8767	9.8308	9.7879

TABLE A-5.—Continued.

(d) Sample sizes 30 to 100

Safety margin, S_M	Probability, P_x	Sample size, N							
		30	40	50	60	70	80	90	100
−5.0	0	−4.2753	−4.3596	−4.4191	−4.4640	−4.4996	−4.5286	−4.5530	−4.5738
−4.0	0	−3.4065	−3.4757	−3.5245	−3.5613	−3.5905	−3.6143	−3.5343	−3.6514
−3.0	.0013	−2.5332	−2.5879	−2.6264	−2.6555	−2.6785	−2.6973	−2.7130	−2.7265
−2.0	.0227	−1.6500	−1.6916	−1.7208	−1.7427	−1.7601	−1.7742	−1.7861	−1.7962
−1.0	.1586	−.7411	−.7732	−.7954	−.8121	−.8251	−.8358	−.8446	−.8522
−0	.5000	.2394	.2061	.1837	.1673	.1547	.1445	.1361	.1290
.1	.5398	.3439	.3094	.2864	.2696	.2566	.2462	.2376	.2304
.2	.5792	.4496	.4138	.3901	.3727	.3594	.3487	.3399	.3325
.3	.6179	.5565	.5193	.4946	.4767	.4629	.4519	.4428	.4352
.4	.6554	.6646	.6257	.6000	.5814	.5671	.5557	.5464	.5385
.5	.6914	.7737	.7331	.7063	.6869	.6721	.6602	.6505	.6424
.6	.7257	.8840	.8414	.8134	.7931	.7777	.7654	.7553	.7468
.7	.7580	.9951	.9505	.9211	.9000	.8839	.8710	.8605	.8517
.8	.7881	1.1072	1.0603	1.0296	1.0075	.9906	.9773	.9663	.9571
.9	.8159	1.2201	1.1708	1.1386	1.1155	1.0979	1.0839	1.0725	1.0630
1.0	.8413	1.3337	1.2820	1.2482	1.2240	1.2056	1.1911	1.1791	1.1692
1.1	.8643	1.4480	1.3937	1.3583	1.3330	1.3138	1.2986	1.2861	1.2757
1.2	.8849	1.5628	1.5059	1.4689	1.4424	1.4223	1.4064	1.3935	1.3826
1.3	.9031	1.6782	1.6186	1.5799	1.5522	1.5312	1.5146	1.5011	1.4898
1.4	.9192	1.7941	1.7317	1.6912	1.6623	1.6404	1.6231	1.6090	1.5972
1.5	.9331	1.9105	1.8452	1.8029	1.7727	1.7499	1.7319	1.7171	1.7049
1.6	.9452	2.0272	1.9590	1.9149	1.8834	1.8596	1.8408	1.8255	1.8127
1.7	.9554	2.1442	2.0731	2.0271	1.9944	1.9696	1.9500	1.9341	1.9208
1.8	.9640	2.2616	2.1875	2.1396	2.1055	2.0797	2.0594	2.0428	2.0290
1.9	.9712	2.3793	2.3021	2.2523	2.2169	2.1901	2.1689	2.1517	2.1374
2.0	.9772	2.4972	2.4170	2.3652	2.3284	2.3006	2.2786	2.2608	2.2459
2.1	.9821	2.6154	2.5320	2.4782	2.4401	2.4112	2.3885	2.3700	2.3545
2.2	.9860	2.7337	2.6472	2.5915	2.5519	2.5220	2.4984	2.4792	2.4633
2.3	.9892	2.8523	2.7626	2.7049	2.6639	2.6329	2.6085	2.5887	2.5721
2.4	.9918	2.9710	2.8782	2.8184	2.7759	2.7439	2.7187	2.6982	2.6810
2.5	.9937	3.0899	2.9938	2.9320	2.8881	2.8550	2.8290	2.8078	2.7901
2.6	.9953	3.2089	3.1096	3.0457	3.0004	2.9663	2.9393	2.9174	2.8992
2.7	.9965	3.3280	3.2255	3.1596	3.1128	3.0776	3.0498	3.0272	3.0084
2.8	.9974	3.4473	3.3416	3.2735	3.2253	3.1889	3.1603	3.1370	3.1176
2.9	.9981	3.5667	3.4577	3.3875	3.3378	3.3004	3.2709	3.2469	3.2269
3.0	.9986	3.6861	3.5738	3.5016	3.4505	3.4119	3.3815	3.3568	3.3362
3.1	.9990	3.8057	3.6901	3.6158	3.5631	3.5234	3.4922	3.4668	3.4456
3.2	.9993	3.9253	3.8064	3.7300	3.6759	3.6351	3.6030	3.5768	3.5551
3.3	.9995	4.0451	3.9228	3.8443	3.7887	3.7467	3.7137	3.6869	3.6646
3.4	.9996	4.1649	4.0393	3.9586	3.9015	3.8585	3.8246	3.7970	3.7741
3.5	.9997	4.2847	4.1558	4.0730	4.0144	3.9702	3.9355	3.9072	3.8837
3.6	.9998	4.4047	4.2724	4.1874	4.1273	4.0820	4.0464	4.0174	3.9933
3.7	.9998	4.5247	4.3891	4.3019	4.2403	4.1939	4.1573	4.1276	4.1029

TABLE A-5.—Concluded.

(d) Concluded.

Safety margin, S_M	Probability, P_x	Sample size, N							
		30	40	50	60	70	80	90	100
3.8	0.9999	4.6447	4.5057	4.4165	4.3433	4.3057	4.2683	4.2379	4.2126
3.9		4.7648	4.6224	4.5310	4.4664	4.4177	4.3793	4.3482	4.3223
4.0		4.8849	4.7392	4.6456	4.5795	4.5296	4.4904	4.4585	4.4320
4.1		5.0051	4.8560	4.7603	4.6926	4.6416	4.6014	4.5688	4.5417
4.2		5.1253	4.9728	4.8749	4.8057	4.7536	4.7125	4.6792	4.6516
4.3		5.2456	5.0897	4.9896	4.9189	4.8656	4.8237	4.7896	4.7612
4.4		5.3659	5.2066	5.1044	5.0321	4.9776	4.9348	4.9000	4.8710
4.5		5.4862	5.3235	5.2191	5.1453	5.0897	5.0460	5.0104	4.9809
4.6		5.6066	5.4405	5.3339	5.2585	5.2018	5.1572	5.1209	5.0907
4.7		5.7270	5.5575	5.4487	5.3718	5.3139	5.2684	5.2314	5.2006
4.8		5.8474	5.6745	5.5635	5.4851	5.4260	5.3796	5.3418	5.3104
4.9	1.0000	5.9679	5.7915	5.6784	5.5984	5.5882	5.4908	5.4523	5.4203
5.0		6.0883	5.9086	5.7932	5.7117	5.6503	5.6021	5.5628	5.5302
5.1		6.2088	6.0256	5.9081	5.8250	5.7625	5.7133	5.6734	5.6401
5.2		6.3294	6.1427	6.0230	5.9384	5.8747	5.8246	5.7839	5.7500
5.3		6.4499	6.2598	6.1379	6.0518	5.9869	5.9359	5.8945	5.8600
5.4		6.5705	6.3770	6.2528	6.1651	6.0991	6.0472	6.0050	5.9599
5.5		6.6911	6.4941	6.3678	6.2785	6.2113	6.1585	6.1156	6.0799
5.6		6.8117	6.6113	6.4828	6.3919	6.3236	6.2698	6.2262	6.1898
5.7		6.9323	6.7285	6.5977	6.5054	6.4358	6.3812	6.3368	6.2998
5.8		7.0529	6.8456	6.7127	6.6188	6.5481	6.4925	6.4474	6.4098
5.9		7.1735	6.9628	6.8277	6.7322	6.6694	6.6039	6.5580	6.5198
6.0		7.2942	7.0801	6.9427	6.8457	6.7727	6.7152	6.6686	6.6298
6.1		7.4149	7.1973	7.0577	6.9592	6.8850	6.8266	6.7792	6.7398
6.2		7.5356	7.3145	7.1728	7.0726	6.9973	6.9380	6.8899	6.8498
6.3		7.6563	7.4318	7.2878	7.1861	7.1096	7.0494	7.0005	6.9598
6.4		7.7770	7.5490	7.4029	7.2996	7.2219	7.1608	7.1112	7.0699
6.5		7.8978	7.6663	7.5179	7.4131	7.3342	7.2722	7.2218	7.1799
6.6		8.0185	7.7836	7.6330	7.5266	7.4466	7.3836	7.3325	7.2899
6.7		8.1393	7.9009	7.7481	7.6401	7.5589	7.4950	7.4432	7.4000
6.8		8.2600	8.0182	7.8632	7.7537	7.6712	7.6064	7.5538	7.5100
6.9		8.3808	8.1355	7.9783	7.8672	7.7836	7.7179	7.6645	7.6201
7.0		8.5016	8.2528	8.0934	7.9807	7.8960	7.8283	7.7752	7.7302
7.1		8.6224	8.3701	8.2085	8.0943	8.0083	7.9408	7.8859	7.8402
7.2		8.7432	8.4875	8.3236	8.2078	8.1207	8.0522	7.9966	7.9503
7.3		8.8640	8.6048	8.4387	8.3214	8.2331	8.1637	8.1073	8.0604
7.4		8.9848	8.7222	8.5538	8.4349	8.3455	8.2751	8.2180	8.1705
7.5		9.1056	8.8395	8.6690	8.5485	8.4579	8.3866	8.3287	8.2806
7.6		9.2264	8.9569	8.7841	8.6621	8.5702	8.4981	8.4394	8.3906
7.7		9.3473	9.0743	8.8992	8.7756	8.6826	8.6095	8.5502	8.5007
7.8		9.4681	9.1916	9.0144	8.8892	8.7950	8.7210	8.6609	8.6108
7.9		9.5890	9.3090	9.1296	9.0028	8.9075	8.8325	8.7716	8.7209
8.0		9.7098	9.4264	9.2447	9.1164	9.0199	8.9440	8.8823	8.8310

Appendix B
Project Managers Guide on Product Assurance

This concise, practical appendix on product assurance management aims to convince you that reliability and quality assurance are major components of project success. It is especially useful to newly appointed project managers and others concerned with specifying product assurance provisions. It begins with a general discussion of the product assurance manager and his or her roles, duties, and functions and then provides condensed descriptions, with illustrations, of frequently applied reliability and quality assurance requirements. NASA NHB 5300.4 and Department of Defense MIL-STD-785 series documents (refs. B-1 to B-11) cover the same subjects.

Product Assurance Manager

Role.—Product assurance managers in NASA Lewis' Office of Reliability and Quality Assurance advise the various project offices on R&QA matters. Their leadership is extremely important during the preparation of a project plan, the generation of a statement of work, the review of a bidder's proposals, and the final contract negotiations. The assigned product assurance manager is normally included on the project organization chart in a staff reporting position. A product assurance manager works closely with the project office that he or she is supporting to develop R&QA requirements that are in consonance with the uniqueness of the project and that are significantly cost effective.

Responsibilities.—The product assurance manager supports projects by providing technical management leadership in applying R&QA principles to the design, manufacture, test, handling, installation, and operation of aeronautics, space, and energy projects. To accomplish this duty, he or she performs the following functions:

(1) Formulates R&QA requirements for assigned projects.

(2) Incorporates appropriate NASA NHB 5300.4 or MIL-STD-785 series (refs. B-1 to B-11) requirements into statements of work.

(3) Evaluates proposals and then participates in contract negotiations.

(4) Serves on source evaluation boards when assigned.

(5) Prepares letters of delegation for R&QA functions and mandatory inspection points to cognizant Government inspection agencies.

(6) Reviews and evaluates R&QA plans, fabrication and test inspection procedures, process specifications, failure reports, corrective actions, equipment history records, and other documents relating to R&QA.

(7) Monitors activities of contractor and Government inspection agencies to assure compliance with R&QA requirements.

(8) Arranges and coordinates problem investigations and analyses with interdirectorate reliability and quality engineering support groups.

(9) Attends directorate and project management meetings.

(10) Supports project design reviews and program status meetings.

(11) Supports the project in the final acceptance of equipment, when planned.

Economics of R&QA

Classical curves (fig. B-1) show the relationship of product quality cost to operational cost. When the percentage of defects is low, the product quality cost is extremely high. Conversely, when the percentage of defects is high, the operational cost

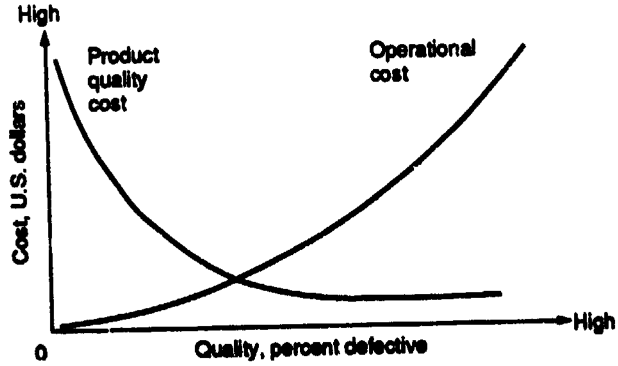


Figure B-1.—Relationship of product quality cost to operational cost.

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TABLE B-1.—RELIABILITY AND QUALITY ASSURANCE REQUIREMENTS IMPOSED ON VARIOUS PROGRAM TYPES

[Composite wind turbine blades, C; global air sampling program, G; lift/cruise fan, L; materials for advanced turbine engines, M; electrical power processor, P; quiet, clean, short-haul experimental engine, Q; JT8D refan engines, R; space experiments, S; variable-cycle engine, V; 200-kW wind turbine generators, W.]

Requirement	Aeronautics				Space		Energy	
	Study	Advanced technology	Development	Flight	Development	Flight	Development	Operational
Reliability program plan					P			
Reliability program control						S		
Reliability program reporting						S		
Reliability training						S		
Supplier control						S		
Reliability of Government-furnished property						S		
Design specifications						S		
Reliability prediction					P			
Failure mode and effects analysis				G				
Maintainability and human-induced failures	L					S		
Design reviews				G				
Failure reporting and corrective action			Q	R,G		S		
Standardization of design practices						S		
Parts program					P			W
Reliability evaluation plan						S		
Testing					P			
Reliability assessment						S		
Reliability inputs to readiness review						S		
Reliability evaluation program reviews						S		
Quality status reporting						S		
Government audits; quality program audits			Q	R		S		W
Quality program plan			Q	R				W
Technical documents; quality support/design reviews		M					C	
Change control			Q	R,G				
Identification control			Q	R,G		S		
Data retrieval						S		
Source selection		M	Q	R,G			C	W
Procurement documents			Q	R,G			C	
Quality assurance at source			Q	R				W
Receiving inspection		M	Q	R,G		S		
Receiving inspection records		M	Q	R,G		S		
Supplier rating system						S		
Postaward surveys						S		
Coordinate supplier inspection and tests						S		
Nonconformance information feedback						S		
Fabrication operations			Q	R,G				
Article and material control		M	Q	R		S	C	W
Cleanliness control							C	W
Process control			Q	R,G			C	W
Workmanship standards		M					C	

TABLE B-1.—Concluded.

Requirement	Aeronautics				Space		Energy	
	Study	Advanced technology	Development	Flight	Development	Flight	Development	Operational
Inspection and test planning			Q	R				
Inspection records; inspection and test performance		M	Q	R,G		S	C	W
Contractor quality control actions						S		
Nonconformance control		M	Q	R,G		S	C	
Nonconformance documentation		M	Q	R		S	C	
Failure analysis and corrective action		M	Q	R,G		S		
Material review			Q	R		G	C	W
Material review board			Q	R		S		
Contracting officer approval						S		
Supplier material review board						S		
Inspection of test equipment and standards								
Evaluation of standards and test equipment		M				S		
Measurement accuracy						S		
Calibration accuracy		M				S		
Calibration control	V	M	Q	R,G		S	C	W
Environmental requirements								
Remedial and preventive action (calibration)				R		S		
Stamp control system			Q	R				W
Stamp restriction						S		
Handling and storage			Q	R,G		S		W
Preserving, marking, packaging, and packing			Q	R		S		W,C
Shipping						S		
Sampling plans				R				
Statistical planning and analysis				G		S		
Contractor's responsibility for Government property			Q	R				W
Unsuitable Government property			Q	R,G				W

is extremely high. The intersection of these curves gives the optimum goal from a cost viewpoint.

The product assurance manager has the optimum cost goal in mind when selecting the R&QA program requirements. However, there may be some critical items, from an engineering viewpoint, where additional safeguards must be established, and the need for close R&QA control is mandatory. Under such a condition economics is still a major consideration.

Development of R&QA Requirements

Reliability and quality assurance are broad and diverse disciplines that have some overlapping authority with procurement, engineering, manufacturing, and testing. This overlap problem is lessened at Lewis by assigning a product assurance manager at the beginning of each project phase when R&QA requirements must be formulated.

The product assurance manager is qualified to sell the need for R&QA controls. He or she has the proper skills, training, and project experience to work out the various organizational relationships and can tailor the many R&QA tasks in the NHB 5300.4 or MIL-STD-785 series documents (refs. B-1 to B-11) into something realistic, reasonable in scope, and easily understood. In addition, the product assurance manager ultimately has the responsibility for assuring that the R&QA program is consistent with the project objectives and that the program will satisfy the overall mission requirements. As an example, table B-1 lists the actual requirements imposed on 10 Lewis contracts. The particular project phase associated with each contract is also identified.

Parts Selection and Screening

The costs incurred during subsystem and system testing are inversely proportional to the money that is spent for examining and testing the parts. Success is directly related to the part screening costs. For example, the exceptional operational life of the Space Electric Rocket Test (SERT) II satellite is no doubt attributable to the extensive parts selection and screening program.

Other factors influence parts selection and screening, such as the criticality of the hardware application, unusual environments, contractor experience, and in-house resources (R&QA parts screening laboratory, etc.). The selection can range from a high-reliability part (identified in a Government- or industry-preferred parts handbook) to an off-the-shelf commercial part (fig. B-2). Likewise, screening is a selective process as called out in the source control document. Reference B-6, paragraph IF302, explains in detail how screening can be done.

1. PART USED IN (ASSEMBLY, COMPONENT, AND SYSTEM) TRANSMITTER EQUIPMENT PACKAGE (TEP)	
2. LeRC REQUESTER, CONTRACTOR, AND PROJECT TRW Systems	3. CONTRACT NO. (IF APPLICABLE) NAS3-15839
4. DESCRIPTION OF PART Hybrid Driver, High Voltage, High Current	

5. DRAWING SPEC NO.	6. PART NO.	7. MFR. AND MFR'S EQUIVALENT COMMERCIAL PART STYLE DESIGNATION
PT4-4145	PT4-4145-011	National DH0008H

8. PREVIOUS APPROVAL (AGENCY)	FOR USE IN	ON CONTRACT
NASA	Pioneer	TRW
SAMSO/USAF	FLTSATCOM	TRW

9. COMPARISON BETWEEN NON-STANDARD PART AND STANDARD PART:
Standard part not available. This part selected based on previous successful use at TRW in similar applications.

10. TEST DATA AND APPLICATION INFORMATION

Part qualified by Group B & C testing on production lot.

11. THIS PART SHOULD BE CONSIDERED FOR INCLUSION INTO THE NSPL(MIL-STD-975)
STATE REASONS FOR RECOMMENDATION ON REVERSE SIDE

12. CONTRACTOR CERTIFICATION

I CERTIFY THAT, TO THE BEST OF MY KNOWLEDGE, THE ABOVE INFORMATION AND DATA ARE CORRECT.

PARTS OR RELIABILITY ENGINEER (SIGNATURE)__________________ DATE__________

PROJECT MANAGER OR DESIGNATED REPRESENTATIVE__________________ DATE__________

13. FOR LeRC USE ONLY APPROVALS

	SIGNATURE	DATE
THE PARTS BRANCH (DOES, DOES NOT) CONCUR WITH THE USE OF THIS PART		
THIS REQUEST (IS, IS NOT) APPROVED BY THE PROJECT MANAGER OR DESIGNATED REPRESENTATIVE		
PROJECT DIRECTOR (IF REQUIRED) DIRECTOR OF SYSTEMS RELIABILITY (IF REQUIRED)		

Figure B-2.—Typical nonstandard parts approval request

Identification of Parts and Materials

It is good engineering practice to identify parts, components, and materials with a part number, a serial number, and a date code, as applicable. Furthermore, the marking on parts and components should be affixed in a location that is easily seen when the item is installed on an assembly. The identification method (paint, electrochemical, etc.) and location on the item are included on a drawing, a specification, or other associated engineering document (fig. B-3, note 6). During the period of fabrication, assembly, and testing the system of marking and recordkeeping will provide a way of tracing backward from an end item to the part or material level.

Material Certification

There are applications in which the certification of metallic and nonmetallic materials is essential to assure that the chemical and physical properties of the materials are compatible with the design requirements. Once a material is selected by the engineer and precisely defined by a specification (Federal, Society of Automotive Engineers, American Society for Testing and Materials, etc.), the purchase order for materials such as steels, aluminum alloys, brass, welding rods, solder, metal coatings, gases, and potting compounds should require that a test report, a certificate of conformance (fig. B-4), or both accompany the vendor's shipment. In addition

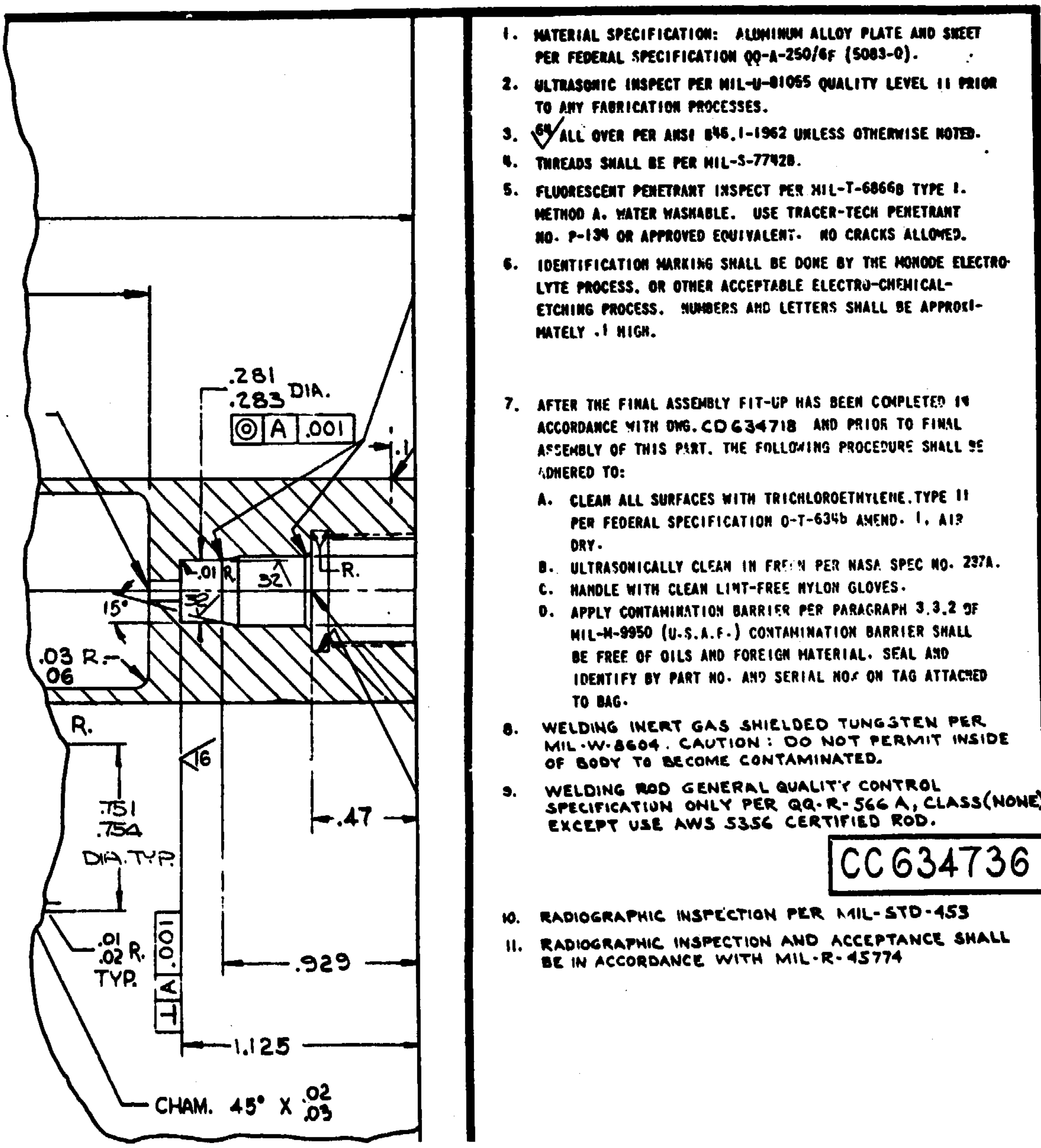


Figure B-3.—Typical drawing specifications.

to the vendor's certification it may be necessary to perform periodic in-house testing of metallic and nonmetallic materials to assure their continued validity.

Review of Drawings

Before releasing the engineering drawings to the manufacturer, design engineers may avail themselves of the technical services provided by quality engineers when developing

CASTECH

CAST TECHNOLOGY INCORPORATED
1482 ERIE BOULEVARD
SCHENECTADY, NEW YORK 12305

LABORATORY REPORT OF CHEMICAL ANALYSIS AND MECHANICAL TESTS
(Job 1365)

SOLD TO
Financo Division (MS500-302)
NASA-Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135

SHIPPED TO
NASA-Lewis Research Center
21000 Brookpark Road
Cleveland, Ohio 44135

Specification	Purchase Order No.	Part No.	Date Shipped
AMS 5391* (Inco 713 LC)	NAS 3-14991	CR655729	8/27/87
Shipper No. A 2987	No. Pcs. 1 pc	Heat No. V4271	

CHEMICAL ANALYSIS

HEAT NO.	C	Mn	Si	P	S	Ni	Cr	B	Mo	Cu	Fe	Co	Cb Ta	Ti	Al
V4271	.05	.05*	.05*	.015*	.006	Bal	12.0	.005	4.76	.05*	.05*	.08	1.96	.76	6.17
								Zr –	.10						

* Less than

MECHANICAL TESTS

HEAT NO.	TEST TEMP. °F.	TENSILE STRENGTH PSI	YIELD STRENGTH PSI	RUPTURE STRESS, PSI	RUPTURE LIFE, HRS.	% LONG. % IN 1"	R. A., %	ROCKWELL C HARDNESS AS CAST	AGED

As Cast
S/N 2

* OK PER PRINT JPW 9-14-87

DATE 8/27/87
Subscribed to and sworn before me

WILLIAM W. LATIMER
Notary Public in State of New York
Qualified in Schenectady County
My Commission Expires March 30, 1988

William W. Latimer
NOTARY PUBLIC

We hereby certify that the above data is a true copy of the data resulting from tests performed in our laboratory or of the data furnished us by the laboratory performing the tests.

BY Cara Mauro
AUTHORIZED AGENT
C. Mauro

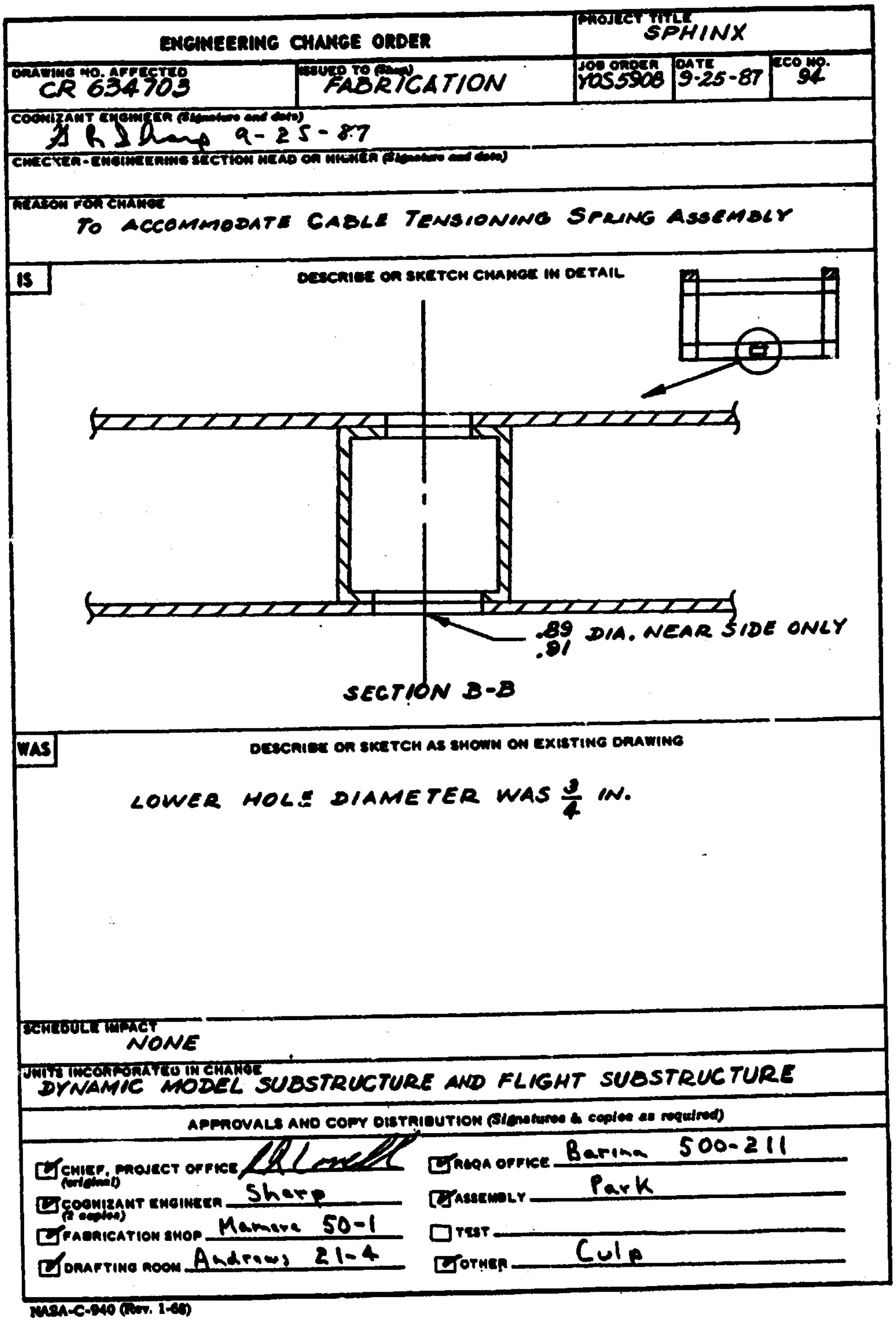
CTI-22 (11-66)

Figure B-4.—Typical material certification.

specification callouts in the note section of the drawings (fig. B-3). Precise information on materials, surface finish, processing, nondestructive testing, cleanliness, identification, packaging, special instructions, etc., is important in obtaining a quality product.

Changes in Engineering Documents

Starting early in the design phase a system is established to control changes (fig. B-5) in engineering documents. Changes in released drawings, specifications, test procedures,



ENGINEERING CHANGE ORDER

PROJECT TITLE: SPHINX

DRAWING NO. AFFECTED: CR 634703

ISSUED TO (Shop): FABRICATION

JOB ORDER: Y0S5908

DATE: 9-25-87

ECO NO.: 94

COGNIZANT ENGINEER (Signature and date): [signature] 9-25-87

CHECKER-ENGINEERING SECTION HEAD OR HIGHER (Signature and date):

REASON FOR CHANGE: TO ACCOMMODATE CABLE TENSIONING SPRING ASSEMBLY

IS — DESCRIBE OR SKETCH CHANGE IN DETAIL

WAS — DESCRIBE OR SKETCH AS SHOWN ON EXISTING DRAWING

LOWER HOLE DIAMETER WAS $\frac{3}{4}$ IN.

SCHEDULE IMPACT: NONE

UNITS INCORPORATED IN CHANGE: DYNAMIC MODEL SUBSTRUCTURE AND FLIGHT SUBSTRUCTURE

APPROVALS AND COPY DISTRIBUTION (Signatures & copies as required)

☑ CHIEF, PROJECT OFFICE (original) [signature]	☑ R&QA OFFICE Barina 500-211
☑ COGNIZANT ENGINEER (2 copies) Sharp	☑ ASSEMBLY Park
☑ FABRICATION SHOP Mamone 50-1	☐ TEST
☑ DRAFTING ROOM Andrews 21-4	☑ OTHER Culp

NASA-C-940 (Rev. 1-68)

Figure B-5.—Typical engineering change order.

Solar Array Failure Mode and Effects Analysis of Mounting and Mechanical Deployment Assembly for Space Electric Rocket Test II

Component	Failure mode	Cause	Effect	Criticality	Action	Status
Actuator assembly	Binding	Needle valve plugged	Degraded deployment	Minor	Spring stiffness adequacy and tolerances reviewed; tests carefully evaluated	Completed
	Operation is erratic	Tolerance buildup; O-ring damage; workmanship	Partial deployment	Major	Workmanship inspected	Specified
	Actuation stops	Spring failure	No deployment	Critical	Data packages will be prepared	Planned
Linkage (mechanism assembly)	Motion stops prematurely	Binding and lockup	Partial deployment	Major	Kinematics study disclosed source of binding; redesigned	Completed
		Design weakness; poor workmanship; damage	Slow deployment	Minor	Confidence tests will verify elimination of failure mode	Planned
Pin-puller assembly	Tie-rod is not released	Excessive load; squib failure; corrosion of pin puller; jamming of catch	Solar array does not deploy	Critical	Need study to develop alternative design with adequate redundancy	Open
Mechanical assembly	Attachment point of solar arrays to Agena bends or breaks	Excessive loads	Partial deployment	Major	Cold gas attitude control system to be programmed; low mode to avoid excessive load	Planned
	Hinges bind spring	Workmanship	Slow deployment	Minor	Confidence tests	Planned
		Tolerance stackup			Tolerances reviewed	Completed

Figure B-6.—Typical failure mode and effects analysis.

and related documents can be critical, particularly during the building and testing phases. For this reason the latest engineering data are processed early, and their distribution is expedited to the participating line organizations. In addition, the system must provide for removing obsolete documents.

Failure Mode, Effects, and Criticality Analysis

The fundamental objective of a failure mode, effects, and criticality analysis is to identify the critical failure areas in a design. In order to accomplish this, each functional component (or higher level, if adequate to attain the intended purpose) is sequentially assumed to fail, and the broad effects of each such failure on the operation of the system (fig. B-6) are traced. More details on this subject are available in MIL-STD-1629 (ref. B-12).[1]

[1]Use the latest document that has been issued.

Use of a Process Plan

It is good quality assurance practice to identify in a plan (fig. B-7) those manufacturing operations that must be performed in a particular sequence. The most commonly used processes are machining, mechanical fastening, grinding, brazing, welding, soldering, polishing, coating, plating, radiography, ultrasonics, fluorescent penetrant inspection, magnetic particle inspection, painting, bonding, heat treating, identification marking, and safety wiring.

Calibration of Measuring Devices

The calibration of instruments is necessary where physical quantities are to be measured with any degree of accuracy. The instruments considered, which use standard units of measure, include test and measuring instruments, various accessories, and gages. As defined herein, calibration includes repairing, periodic (recall) maintenance, and determining the accuracy (adjustments made as required) of the measuring

PROJECT NAME NAVY ORD. ENGINE	NASA - LEWIS RESEARCH CENTER	PAGE 1 OF 2
PROJECT NO. YOD 4110	PROCESS PLAN	PREPARED BY P. J. BARINA
PART NAME COMPRESSOR & TURBINE ROTOR BALANCING ASSY.		DATE OF ISSUE 6/28/89
PART NO. 655771D / DWG. NO. CR 655771C — SERIAL NO. 1		REVISION DATE
HEAT CODE NO. — CATEGORY (Exper., proto., etc.)		

OPERATION NO.	OPERATION	DRAWING SPECIFICATION REQUIREMENT	RESPONSIBLE ORGANIZATION (CODE)	OPERATION COMPLETED BY (Signature)	PART MEETS SPEC. REQ'MT. (Check approp. col.) ACCEPTABLE	NOT ACCEPTABLE	INSPECTED BY (Signature or insp. stamp)	REMARKS
000	REMOVE BALSA FAIRING	REMOVE BALSA FROM ROTOR HUB	7443	SCOPPA by [illegible]				
001	REMOVE TURBINE WHEEL FROM ASSY.		7443	SCOPPA by [illegible]				
002	BEND 3RD STAGE ROTOR BLADES	PER ENGINEERING ECO # 3	7443	[illegible] 6-28-89	✓		[illegible] 6-28-89	
003	PENETRANT INSPECT ALL BLADES	PER AMS 2645. USE TRACER-TECH PENETRANT NO. P-136 OR APP'D. EQV. (NO CRACKS ALLOWED)	7412	J. M	✓		6-28-89 J. Malcolm	NO CRACKS INDICATED
004	CLEAN	USE TRICHLOROETHYLENE PER PED SPEC. ET-6366	7412	J. M			6-28-89 J. Malcolm	RINSE & DRY WELL
005	INSPECTION	INSPECT TWIST ANGLE ON 3RD STAGE ONLY PER ECO ATTACHED	7462	Waived →				WAIVED BY H. GOLD & C. [illegible] 6/28/89 [illegible]
005A	IDENTIFICATION	USE [illegible] PROCESS & MARK AS NOTED IN "REMARKS"	7462	[illegible] 6/28/89				ASSY 655771D-1 PER ECO NO. 3 6/28/89 [illegible]
006	ASSEMBLE BALANCED TURBINE ROTOR	ASSEMBLE BALANCED TURBINE ROTOR (CR 657729) PER COMPRESSOR ROTOR BALANCING ASSY. CF 657776A	7443	[illegible]				USE TURBINE ROTOR 655729F-11CT
				[illegible]	✓		[illegible]	TORQUE TURBINE NUT TO 190-200 in-lb FRONT BRG. NUT TO 600 in-lb.
				[illegible]	✓		[illegible]	REAR BRG. NUT TO 600 in-lb

APPROVED BY:	PROJECT OFFICE	DATE	OFFICE OF RELIABILITY AND QUALITY ASSURANCE,	DATE
	L. E. Macein for H. Gold	6-28-89	P. J. Barina	6/28/89

NASA-C-902 (3-71) NASA-Lewis

Figure B-7.—Typical process plan.

devices when compared with known standards from the National Institute of Standards and Technology. Figure B-8 shows a typical certificate of calibration.

Inspection of Hardware Items

Quality control inspectors check in-process items against acceptable quality standards and engineering documents (fig. B-9). Minor deviations from good quality practices are normally resolved at the work site; otherwise they are brought to the attention of the inspection supervisor. If the quality standard being violated is not contained in an engineering document, the supervisor may review the inspector's decision if the risks are involved. If the discrepancy is a characteristic defined by an engineering document, the final decision is made by material review engineering and product assurance representatives or the material review board (engineering, product assurance, and Government representatives).

Nonconforming Hardware

When hardware is to be built, some provision must be made for the orderly review and disposition of all items that are determined by inspection or test not to conform to the drawing, specification, or workmanship requirements. The system most frequently used comprises two basic methods:

(1) It provides for review and disposition of hardware that can be reworked into a conforming condition without an engineering change, an instruction, or both. Traditionally, an engineer or a product assurance manager is authorized to make this decision.

(2) If the item cannot be reworked to meet the engineering specifications, the material review board reviews the problem. This board consists of engineering, product assurance, and, when required, Government representatives. In difficult situations the board members are not reluctant to consult with other organizations and persons to arrive at the best decision.

WESTERN AUTOMATIC TEST SERVICES
991 Commercial Street
Palo Alto, California, 94303
(415) 328-6086

CERTIFICATE OF CALIBRATION

DATE: 21 July 1988

TO: Litton Industries
960 Industrial Way
San Carlos, CA

Reference: Your Order No. 49721

WATS Order No. 8526

TO WHOM IT MAY CONCERN:

The equipment listed below has been duly calibrated by Wavecom Industries/WATS Group per your instructions.

Wavecom Industries/WATS Group calibration measurements are traceable to the National Bureau of Standards to the extent allowed by the Bureau's Calibration facilities.

WAVECOM INDUSTRIES/WATS Group

Quantity	Description	Serial No.
1	NASA INPUT SYSTEM	#1
1	NASA INPUT SYSTEM	#2
1	NASA OUTPUT SYSTEM	#1
1	NASA OUTPUT SYSTEM	#2

LITTON
57

N. Cazares
July 22, 1988

PROVISION OF WAVECOM INDUSTRIES, SOLID STATE CIRCUITS DIVISION 687 N. PASTORIA, SUNNYVALE CALIFORNIA 94086 • 245-6031

Figure B-8.—Typical certificate of calibration.

L-5394 S/N 2029 **BODY SECTIONS ASSEMBLY** PROCEDURE CFPA-171

NASA 200 WATTS CENTER SECTION ASSY DATA SHEET PAGE 6 OF 6

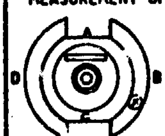
PARA NO.	DESCRIPTION	DATE	TECH	ENGR	QC
1	DEBURRING AND INSPECTION UNDER SCOPE OF ALL PARTS	1/20/5	BJ		
2	LAYOUT OF CIRCUIT PARTS ON CIRCUIT LAYOUT SHEET	1/20/5	BJ		NC
3	COLD TEST DATA (A) RETURN LOSS FREQUENCY @ 5db DOWN 12019.4 MHz (B) NOMINAL RETURN LOSS 23 db (C) WORST SPIKE 22 db @ 12100 MHz (D) I_L @ 12038 MHz 3.2 db, I_L @ 12080 MHz 2.2 db I_L @ 12123 MHz 1.8 db, I_L = 20 db @ 11989.7 MHz COMMENTS: ⓧ MARK AT OUTPUT	1/24/5	BJ	CLJ	NC
4	CLEAN PARTS PER LBPC-171	2-3-5	DG		
5	INSPECT PARTS BEFORE STACKING	2-3-5	DG		NC
6	STACK CIRCUIT PARTS ON BRAZING FIXTURE	2-3-5	DG		
7	MEASUREMENT OF CIRCUIT HEIGHTS BEFORE BRAZE (WITHOUT ALLOY)  A 1.7235 B 1.7236 ↑ C 1.7232 ↓ D 1.7232 ↓ Δ = .0004	2-3-5	DG		JD
8	REMOVE 0.048" ϕ CERAMIC ROD QC VERIFY ORIENTATION AND BRAZING FIXTURE NUMBERS FURNACE TYPE Line H₂ NO 9 SOAK DURATION 1.5 MIN. TAP POSITION 5	2-5-5 2-5-5 2-5-5	DG DG		 JD
9	CONDITION OF ALLOY AFTER BRAZE GOOD MEASUREMENT OF CIRCUIT HEIGHTS AFTER BRAZE Δ = .0006 A 1.7239 ↓ B 1.7241 C 1.7245 ↑ D 1.7241 RECORD THE DIFFERENCE BETWEEN PARAGRAPH 7 AND 9 A +.0004 B +.0005 C +.0013 D +.0009	2-5-5 2-5-5	DG DG		 NC
10	SIZE OF MANDREL DROPPED THROUGH BEAM HOLE .048 + INCH	2-5-5	DG		JD
11	VERIFY PERPENDICULARITY .002 INCHES OFF VERTICAL @ .650 INCHES. MAXIMUM RUN OUT .001 INCHES @ .470 INCHES FROM TOP OF SPACER (ⓧ MARK UP)	2-6-5	DG		NC
12	LEAK CHECK (✓)	2/6/5	[illegible]		
13	FINAL COLD TEST DATA (A) RETURN LOSS FREQUENCY @ 5db DOWN 12004.3 MHz (B) NOMINAL RETURN LOSS 21 db (C) WORST SPIKE 19 db @ 12100 MHz (D) I_L @ 12038 MHz 2.0 db I_L @ 12080 MHz 1.5 db I_L @ 12123 MHz 1.5 db I_L = 20 db @ 11965.0 MHz COMMENTS: ⓧ MARK AT OUTPUT	2/6/5	BJ	CLJ	NC
14	DISPOSITION OF ASSEMBLY USE: ✓ REJECT: DISPOSITION IF REJECT: LITTON 5	2/6/5		CLJ	NC

Figure B-9.—Typical mandatory quality control inspection points.

Documenting Equipment Discrepancies

Certain characteristics in a design are distinct, describable, and measurable in engineering units. The critical characteristics are generally identified by engineering documents and are closely controlled by quality assurance personnel.

Whenever any characteristic is determined not to conform to released engineering requirements, one of the following two reporting methods applies:

(1) A minor discrepancy is recorded in a discrepancy log (fig. B-10). A disposition must be made by an engineer, an inspector, or both if the condition is a minor discrepancy (e.g., a scratch on a metal surface or excess material) that does not adversely affect form, fit, or function and can be used "as is" or reworked to engineering requirements.

(2) A failure discrepancy report is written. A disposition is obtained through the engineering review board (ERB) if a mechanical, electrical, or electronic system or subsystem has failed to perform within the limits of a critical characteristic identified by an engineering drawing, specification, test procedure, or related engineering document.

Failure Analysis of Parts

Some failed parts are analyzed and investigated to determine the cause of the failure (fig. B-11). Corrective action is worked out to assure that the problem does not recur. The corrective action is verified by testing. The problem is closed by ERB review. Sometimes corrective action may change a component application criterion, improve a packaging technique, or revise a test procedure. Often the detailed physical and chemical examination reveals that a refinement is needed in the materials used during the manufacturing of a part or that an improvement in the parts screening process is necessary.

Quality Assurance Recording of Production, Inspection, and Test Operations

Manufacturing, inspecting, testing, and related operations on major assemblies and subassemblies should be documented for several reasons. Such documentation can provide a status record of the work in progress as well as the work completed. Also, it can become a part of the permanent record of production, inspection, and test operations. The sophistication of the format and the entries in the log can be adjusted to suit the type of contract—research, development, or production. These chronological entries in the log can be summarized and included in an acceptance data package, which contains information that is helpful to review during a contractor's acceptance of a supplier's equipment or during final Government acceptance of a contract end item. Figure B-12 shows a list used to check an item's conformance to specifications.

VEHICLE DISCREPANCY LOG

VEHICLE SERIAL #6632 SUBSYSTEM TELEMETRY SHEET 1 OF 10

ITEM NO.	PA STAMP	DATE DISC.	DESCRIPTION OF DISCREPANCY	RESP. CODE	ASSEMBLY STA	ASSEMBLY QUAD	METHOD OF CORRECTION	COMPL DATE	ORG STAMP	PA STAMP
1	MSC 57 4/27/86	4/27/86	Accelerometer P/N 161536-1 (S/N 109) cable breakout leads have insufficient bend radii. Prod Assurance Std 1-6.4 requires a min. radius of 3 times outside diameter of lead.	44-52	102	IV	Remove five spot ties at breakout points with diagonal cutting plier tool & re-dress to conform with PA Std 1-6.4	4/28/86	4/28/86	DISC 63 4/28/86

FORM LMSC 1257 B
LOCKHEED MISSILES & SPACE COMPANY

LOG SHEET CLEARED BY: Otto Evans ORGN 66-53

Figure B-10.—Typical discrepancy log.

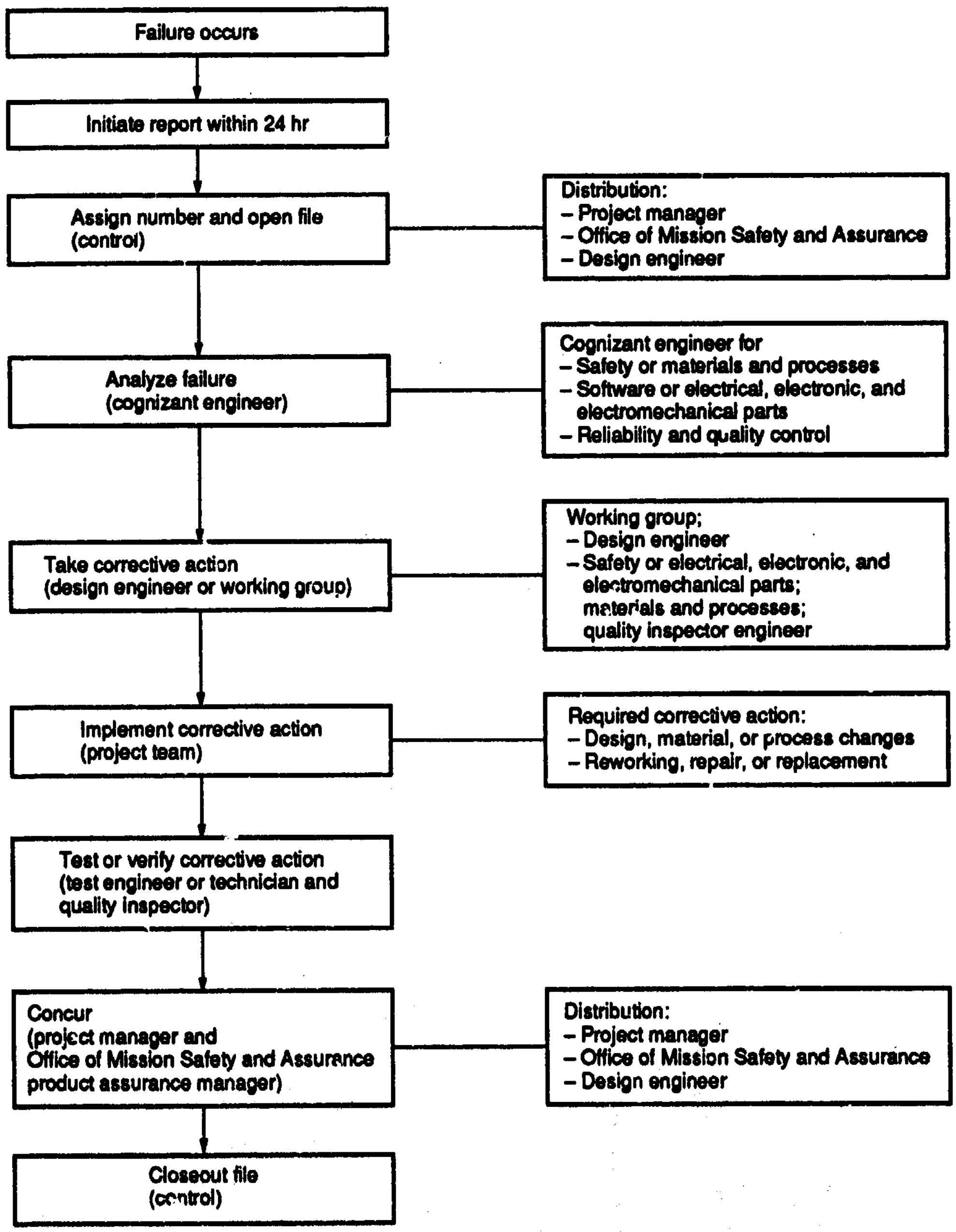


Figure B-11.—Failure report, analysis, and corrective action flowchart.

2.0 Quality assurance checklist for conformance to specifications of Communications Technology Satellite (CTS) output stage tube (OST)

OST S/N: 2021 Classification: QTM-2(QF-2)

2.1 Overall efficiency

Specification: 50 percent minimum over CTS band of 12.038 GHz to 12.123 GHz, at saturation	Actual: 40.7 percent minimum at 12.040 GHz. Out of specification. (Waiver required.)

2.2 Center frequency

Specification: 12.0805 GHz	Actual: 12.0805 GHz

2.3 RF power output

Specification: 200 W minimum at saturation over CTS band of 12.038 to 12.123 GHz	Actual: 170 W minimum at 12.040 GHz. Out of specification. (Waiver required)

2.4 Small signal bandwidth

Specification: 3 dB maximum peak to peak measured at 10 dB below peak saturation over the CTS band, 12.038 to 12.123 GHz	Actual: 2.4 dB maximum peak to peak

Figure B-12.—Checklist for item conformance to specifications.

Quality Assurance for Suppliers of Materials and Services

Materials and services acquired by the user from outside sources must satisfy, as applicable, either contract, Government, or company reliability and quality assurance requirements. The user needs a system of control that involves

(1) Selecting acceptable or qualified sources
(2) Performing surveys and audits of the supplier's facilities
(3) Inspecting supplier's products received
(4) Taking corrective action on problems that occur

References

B-1. Reliability Program Requirements for Aeronautical and Space System Contractors. NHB 5300.4 (1A-1), NASA, Jan. 21, 1987.

B-2. Maintainability Program Requirements for Space Systems. NHB 5300.4 (1E), NASA, Mar. 10, 1987.

B-3. Quality Program Provisions for Aeronautical and Space System Contractors. NHB 5300.4 (1B), NASA, Apr. 1, 1969.

B-4. Inspection System Provisions for Aeronautical and Space System Materials, Parts, Components and Services. NHB 5300.4 (1C), NASA, July 1, 1971.

B-5. Quality Assurance Provisions for Delegated Government Agencies. NHB 5300.4 (2B-1), NASA, June 1, 1985.

B-6. Requirements for Soldered Electrical Connections. NHB 5300.4 (3A-1), NASA, Dec. 1, 1976.

B-7. Qualified Products Lists Requirements for Microcircuits. NHB 5300.4 (3F), NASA, June 1, 1972.

B-8. Requirements for Printed Wiring Boards. NHB 5300.4 (3I), NASA, May 1, 1984.

B-9. Requirements for Conformal Coating and Stacking of Printed Wiring Boards and Electronic Assemblies. NHB 5300.4 (3J), NASA, Apr. 1, 1985.

B-10. Design Requirements for Rigid Printed Wiring Boards and Assemblies. NHB 5300.4 (3K), NASA, Jan. 7, 1986.

B-11. Reliability Program for Systems and Equipment Development and Production. MIL-STD-785B (plus change notices), Sept. 15, 1980.

B-12. Procedures for Performing a Failure Mode, Effects and Criticality Analysis. MIL-STD-1629, Nov. 1984.

Appendix C
Reliability Testing Examples

A great deal of work has been done by various researchers to develop probabilistic methods suitable for reliability problems (ref. C-1). Probabilistic methods that apply discrete and continuous random variables to user problems are not as well covered in the literature.

This appendix concentrates on four useful functions: (1) failure $f(t)$, (2) reliability $R(t)$, (3) failure rate λ, and (4) hazard rate λ'. Because we usually need to know how well a point estimate has been defined, some consideration is given to confidence intervals for these functions. The appendix also explains methods for planning events at the critical delivery milestone and closes with a brief explanation of two reliability case histories.

Useful Distribution Functions

The failure function $f(t)$, which defines failures as a function of time or number of cycles, is important knowledge obtained from reliability testing. Failure records are kept on a particular piece of hardware to obtain a histogram of failures against time. This histogram is studied to determine which failure distribution fits the existing data best. Once a function $f(t)$ is obtained, reliability analysis can proceed. In many cases sufficient time is not available to obtain large quantities of failure density function data. In these cases experience can be used to determine which failure frequency function best fits a given set of data. Table C-1 lists seven distributions—five continuous and two discrete. These distributions can be used to describe the time-to-failure functions for various components. The derivation of the four reliability functions for the seven listed distributions is explained in the next section (ref. C-2).

Derivation of Q(t), R(t), λ, and λ' functions.—The unreliability function $Q(t)$ is the probability that in a random trial the random variable is not greater than t; hence,

$$Q(t) = \int_0^t p(t)\, dt$$

When time is the variable, the usual range is 0 to t, implying that the process operates for some finite time interval. This integral is used to define the unreliability function when failures are being considered.

The reliability function $R(t)$ is given by

$$R(t) = 1 - Q(t)$$

In integral form $R(t)$ is given by

$$R(t) = \int_t^\infty p(t)\, dt$$

Differentiation yields

$$\frac{dR(t)}{dt} = -\frac{dQ(t)}{dt} = -p(t)$$

The a posteriori probability of failure p_f in a given time interval, t_1 to t_2, can be calculated by using these equations and is given by

$$p_f = \frac{1}{R(t_1)}\left[\int_{t_1}^{t_2} p(t)\, dt\right]$$

$$= \frac{1}{R(t_1)}\left[\int_{t_1}^{\infty} p(t)\, dt - \int_{t_2}^{\infty} p(t)\, dt\right]$$

Substituting and simplifying gives

$$p_f = 1 - \frac{R(t_2)}{R(t_1)}$$

The rate at which failures occur in a time interval is defined as the ratio of the probability of failure in the interval to the interval length. Thus, the equation for failure rate λ is given by

$$\lambda = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)} = \frac{1}{t_2 - t_1}\left[1 - \frac{R(t_2)}{R(t_1)}\right]$$

Substituting $t_1 = t$ and $t_2 = t + h$ into this equation gives

$$\lambda = \frac{R(t) - R(t+h)}{(t+h-t)R(t)} = \frac{R(t) - R(t+h)}{hR(t)}$$

TABLE C-1.—FIT DATA FOR FAILURE FUNCTIONS

Distribution	Failure fit
Continuous distribution	
Exponential	Complex electrical systems
Normal	Mechanical systems subject to wear
Weibull	Mechanical, electromechanical, or electrical parts: bearings, linkages with fatigue loads, relays, capacitors, and semiconductors. Reduces to exponential distribution if $\alpha = t$, $\beta = 1$, and $\gamma = 0$
Gamma	Combined mechanical and electrical systems
Log normal	Mechanical parts under stress rupture loading
Discrete distribution	
Poisson	One-shot parts
Binomial	Complex electrical systems for probability of N_f defects

The instanteous failure rate in reliability literature is often called the hazard rate. The hazard rate λ' is by definition the limit of the failure rate as $h \to 0$. Using a previous equation and taking the limit of the failure rate as $h \to 0$ gives

$$\lambda' = \lim_{h \to 0} \frac{R(t) - R(t+h)}{hR(t)}$$

Letting $h = \Delta t$ in this equation gives

$$\lambda' = \lim_{\Delta t \to 0} - \frac{1}{R(t)} \left[\frac{R(t+\Delta t) - R(t)}{\Delta t} \right]$$

The term in brackets is recognized from the calculus to be the derivation of $R(t)$ with respect to time, and the negative of this derivation is equal to $p(t)$. Substituting these values gives

$$\lambda' = -\frac{1}{R(t)} \left[\frac{dR(t)}{dt} \right] = \frac{p(t)}{R(t)}$$

As an example consider a jet airplane traveling from Cleveland to Miami. This distance is about 1500 miles and could be covered in about 2.5 hours. The average rate of speed would be 1500 miles divided by 2.5 hours, or 600 miles per hour. The instantaneous speed may have varied anywhere from 0 to 700 miles per hour. The air speed at any given instant could be determined by reading the speed indicator in the cockpit. Replacing the distance continuum by failures, failure rate is analogous to average speed, 600 miles per hour in this example, and hazard rate is analogous to instantaneous speed, the speed indicator reading in this example.

Figure C-1 (pp. 192 and 193) shows a summary of the useful frequency functions for the failure distributions given in table C-1. These functions were derived by using the defining equations given previously. Choose any failure function and verify that $R(t)$, λ, and λ' are properly defined by going through the derivation yourself. Five reliability problems using the continuous distributions given in figure C-1 are solved in the next section.

Estimation using the exponential, normal, Weibull, gamma, and log normal distributions.—As an illustration of how to use these equations for an electrical part that experience indicates will follow the exponential distribution, consider example 1:

Example 1: Testing of a particular tantalum capacitor showed that the failure density function was exponentially distributed. For the 100 specimens tested, it was found that the mean time between failures $\bar{t}$ was 1000 hours.

(1) What is the hazard rate?

(2) What is the failure rate at 100 hours and during the next 10-hour interval?

(3) What are the failure and reliability time functions?

Solution 1:

(1) Using the equations given in figure C-1 for exponential distribution, the hazard rate is given by

$$\lambda' = \frac{1}{\bar{t}} = \frac{1}{1000 \text{ hours/failure}}$$

or

$$\lambda' = 1 \times 10^{-3} \text{ failure/hour}$$

(2) The failure rate is given by

$$\lambda = \frac{1}{h}\left(1 - \frac{e^{-t_2/\bar{t}}}{e^{-t_1/\bar{t}}}\right)$$

For this case the time interval is given by

$$h = t_2 - t_1 = 110 - 100 = 10 \text{ hours}$$

The necessary reliability functions are given by

$$e^{-t_2/\bar{t}} = e^{-110/1000} = e^{-0.11} = 0.896$$

and

$$e^{-t_1/\bar{t}} = e^{-100/1000} = e^{-0.1} = 0.905$$

Substituting these values gives

$$\lambda = \frac{1}{10}\left(1 - \frac{0.896}{0.905}\right) = 1 \times 10^{-3} \text{ failure/hour}$$

This is to be expected for the exponential case because the failure rate is constant with time and always equal to the hazard rate.

(3) The failure and reliability time functions are given by

$$p(t) = \frac{1}{1000} e^{-t/1000}$$

$$R(t) = e^{-t/1000}$$

As an illustration of how to use the equations given in figure C-1 for mechanical parts subject to wear using the normal distribution, consider example 2:

Example 2: A gimbal actuator is being used where friction, mechanical loading, and temperature are the principal failure-causing stresses. Assume that tests to failure have been conducted on the mechanical parts, resulting in the data shown in table C-2.

(1) What is the mean time between failures and the standard deviation?

(2) What are the hazard rate at 85 300 hours and the failure rate during the next 10 300-hour interval?

(3) What are the failure and reliability time functions?

Solution 2:

(1) The mean time between failures is given by

$$\bar{t} = \frac{\sum_{f=1}^{n} t_f}{n}$$

where

$\bar{t}$ mean time between failures, hours

t_f time to failure, hours

n number of observations

TABLE C-2.—TEST DATA FOR GIMBAL ACTUATORS

Ordered sample number	Time to failure, t_f, hr	Time to failure squared, t_f^2, $(10^3\ \text{hr})^2$
1	60×10^3	3600
2	65	4225
3	68	4624
4	70	4900
5	75	5625
6	75	5625
7	80	6400
8	83	6889
9	85	7225
10	90	8100
Total	750×10^3	57 213

Therefore, using the data from table C-2,

$$\bar{t} = \frac{750\ 000}{10} = 75\ 000 \text{ hours}$$

The unbiased standard deviation σ is given by

$$\sigma = \left[\frac{\sum_{f=1}^{n} t_f^2 - \frac{\left(\sum_{f=1}^{n} t_f\right)^2}{n}}{n-1}\right]^{1/2}$$

The sum terms required for this calculation are given by

$$\sum_{f=1}^{n} t_f^2 = 57\ 213\ (10^3 \text{ hours})^2 \text{ (column 3, table C-2)}$$

and

$$\left(\sum_{j=1}^{n} t_f\right)^2 = (750)^2 = 562\ 500\ (10^3 \text{ hours})^2$$

$$\sigma = \left(\frac{57\ 213 - 56\ 250}{9}\right)^{1/2} = \left(\frac{963}{9}\right)^{1/2} = 10\ 300 \text{ hours}$$

(2) The hazard rate λ' is given by

$$\lambda' = \frac{\text{Scaled ordinate at 85 300 hours}}{\text{Normal area from 85 300 hours to } \infty}$$

Let Y_1 be the normal ordinate at 85 300 hours and Z_1 be the standardized normal variable, which is given by

$$Z_1 = \frac{t - \bar{t}}{\sigma} = \frac{(85\ 300 - 75\ 000) \text{ hours}}{10\ 300 \text{ hours}}$$

Existing tables for the normal ordinate values for $Z = 1.0$ gives $Y_1' = 0.242$. The scale constant K_s to modify this ordinate value for this problem is given by (ref. C-3)

$$K_s = \frac{n\theta}{\sigma}$$

where θ is the class interval. Substituting values and solving for Y_1 gives

$$Y_1 = f(t_1) = K_s Y_1' = \frac{10 \times 1 \text{ failures}}{10\ 300 \text{ hours}} \times 0.242$$

$$= 2.35 \times 10^{-4} \text{ failure/hour}$$

C-3

Distribution	$p(t)$	$R(t)$
Exponential	$\frac{1}{\bar{t}} \exp [-t/\bar{t}]$	$\exp [-t/\bar{t}]$
Normal	$\frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-\bar{t}}{\sigma}\right)^2\right]$	$\frac{1}{\sigma\sqrt{2\pi}} \int_t^\infty \exp\left[-\frac{1}{2}\left(\frac{t-\bar{t}}{\sigma}\right)^2\right] dt$
Weibull	$\frac{\beta(t-\gamma)^{\beta-1}}{\alpha^\beta} \exp\left[\frac{-(t-\gamma)^\beta}{\alpha}\right]$	$\exp\left[\frac{-(t-\gamma)^\beta}{\alpha}\right]$
Gamma	$\frac{1}{\alpha^\beta \Gamma(\beta)} (t-\gamma)^{\beta-1} \exp\left[\frac{-(t-\gamma)}{\sigma}\right]$	$\frac{1}{\alpha^\beta \Gamma(\beta)} \int_t^\infty (t-\gamma)^{\beta-1} \exp\left[\frac{-(t-\gamma)}{\sigma}\right] dt$
Log normal	$\frac{1}{t'\sigma_{t'}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t'-\bar{t}'}{\sigma_{t'}}\right)^2\right]$	$\frac{1}{\sigma_{t'}\sqrt{2\pi}} \int_{t'}^\infty \exp\left[-\frac{1}{2}\left(\frac{t'-\bar{t}'}{\sigma_{t'}}\right)^2\right] dt'$

Distribution	$p(N_f)$	$R(N_f)$
Poisson	$\frac{(t/\bar{t})^{N_f} \exp[-t/\bar{t}]}{N_f!}$	$\sum_{j=N_f}^{0} \frac{(t/\bar{t})^j \exp[-t/\bar{t}]}{j!}$
Binomial	$\frac{n!}{(n-N_f)!N_f!} p^{N_f} q^{n-N_f}$	$\sum_{j=N_f}^{n} \frac{n!}{(n-j)!j!} p^j q^{n-j}$

Figure C-1.—Summary of useful

λ	λ'	Remarks
$\frac{1}{h}\left\{1-\frac{\exp\left[\frac{-t_2}{\bar{t}}\right]}{\exp\left[\frac{-t_1}{\bar{t}}\right]}\right\}$	$1/\bar{t}$	$h = t_2 - t_1$ Complex electrical systems
$\frac{1}{h}\left[1-\frac{R(t_2)}{R(t_1)}\right]$	$\frac{\text{Normal ordinate at } t}{\text{Normal area } t_1 \text{ to } \infty}$	Mechanical systems
$\frac{1}{h}\left\{1-\frac{\exp\left[-\frac{(t_2-\gamma)^{\beta}}{\alpha}\right]}{\exp\left[-\frac{(t_1-\gamma)^{\beta}}{\alpha}\right]}\right\}$	$\frac{\beta}{\alpha}(t-\gamma)^{\beta-1}$	α = scale parameter β = shape parameter γ = location parameter Mechanical or electrical systems. If $\alpha = \bar{t}$, $\beta = 0$, and $\gamma = 0$, reduces to exponential. If $\beta = 3.5$, approximates normal.
$\frac{1}{h}\left\{1-\frac{(t_2-\gamma)^{\beta-1}\exp\left[\frac{-(t_2-\gamma)^{\beta}}{\alpha}\right]}{(t_1-\gamma)^{\beta-1}\exp\left[\frac{-(t_1-\gamma)^{\beta}}{\alpha}\right]}\right\}$	$\frac{\text{Gamma ordinate at } t}{\text{Gamma area } t_1 \text{ to } \infty}$	Same as Weibull parameters but may be harder to use. $\Gamma(\beta) = \int_0^{\infty} t^{\beta-1}e^{-t}\,dt$ $\Gamma(\beta) = (\beta-1)\Gamma(\beta-1)$ Combined mechanical and electrical systems
$\frac{1}{h}\left[1-\frac{R(t_2)}{R(t_1)}\right]$	$\frac{\text{Log normal ordinate at } t}{\text{Log normal area } t_1 \text{ to } \infty}$	Mechanical parts that fail due to some wearout mechanism

λ	λ'	Remarks
Not applicable	Not applicable	N_f = number of failures One-shot devices
Not applicable	Not applicable	p = defectives q = effectives n = trials (sample size) Complex systems for probability of N_f defects

frequency functions.

Note that the denominator required to calculate λ' is $R(t_1)$, which is the normal area from 85 300 hours to ∞. Existing tables for the normal area for $Z_1 = 1.0$ (ref. C-3) give the area from $-\infty$ to Z_1, so that the unreliability $Q(t_1)$ is given by

$$Q(t_1) = 0.841 \times (\text{Area from } -\infty \text{ to } Z_1)$$

Because $Q(t_1) + R(t_1) = 1.000$,

$$R(t_1) = 1.000 - 0.841 = 0.159$$

and the hazard rate is given by

$$\lambda' = \frac{2.35\times10^{-4} \text{ failure/hour}}{1.59\times10^{-1}} = 1.47\times10^{-3} \text{ failure/hour}$$

The failure rate is given by

$$\lambda = \frac{1}{h}\left[1 - \frac{R(t_2)}{R(t_1)}\right]$$

In this case h is given as 10 300 hours. The reliability at 95 600 hours is given by

$$R(t_2) = \text{Normal area from 95 600 hours to } \infty$$

Using the preceding procedure results in

$$R(t_2) = 0.023$$

Substituting values gives

$$\lambda = \frac{1}{10\ 300 \text{ hours}}\left(1 - \frac{0.023}{0.159}\right) = \frac{8.56\times10^{-1}}{1.03\times10^{4}}$$

$$= 8.31\times10^{-5} \text{ failure/hour}$$

(3) The constants required to write expressions for $p(t)$ and $R(t)$ are calculated as follows:

$$\frac{1}{\sigma(2\pi)^{1/2}} = \frac{1}{(1.03\times10^4)\times2.52} = 3.87\times10^{-5}$$

$$2\sigma^2 = 2\times(1.03\times10^4)^2 = 2.12\times10^8$$

Using the constants and substituting values gives

$$p(t) = 3.87\times10^{-5}\, e^{-(t-7.5\times10^4)^2/2.12\times10^8}$$

$$R(t) = 3.87\times10^{-5}\int_t^\infty e^{-(t-7.5\times10^4)^2/2.12\times10^8}\,dt$$

As an illustration for the Weibull distribution, consider example 3:

Example 3: A lot of 100 stepping motors was tested to see what their reliability functions were. A power supply furnished electrical pulses to each motor. Instrumentation recorded the number of continuous steps a motor made before it failed to step even though a pulse was provided. All testing was stopped at 1×10^6 steps. The step failure data are given in table C-3.

(1) Calculate the frequency functions.
(2) Plot the hazard rate function on log-log paper.
(3) What conclusions can be drawn from this graph?

Solution 3: Because there are 100 motors in this lot, the data give ordered plotting positions suitable for plotting on Weibull probability paper. Figure C-2 shows a plot of these data. From the shape of the data in figure C-2 it appears as though two straight lines are necessary to fit this failure density function. This means that different frequency functions exist at different times. These frequency functions are said to be separated by a partition parameter δ.

From figure C-2 the Weibull scale, shape, and location parameters can be estimated by following the steps listed here:

(1) Estimate the partition parameter δ. This estimate can be obtained directly from figure C-2. The two straight lines

TABLE C-3.—WEIBULL DATA FOR STEPPING MOTORS

Number of steps to failure	Cumulative number of failures		Median rank	5-Percent rank	95-Percent rank
	Problem 3	Problem 9	Scaled time to failure, $\hat{t}_i$		
0.2×10^3	2	1	6.70	0.51	25.89
.4	4	2	16.23	3.68	39.42
.9	5	3	25.86	8.73	50.69
4.0	16	4	35.51	15.00	60.66
10.0	20	5	45.17	22.24	69.65
18.0	50	6	54.83	30.35	77.76
30.9	90	7	64.49	39.34	85.00
50.0	97	8	74.14	49.30	91.27

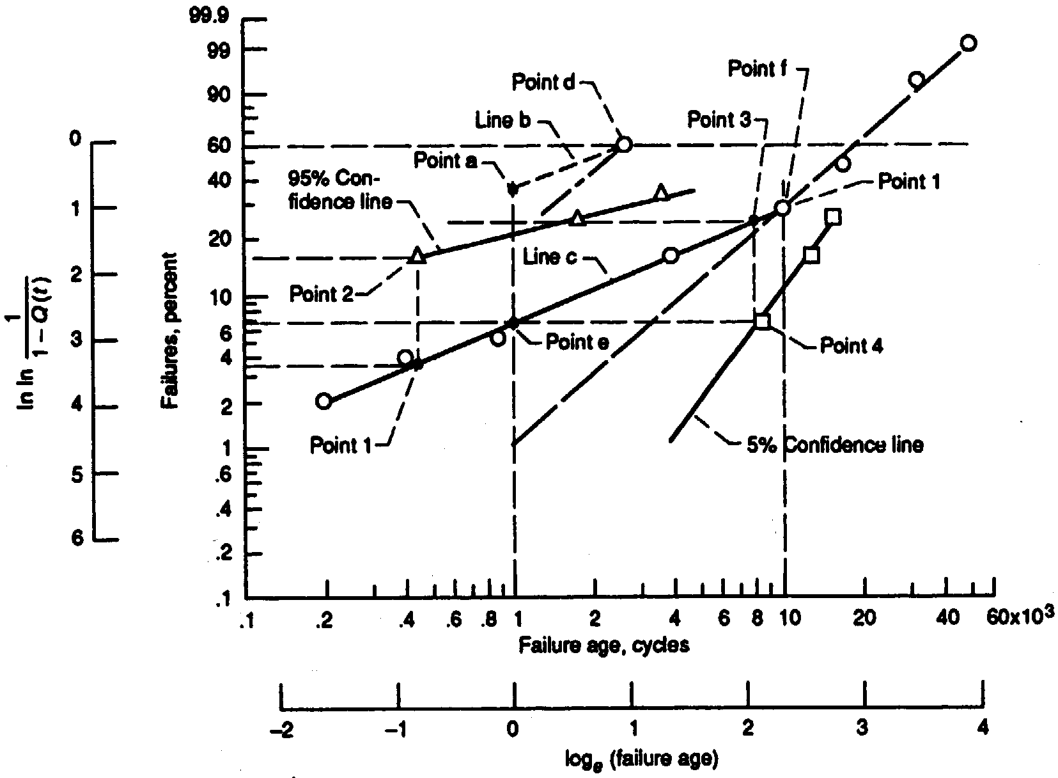


Figure C-2.—Weibull plot for stepping motors.

that best fit the given data intersect at point f. Projecting this point down to the abscissa gives a failure age of 10 000 cycles for the partition parameter δ.

(2) Estimate the location parameter γ. This parameter is used as a straightener for $p(t)$. Because $p(t-0)$ is already a straight line for both regions, it is clear that $\gamma_1 = \gamma_2 = 0$. In general, several tries at straightening may be required before the one yielding a straight line for $p(t-\gamma)$ is found.

(3) Estimate the shaping parameter β. The intercept point a for line b, drawn parallel to line c and passing through point d, where $\ln(t-\gamma) = 1$ is equal to β. Thus, $\beta_1 = 0.75$ and $\beta_2 = 1.50$.

(4) Estimate the scale parameter α. At point e for line c,

$$\ln \alpha = -\ln \ln \frac{1}{1-Q(t)}$$

so that

$$\alpha = \exp\left[-\ln \ln \frac{1}{1-Q(t)}\right]$$

Therefore,

$$\alpha_1 = e^{2.75} = 15.7$$

$$\alpha_2 = e^{4.6} = 100$$

By using the parameters just estimated and the equations given in figure C-1 for the Weibull distribution, the following failure frequency functions can be expressed: The partition limits on the number of steps c are $0 \le c \le 10$ and $c > 10$. The frequency functions are given by

$$f(c) = \frac{\beta}{\alpha}(c-\gamma)^{\beta-1}e^{-(c-\gamma)^{\beta/\alpha}}$$

Substituting values results in

$$f_1(c) = \frac{0.75}{15.7}c^{0.75-1}e^{-(c/15.7)^{0.75}}$$

or

$$f_1(c) = 0.047c^{-0.25}e^{-c^{0.75/15.7}} \qquad \text{for } 0 \le c \le 10$$

Similarly,

$$f_2(c) = 0.015c^{0.5}e^{-c^{1.50/100}} \qquad \text{for } c > 10$$

The reliability functions are given by

$$R(c) = e^{-(c-\gamma)^{\beta/\alpha}}$$

Therefore, substituting values gives

$$R_1(t) = e^{-c^{0.75/15.7}} \qquad \text{for } 0 \le c \le 10$$

and

$$R_2(t) = e^{-c^{1.5/100}} \qquad \text{for } c > 10$$

The failure rate functions are given by

$$\lambda = \frac{1}{h}\left[1 - \frac{e^{-(c_2-\gamma_1)^{\beta_1/\alpha_1}}}{e^{-(c_1-\gamma_1)^{\beta_1/\alpha_1}}}\right]$$

Therefore, substituting values gives

$$\lambda_1 = \frac{1}{h}\left[1 - \frac{e^{-(c_2)^{0.75/100}}}{e^{-(c_1)^{0.75/100}}}\right] \qquad \text{for } 0 \le c \le 10$$

and

$$\lambda_2 = \frac{1}{h}\left[1 - \frac{e^{-(c_2)^{1.5/100}}}{e^{-(c_1)^{1.5/100}}}\right] \qquad \text{for } c > 10$$

The hazard rate functions are given by

$$\lambda' = \frac{\beta}{\alpha}(c-\gamma)^{\beta-1}$$

Therefore, substituting values gives

$$\lambda_1' = 0.047\ c^{-0.25} \qquad \text{for } 0 \le c \le 10$$

and

$$\lambda_2' = 0.015\ c^{0.5} \qquad \text{for } c > 10$$

(2) By using two-cycle log-log paper and the following calculation method, a graph of λ' against c can be obtained:

$$\lambda_1' = 0.047\ c^{-0.25}$$

Taking logarithms to the base 10 gives

$$\log \lambda_1' = \log 0.047 + (-0.25) \log c$$

Useful corollary equations are

$$10^x = y$$

$$x = \log Y$$

$$10^0 = 1$$

and

$$\log 0.047 = \log 4.7\times10^{-2} = \log 4.7 + (-2)\log 10$$

$$= \bar{2}.672, \text{ or } 8.672 - 10$$

For $c = 1$

$$\log \lambda_1' = \log 0.047 + (-0.25) \log 1$$

$$\lambda_1' = 0.047$$

For $c = 10$

$$\log \lambda_1' = \log 0.047 + (0.25) \log 10 = \bar{2}.672 - 0.25 = \bar{2}.422$$

$$\lambda_1' = 0.0264$$

In a similar manner solving for λ_2' gives the data points shown in table C-4. These data are plotted in figure C-3.

TABLE C-4.—HAZARD RATE DATA FOR STEPPING MOTORS

Number of steps, c	Failures per cycle, λ'
1×10^3	0.047
10	.026
10	.015
100	.150

(3) Figure C-3 indicates that the hazard rate is decreasing by 0.25 during the first interval and increasing by 0.50 during the second interval for each logarithmic unit change of c. It appears that step motors, for first misses, jump from the "infant mortality" stage into the wearout stage without any transition period of random failures with a constant failure rate (ref. C-4).

As an illustration of combined mechanical and electrical systems that follow the gamma distribution, consider example 4:

Example 4: Environmental testing of 10 electric rockets with associated power conditioning has resulted in the ordered time-to-failure data given in table C-5.

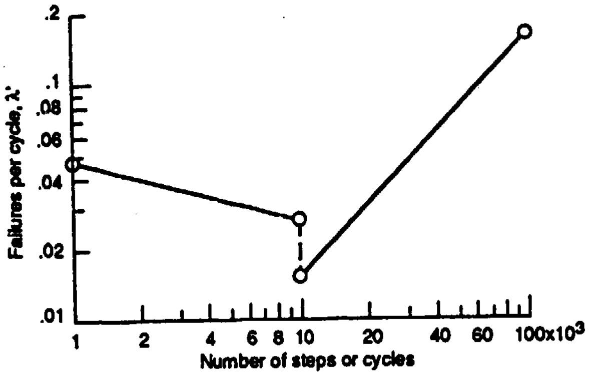


Figure C-3.—Hazard rate plot for stepping motors.

TABLE C-5.—ELECTRIC ROCKET RELIABILITY DATA

Ordered sample number	Time to failure, t_f, hr	Median rank	Scaled time to failure	Linear scale rank
		Scaled time to failure, $\hat{t}_i$		
1	1 037.8	6.70	7.2	5.0
2	1 814.4	16.23	12.6	15.0
3	2 332.8	25.86	16.3	25.0
4	3 124.8	35.51	21.7	35.0
5	3 614.4	45.71	25.1	45.0
6	4 579.2	54.83	31.8	55.0
7	5 342.4	64.49	37.1	65.0
8	6 292.8	74.14	43.7	75.0
9	7 920.0	83.77	55.0	85.0
10	11 404.8	93.30	79.2	95.0

(1) What is the mean time between failures?
(2) Write the gamma failure and the reliability functions.
(3) What is the hazard rate at 5000 hours?
(4) What is the failure rate at 5000 hours during the next 1000-hour interval?

Solution 4: The essential steps for the graphical solution of this problem are as follows (ref. C-5):

(1) Obtain the median ranks for each ordered position; see table C-5.

(2) Plot on linear graph paper (10 × 10 to the inch) median rank against time to failure for the range around 80-percent median rank.

(3) Fit a straight line to the plotted points. For a median rank of 80 read the corresponding time to failure t_{80} in hours. Figure C-4 gives a t_{80} of 7200 hours.

(4) The time-to-failure data are scaled by using the equation

$$\hat{t}_i = \frac{50}{t_{80}} t_i$$

where

$\hat{t}_i$ i^{th} scaled time to failure

t_{80} rough estimate of 80-percent failure time

t_i i^{th} time to failure, hours

Table C-5 gives $\hat{t}_i$ for each ordered sample.

(5) Plot on linear graph paper (10 × 10 to the inch) median rank against scaled time to failure $\hat{t}_i$. Figure C-5 shows the plotted data points for this problem.

(6) These data points fit the gamma curve well with a β estimate of 2.0; hence, it appears as though a two-parameter gamma distribution is required with the location parameter γ equal to zero. The nonzero location parameter case is covered in the literature (ref. C-5).

(7) Overlay the linear axis (10 spaces to the inch) of a sheet of five-cycle semilog paper corresponding to a β of 2.0. Plot on this special graph paper the linear scale rank against time-to-failure data given in table C-5.

(8) Fit a straight line through the plotted points. Figure C-6 shows the plot for these data. Two additional straight lines are shown in this figure. Line 1 was obtained by plotting two known points (0.5,1) and (20,8) (ref. C-5). Line 2 has one point at (0.5,1) with a slope m. If line 1 were coincident with line 2, the β estimate would be sufficiently accurate.

(9) Because the two lines are not coincident, a closer approximation for β is obtained by taking a new midpoint coordinate estimate of 6.8 from figure C-6. Using existing charts gives $\beta = 2.25$, which satisfies the slope criteria (ref. C-5).

(10) For a shape parameter β of 2.25 a linear scale rank of 20 percent applies. Entering figure C-6 at this point on the ordinate gives a scale parameter α of 2400 hours.

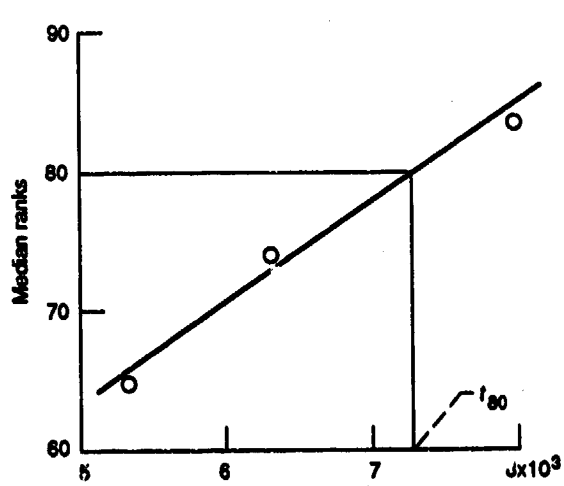


Figure C-4.—Electric rocket life.

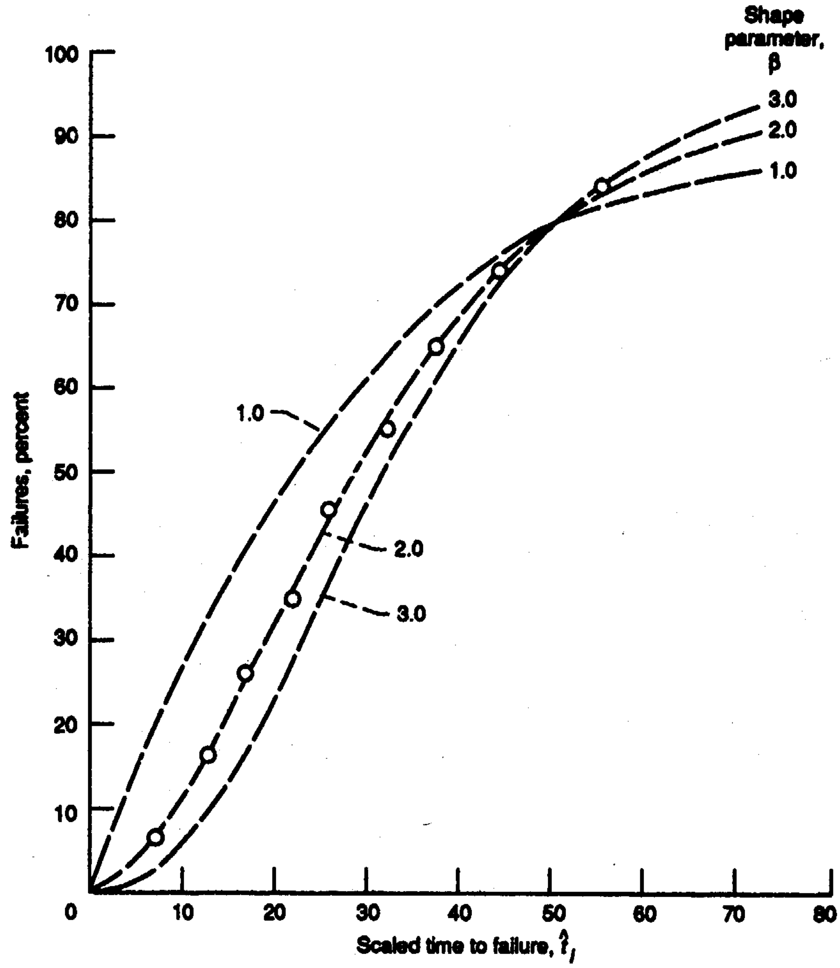


Figure C-5.—Electric rocket shape parameter curves.

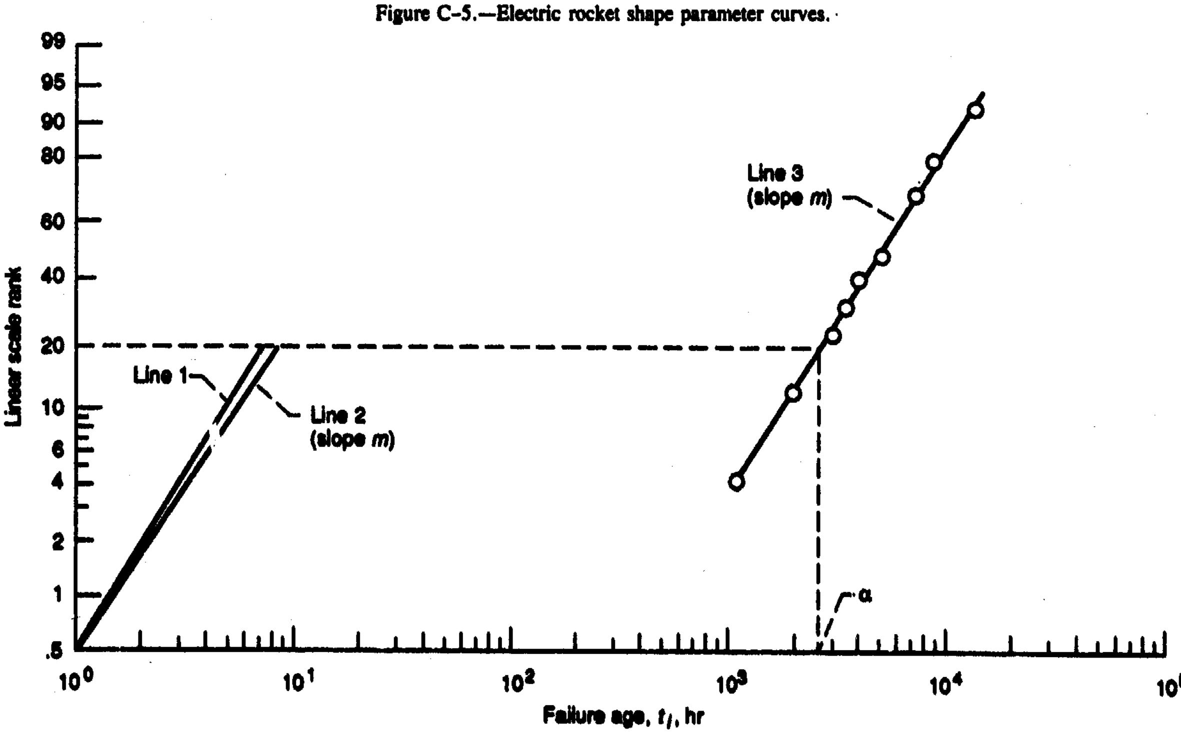


Figure C-6.—Electric rocket parameter diagram.

With these graphical construction aids the solution to the problem is readily achieved:

(1) The mean time between failures is given by

$$\bar{t} = \alpha\beta = 2.4\times10^3 \text{ hours} \times 2.25 = 5.4\times10^3 \text{ hours}$$

(2) The gamma failure and reliability functions are given by

$$p(t) = \frac{1}{\alpha^{\beta}\Gamma(\beta)} (t-\gamma)^{\beta-1} e^{-(t-\gamma)/\alpha}$$

It has been shown that $\gamma = 0$; the other constants are calculated as follows:

$$\alpha^{\beta} = (2.4\times10^3)^{2.25}$$

Using logarithms, $\log\ \alpha^{\beta} = 2.25(\log\ 2.4 + \log\ 10^3)$; performing the indicated operations gives $\log\ \alpha^{\beta} = 7.61$; hence, $\alpha^{\beta} = 4.25\times10^7$.

The second required constant is $\Gamma(\beta) = \Gamma(2.25)$. Using the identity $\Gamma(x+1) = x!$, then $\Gamma(2.25) = \Gamma(1.25+1) = 1.25!$. Using Sterling's formula, $x! = x^x e^{-x}(2\pi x)^{1/2}$. Taking logarithms gives

$$\log x! = x \log x + (-x) \log_e + \left(\frac{1}{2}\right)[\log 2\pi + \log x]$$

$$= \left(x + \frac{1}{2}\right) \log x - 0.434x + 0.399$$

$$\log(1.25!) = 1.75 \log 1.25 - 0.434 \times 1.25 + 0.399 = 0.026$$

Substituting and forming the product gives $\alpha^{\beta}\Gamma(\beta) = (4.24\times10^7) \times 1.06 = 4.5\times10^7$. Using these constants and substituting values gives

$$p(t) = \frac{1}{4.5\times10^7} t^{1.25} e^{-t/2.4\times10^3}$$

and

$$R(t) = \frac{1}{4.5\times10^7} \int_t^{\infty} t^{1.25} e^{-t/2.4\times10^3} dt$$

(3) The hazard rate function at 5000 hours is given by

$$\lambda' = \frac{p(t_1)}{R(t_1)}$$

Here

$$p(t_1) = \frac{1}{4.5\times10^7} (5\times10^3)^{1.25} e^{-5\times10^3/2.4\times10^3}$$

Performing the indicated operations gives

$$p(t_1) = \frac{(4.21\times10^4)\times(1.25\times10^{-1})}{4.5\times10^7} = 1.17\times10^{-4}$$

We can obtain $R(t_1)$ either analytically by using this integral equation or graphically from figure C-6. Enter figure C-6 at a failure age of 5000 hours. Draw a vertical line to line 3. Project the intersection of $f(t)$ and 5000 hours over to the linear scale rank (0.605). Using a previous identity,

$$R(t_1) = 1 - 0.605 = 0.395$$

Substituting values gives

$$\lambda' = \frac{1.17\times10^{-4}}{3.95\times10^{-1}} = 2.71\times10^{-4} \text{ failure/hour}$$

(4) The failure rate function at 5000 hours during the next 1000-hour interval is given by

$$\lambda = \frac{1}{t_2 - t_1}\left[1 - \frac{R(t_2)}{R(t_1)}\right]$$

Following the procedure given previously and substituting values gives

$$R(t_2) = 1 - 0.710 = 0.290$$

and

$$\lambda = \frac{1}{10^3}\left(1 - \frac{0.290}{0.395}\right) = 2.65\times10^{-4} \text{ failure/hour}$$

As an illustration of mechanical parts, consider example 5:

Example 5: A cable used as guy supports for sail experiments in wind tunnel testing exhibited the time-to-failure performance data given in table C-6.

(1) Write the failure and reliability functions.
(2) What is the hazard rate at 5715 hours?
(3) What is the failure rate during the next 3000 hours?

TABLE C-6.—TEST DATA FOR GUY SUPPORTS

Ordered sample number	Time to failure, t_f, hr	Median rank	5-Percent rank	95-Percent rank
1	1 100	6.7	0.5	25.9
2	1 890	16.2	3.7	39.4
3	2 920	25.9	8.7	50.7
4	4 100	35.5	15.0	60.7
5	5 715	45.2	22.2	69.7
6	8 720	54.8	30.3	77.8
7	12 000	64.5	39.3	85.0
8	17 500	74.1	49.3	91.3
9	23 900	83.3	60.6	96.3
10	46 020	93.3	74.1	99.5

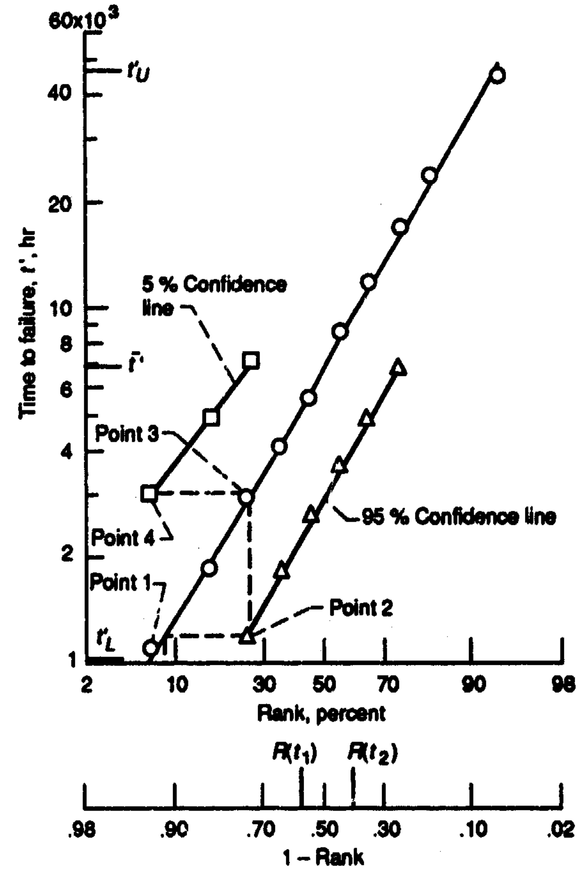


Figure C-7.—Guy support life.

Solution 5:

(1) The essential steps for solving this problem are given here:

(a) Obtain the median rank for each ordered position, see table C-6.

(b) Plot median rank against time to failure on log-normal probability graph paper (probability times two log cycles), as shown in figure C-7.

(c) If a straight line can be fitted to these plotted points, the time-to-failure function is log normal.

(d) The mean time between failures is calculated by $\bar{t}' = \ln(\bar{t})$, where $\bar{t} = 6970$ hours as shown in figure C-7 for a median rank of 50 percent; hence $\bar{t}' = 8.84$.

(e) The standard deviation is given by

$$\sigma_{t'} = \left[\frac{\ln t'_U - \ln t'_L}{3}\right]$$

where $t'_U = 49\,500$ hours and $t'_L = 1020$ hours as shown in figure C-7 for a median rank and a 1 − rank of 93.3 percent; hence, $\sigma_{t'} = (10.81 - 6.93)/3 = 1.28$.

With these constants the expressions for $p(t)$ and $R(t)$ are written as

$$p(t) = \frac{3.21 \times 10^{-1}}{t'} e^{-(t'-8.84)^2/3.28 \times 10}$$

and

$$R(t) = 3.21 \times 10^{-1} \int_{\ln(t)}^{\infty} e^{-(t'-8.84)^2/3.28 \times 10}\, dt$$

(2) The log-normal ordinate required for λ' can be calculated by using the standardized normal variable table as in example 2. The log-normal standardized variable is given by

$$Z_2 = \frac{t' - \bar{t}'}{\sigma_{t'}} = \frac{8.66 - 8.84}{1.28} = -0.143$$

From the normal-curve ordinate tables

$$Y'_2 = 0.395$$

and

$$Y_2 = \frac{NY'_2}{\sigma_{t'}} = \frac{10 \times 0.395}{1.28} = 3.09 \text{ failures}$$

Substituting values gives

$$p(t') = \frac{Y_2}{t} = \frac{3.09}{5.715 \times 10^3} = 5.40 \times 10^{-4} \text{ failure/hour}$$

The log-normal area from t' to infinity can be obtained directly from figure C-7 by using the 1 − rank scale. Enter the time-to-failure ordinate at 5715 hours; project over to the log-normal file function $f(t)$ and down to the 1 − rank abscissa value of 0.638. Therefore, the hazard rate λ' at 5715 hours

is given by

$$\lambda' = \frac{5.40\times10^{-4}}{6.38\times10^{-1}} = 8.46\times10^{-4} \text{ failure/hour}$$

(3) The failure rate during the next 3000 hours is calculated by knowing that $R(t_1) = -0.638$ at a time to failure of 5715 hours and obtaining $R(t_2) = 0.437$ from figure C-7 at 8715 hours. Therefore, the failure rate is given by

$$\lambda = \frac{1}{3\times10^3}\left(1 - \frac{0.437}{0.638}\right) = 1.05\times10^{-4} \text{ failure/hour}$$

Determination of confidence limits.—In the preceding sections statistical estimates of various parameters have been made. Here we determine the methods for defining the confidence to be placed in some of these estimates. In example 1 tantalum capacitors with a one-parameter exponential distribution were studied. For an exponentially distributed population, additional estimates follow the chi-squared distribution. As an illustration of how to determine confidence limits for an exponentially distributed estimate, consider example 6.

Example 6: One hundred tantalum capacitors were tested for 15 000 hours, during which time 15 parts failed.

(1) What is the mean time between failures?

(2) What are the upper and lower confidence limits at 98-percent confidence level?

Solution 6:

(1) The mean time between failures is given by

$$\bar{t} = \frac{T}{r} = \frac{15\ 000 \text{ hours}}{15 \text{ failures}} = 1000 \text{ hours/failure}$$

(2) The upper and lower confidence limits at some confidence level are given by

$$U = \left\{\frac{2r}{\chi^2_{[1-(\alpha/2)];2r}}\right\}\bar{t}$$

and

$$L = \left[\frac{2r}{\chi^2_{(\alpha/2);2r}}\right]\bar{t}$$

where

U	upper confidence limit, hours
L	lower confidence limit, hours
T	total observed operating time, hours
χ^2	percentage points of chi-squared distribution
r	number of failures
$1-\alpha/2$, $\alpha/2$	probabilities that $\bar{t}$ will be in calculated interval

For the 98-percent confidence level required by this problem

$$\frac{\alpha}{2} = 0.01$$

$$1 - \frac{\alpha}{2} = 0.99$$

and

$$2r = 30$$

Therefore, the chi-squared distribution values are given by (available from many existing tables)

$$\chi^2_{0.01;30} = 50.9$$

$$\chi^2_{0.99;30} = 14.9$$

Substituting values gives

$$U = \frac{30\times1000}{14.9} = 2013 \text{ hours}$$

and

$$L = \frac{30\times1000}{50.9} = 589 \text{ hours}$$

Thus, it is known with 98-percent confidence that the limits of the time $\bar{t}$ lie between approximately 590 and 2010 hours.

Determining the percentage values for the chi-squared distribution for values of r greater than 30 may also be useful. It has been shown that when $r \geq 30$,

$$(2\chi^2)^{1/2} = [2(2r) - 1]^{1/2} \pm Z$$

where Z is the area under the normal curve at the specified confidence level. Example 7 illustrates how this equation is used for confidence interval calculations.

Example 7: The tantalum capacitors of example 6 have been operated for 5000 more hours; five additional units have failed. What are the confidence limits on $\bar{t}$ at the 98-percent confidence level for this additional testing?

Solution 7: For the areas under the normal curve from $-\infty$ to Z equal to 0.98 and 0.02, existing area tables give $Z = \pm2.06$ and $r = 15 + 5 = 20$ total failures, with $2r = 40$.

Substituting values gives

$$(\theta_{\chi^2})^{1/2} = (2 \times 40 - 1)^{1/2} \pm 2.06$$

$$\chi^2_{0.01;40} = 59.7, \quad \chi^2_{0.99;40} = 23.4$$

Hence,

$$U = \frac{40 \times 10^3}{23.4} = 1709 \text{ hours}$$

$$L = \frac{40 \times 10^3}{59.7} = 670 \text{ hours}$$

Thus, it can be said, with 98-percent confidence that $\bar{t}$ lies between approximately 670 and 1710 hours; as the test time increases, the estimated-parameter confidence interval decreases.

In example 2 gimbal actuators that exhibited normally distributed time-to-failure data were analyzed. For a normally distributed population, additional mean estimates will also be normal. As an illustration of how to determine confidence intervals for normal estimates, consider example 8.

Example 8: Twenty-five gimbal actuators have been tested. The mean time between failures has been calculated to be 75 000 hours with a standard deviation of 10 300 hours (see example 2). What are the upper and lower confidence limits at a 90-percent confidence level?

Solution 8: The upper and lower confidence limits are given by

$$U = \bar{t} + K_{\alpha/2} \frac{\sigma}{n^{1/2}}$$

$$L = \bar{t} - K_{\alpha/2} \frac{\sigma}{n^{1/2}}$$

where

$\bar{t}$ mean time between failures, hours

$K_{\alpha/2}$ standardized normal variable

σ unbiased standard deviation

n number of samples

$1 - \alpha$ probability that t will be in calculated interval

For this problem

$$1 - \alpha = 0.90$$

$$\alpha = 0.10$$

$$\frac{\alpha}{2} = 0.05$$

and from existing tables for the area under the normal curve $K_{\alpha/2} = 1.64$. Substituting values gives

$$U = 75\ 000 + \frac{1.64 \times 10\ 300}{25^{1/2}} = 78\ 400 \text{ hours}$$

and

$$L = 75\ 000 - \frac{1.64 \times 10\ 300}{25^{1/2}} = 71\ 600 \text{ hours}$$

This means that 90 percent of the time the mean-time-between-failures estimate $\bar{t}$ for 25 gimbal actuators, rather than the original 10, will be between 71 600 and 78 400 hours. Note that the sample size n has been increased to use this technique. This reflects the usual user pressure to learn as much as possible with the least amount of testing. Try to keep $n \geq 25$ in estimating normal parameters with this technique. If $n < 25$, use Student's t distribution (ref. C-6). To determine the effects of reducing sample size on confidence intervals, rework example 2 for the smaller sample size of 10, using Student's t distribution. The upper and lower confidence limits are given by

$$U = \bar{t} + t_{\alpha/2} \frac{s}{n^{1/2}}$$

and

$$L = \bar{t} - t_{\alpha/2} \frac{s}{n^{1/2}}$$

where

$t_{\alpha/2}$ Student's t variable

s standard deviation

For this problem, $r = n - 1 = 9$, $\alpha = 0.10$, and $t_{\alpha/2}$ from existing tables is 1.83. The standard deviation is given by

$$s = \left(\frac{57\ 213 - 56\ 250}{10}\right)^{1/2} = 9820$$

Substituting values gives

$$U = 75\ 000 + \frac{1.83 \times 9820}{10^{1/2}} = 80\ 700 \text{ hours}$$

and

$$L = 75\ 000 + \frac{1.83 \times 9820}{10^{1/2}} = 69\ 300 \text{ hours}$$

Comparing this time interval with that calculated for a sample size of 25 shows that the smaller sample gives a larger interval of uncertainty.

In example 3 stepping motors that exhibited Weibull-distributed time-to-failure data were studied. As a graphical illustration of how to determine confidence intervals for a Weibull-distributed estimate, consider example 9.

Example 9: Another group of stepping motors has been step tested as previously explained in example 3. The Weibull plot of percent failures for a given failure age is the same as that given in figure C-2. During this testing, however, only eight failures have occurred. What is the 90-percent confidence band on the reliability estimate at 4000 cycles?

Solution 9: The data needed for graphical construction of the confidence lines on the Weibull plot are given in table C-3. The steps necessary to construct the confidence lines in figure C-2 are as follows:

(1) Enter the percent failure axis with the first 5-percent rank value hitting $f(t)$; for failure 2 the 5-percent rank is 3.68.

(2) Draw a horizontal line that intersects $f(t)$ at point 1.

(3) Draw a vertical line to cross the corresponding median rank; for failure 2 the median rank is 16.23.

(4) Draw a horizontal line at the median rank, 16.23, for failure 2. The intersection point of the line for step (3) with this line is one point on the 95-percent confidence line.

(5) Repeat steps (1) to (4) until the desired cycle life is covered, 4000 cycles in this case.

(6) The 5-percent confidence line is obtained in a similar manner. Enter the percent failure axis with the 95-percent failure rank; 25.89 for failure 1.

(7) Draw a horizontal line that intersects $f(t)$ at point 3.

(8) Draw a vertical line to cross the corresponding median rank; 6.70 for failure 1.

(9) Draw a horizontal line at the median rank, 6.70, for failure 1. The intersection point of these two lines is one point on the 5-percent confidence line.

(10) Repeat steps (6) to (9) until the desired cycle life is covered.

A 90-percent confidence interval for $f(t)$ at 4000 cycles is, from figure C-2, 1.2 percent to 37.5 percent. Hence, a 90-percent confidence interval for $R(t)$ at 4000 cycles is 0.998 to 0.625.

In example 5 guy supports that exhibited log-normally-distributed time-to-failure data were analyzed. As a final graphical illustration of how to determine confidence intervals for a log-normally-distributed estimate, consider example 10.

Example 10: It has been shown that the guy supports of example 5 exhibited a reliability of 0.638 at a time to failure of 5715 hours. Consider now the procedure for determining the confidence band on this log-normal estimate. The data needed for the graphical construction of the 90-percent confidence lines on the log-normal graph of figure C-7 are also given in table C-6.

Solution 10: The steps necessary to graphically construct the confidence lines in figure C-7 are as follows:

(1) Enter the rank axis with the first 5-percent rank value hitting $f(t)$, the log-normal life function shown in figure C-7; for ordered sample 3 the 5-percent rank is 8.7.

(2) Draw a vertical line to intersect $f(t)$ at point 1 as shown in figure C-7.

(3) Draw a horizontal line to cross the corresponding median rank; for ordered sample 3 the median rank is 25.9.

(4) The intersection point (point 2 in fig. C-7) of step (3) and the median-rank line is one point on the 95-percent confidence line.

(5) Repeat steps (1) to (4) until the desired time to failure is covered; 5715 hours in this case.

(6) The 5-percent confidence line is obtained in a similar manner. Enter the rank axis with the 95-percent-failure rank, 25.9, for ordered sample 1.

(7) Draw a vertical line intersecting $f(t)$ at point 3.

(8) Draw a horizontal line to cross the corresponding median rank; for ordered sample 1 the median rank is 6.7.

(9) The intersection point (point 4 in fig. C-7) of these two lines is one point on the 5-percent confidence line.

(10) Repeat steps (6) to (9) until the desired time to failure is covered.

At 5715 hours the 90-percent confidence interval for $f(t)$ is, from figure C-7, 19.7 to 69.4 percent. Hence, a 90-percent confidence interval for $R(t)$ at 5715 hours is 0.803 to 0.306. Incidentally, this graphical procedure for finding confidence intervals is completely general and can be used on other types of life test diagrams.

Estimation using the Poisson and binomial events.--The binomial and Poisson distributions are discrete functions of the number of failures N_f that occur rather than of the time t.

The Poisson distribution (fig. C-1) is a discrete function of the number of failures. When this distribution applies, it is of interest to determine the probabilities associated with a specified number of failures in the time continuum. As an illustration for a complex electrical component that follows the Poisson distribution, consider example 11.

Example 11: Ten space-power speed controllers were tested during the rotating solar dynamic development program. The time-to-failure test data are given in table C-7.

(1) Write the Poisson failure density and reliability functions.

(2) What is the probability of five failures in 10 000 hours?

(3) What is the probability that 6, 7, 8, 9, or 10 failures will occur? What is the reliability after the fifth failure?

Solution 11:

(1) Reducing the data given in table C-7 gives the mean time between failures as

$$\bar{t} = \frac{\sum_{i=1}^{10} t_i}{N_f} = \frac{8.59\times 10^4}{10} = 8.59\times 10^3 \text{ hours/failure}$$

TABLE C-7.—POISSON DATA FOR SPEED CONTROLLER

Ordered sample number	Time to failure, t_f, hr
1	3 520.0
2	4 671.2
3	6 729.3
4	7 010.0
5	8 510.2
6	9 250.1
7	10 910.0
8	11 220.5
9	11 815.6
10	12 226.4
Total	85 866.3

Hence, the Poisson failure density function is given by

$$p(N_f) = \frac{\left(\dfrac{t}{8.59\times 10^3}\right)^{N_f}}{N_f!} e^{-t/8.59\times 10^3}$$

The reliability function is given by

$$R(N_f) = \sum_{j=1}^{10} \frac{\left(\dfrac{t}{8.59\times 10^3}\right)^{j}}{j!} e^{-t/8.59\times 10^3}$$

(2) To calculate the probability of five failures in 10 000 hours, use the ratio

$$\frac{t}{\bar{t}} = \frac{1.0\times 10^4}{8.59\times 10^3} = 1.16$$

The probability of five failures in 10 000 hours is given by

$$p(5) = \frac{(1.16)^5 e^{-1.16}}{5!} = \frac{2.09\times 0.314}{1.2\times 10^2} = 5.47\times 10^{-3}$$

One easy method of calculating the term $(1.16)^5$ is as follows:

$$\log(1.16)^5 = 5 \log 1.16 = 5(0.148) = 0.740$$

$$(1.16)^5 = 2.09$$

(3) The reliability from the 5th to the 10th failure is the sum of the remaining terms in the Poisson expansion. The Poisson expansion in sum form is given by

$$R(N_f) = \sum_{j=6}^{10} \frac{0.314(1.16)^j}{j!}$$

Calculating each term and summing gives

$$R(6) = 0.[illegible]13$$

The binomial distribution is given in figure C-1 as distribution 7. Considerable work has been done to develop the techniques suitable for using this powerful tool (refs. C-1 and C-3). As an illustration consider a pyrotechnic part described in example 12.

Example 12: A suspicious lot of explosive bolts is estimated to be 15 percent defective due to improper loading density as observed by neutron radiography.

(1) Calculate the probability of one defective unit appearing in a flight quantity of four.

(2) Plot the resulting histogram.

(3) What is the reliability after the first defect?

Not many failure density data are available, but past experience with pyrotechnic devices has shown that the binomial distribution applies. From the given data the per-unit number of effectives q is 0.85, the per-unit number of defectives p is 0.15, the sample size n is 4, and the possible number of failures N_f is 0, 1, 2, 3, or 4. The frequency functions corresponding to these constants are given by

$$p(N_f) = \frac{4!}{(4-N_f)!N_f!} p^{N_f} q^{4-N_f}$$

and

$$R(N_f) = \sum_{j=N_f}^{4} \frac{4!}{(4-j)!j!} p^j q^{n-j}$$

One simple method for obtaining the binomial expansion coefficients is to make use of Pascal's triangle. Pascal found that there was symmetry to the coefficient development and explained it as shown in table C-8. Pascal's triangle (dashed lines) is shown in the last column. The lower number in the

TABLE C-8.—BINOMIAL EXPANSION COEFFICIENTS

Sample size, n	Possible number of failures	Binomial expansion coefficients
1	2	1
2	3	1 2 1
3	4	1 3 3 1
4	5	1 4 6 4 1

dashed triangle is obtained by adding the two upper numbers (i.e., $3 + 3 = 6$).

Using these constants and expanding gives $p(N_f)$ as

$$p(N_f) = q^4 + 4q^3p + 6q^2p^2 + 4qp^3 + p^4$$

The probability of one defective unit appearing in a flight quantity of four is given by the second term in the expansion; hence,

$$4q^3p = 4(0.85)^3(0.15) = 0.37$$

The resulting histogram for this distribution is shown in figure C-8. The probability that 2, 3, or 4 defects will occur, as the reliability after the first defect, is the sum of the remaining terms in the binomial expansion. This probability can be calculated by using the equation for $R(N_f)$. However, it is simpler to use the histogram graph and sum the probabilities over N_f from 2 to 4; hence,

$$R(2) = 0.096 + 0.011 + 0.0011 = 0.108$$

These explosive bolts in their present form are not suitable for use on any spacecraft because the probability of zero defects is only 0.522, much below the usually desired 0.999 for pyrotechnic spacecraft devices.

Determination of confidence limits.—When an estimate is made from discrete distributions, it is expected that additional estimates of the same parameter will be close to the original estimate. It is desirable to be able to determine upper and lower confidence limits at some stated confidence level for discrete distribution estimates just as is done for continuous functions of time. The analytical procedure for determining these intervals is simplified by using specially prepared tables and graphs. Useful tables for the binomial distribution are given in the literature (ref. C-3).

As an example of how confidence intervals can be obtained for Poisson estimates, consider problem 13.

Problem 13: The Poisson estimate of reliability from the 5th to the 10th failure for speed controllers was found to be

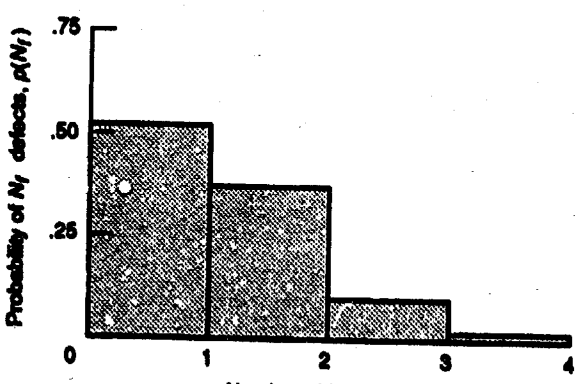


Figure C-8.—Explosive bolts histogram.

0.0013 in a previous problem. What are the upper and lower confidence limits on this estimate at a 95-percent confidence level?

The variation in $\bar{t}$ can be found by using figure C-9. Enter figure C-9 on the 5-percent α line at the left-hand end of the 5 interval. Here $T/\bar{t}_1 = 10.5$; then $\bar{t}_1 = 10\bar{t}/(T/\bar{t}_1) = 8.57\times10^4/10.5 = 8160$ hours. Using the left-hand end of the 4 interval gives $T/\bar{t}_2 = 9.25$; then $\bar{t}_2 = 8.57\times10^4/9.25 = 9530$ hours. One simple method for finding $f(5)$ is to use figure C-10 (ref. C-5). The $t/\bar{t}$ ratios of interest are 1.22, 1.16, and 1.05, respectively. For these ratios with $N_f = 5$ the values of $f(5)$ from figure C-10 are 0.997, 0.9987, and 0.99992, respectively. Because the sum of the last five terms is desired, $R(5)$ is 0.003, 0.0013 and 0.0008, respectively. This means that the probability of the 5th to the 10th failure of a speed control occurring is in the interval 0.0008 to 0.003 at a confidence level of 95 percent.

As an illustration of how confidence intervals can be obtained for a binomial distribution, consider example 14.

Example 14: The probability of one defective unit appearing in a flight quantity of four explosive bolts has been calculated to be 0.37. What are the upper and lower confidence limits on this estimate at a 90-percent confidence level?

If the sample size is n, the number of defectives is r, and the confidence level is γ, this example has the following constraints: $n = 4$, $r = 1$, and $\gamma = 90$ percent. Using these constraints, the upper U and lower L confidence limits can be obtained directly from existing tables as $U = 0.680$ and $L = 0.026$. This means that with a 90-percent confidence level the probability of one defective bolt appearing in a flight quantity of four is in the interval from 0.026 to 0.680.

Sampling

Purpose of sampling.—Sampling is a statistical method used when it is not practical to study the whole population. There are usually five basic reasons why sampling is necessary:

(1) Economy—It usually costs less money to study a sample of an item than the whole population.

(2) Timeliness—A sample can be studied in less time than the whole population, giving prompt results.

(3) Destructive nature of a test—Some tests require that the end item must be used up to demonstrate performance, leaving nothing to use.

(4) Accuracy—A sample survey accomplished by well-trained researchers usually will result in accurate and valid decisions.

(5) Infinite population—In many analytical studies an infinite population is available. If any information is to be used for decision making, it must be based on a sample.

Choosing a sample.—Good judgment must be used in choosing a sample. Subjective methods of choosing samples frequently result in bias. Bias is an expression, either conscious or subconscious, of the selector's preferences. Bias can be held to a minimum by using a nonsubjective method developed just

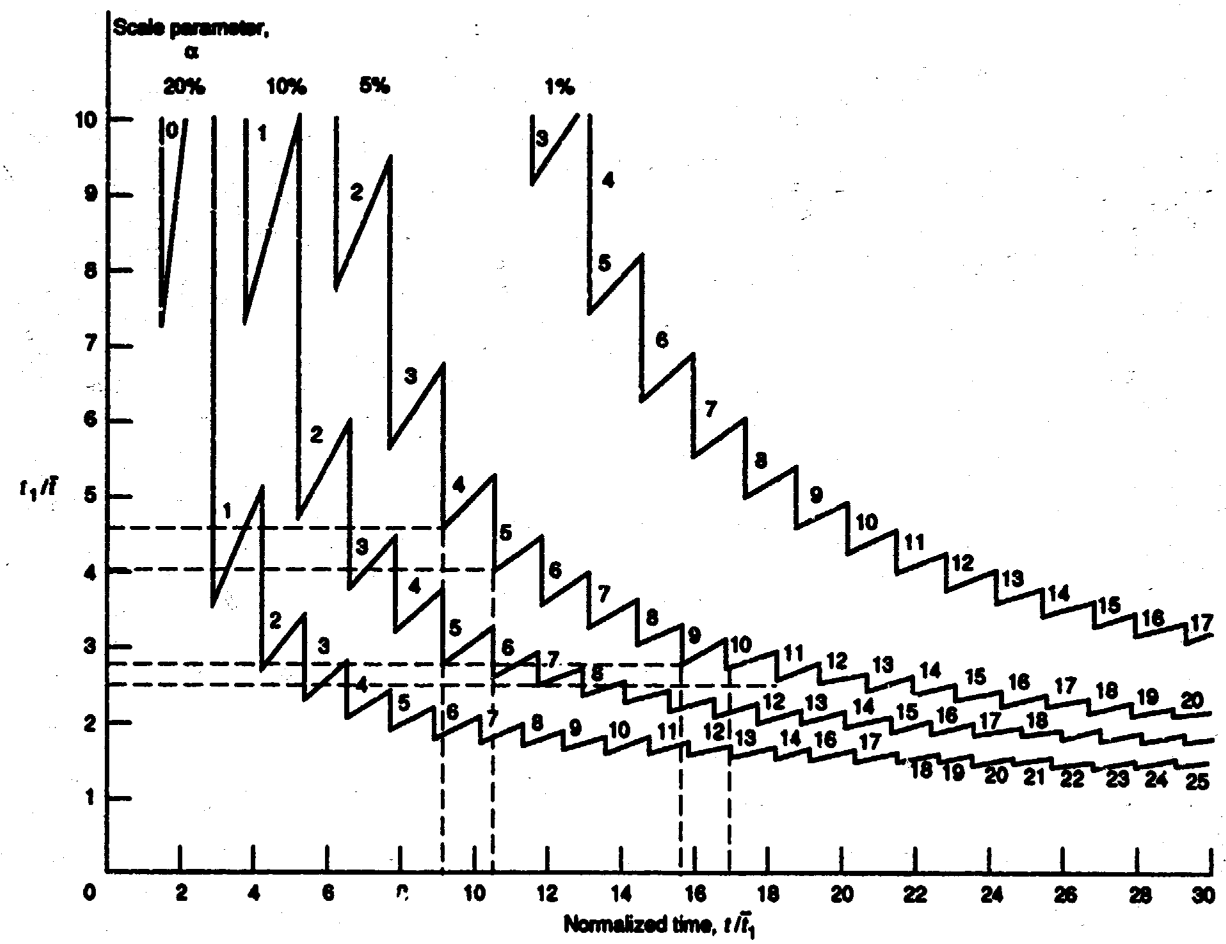


Figure C-9.—Poisson MTBF fixed test time.

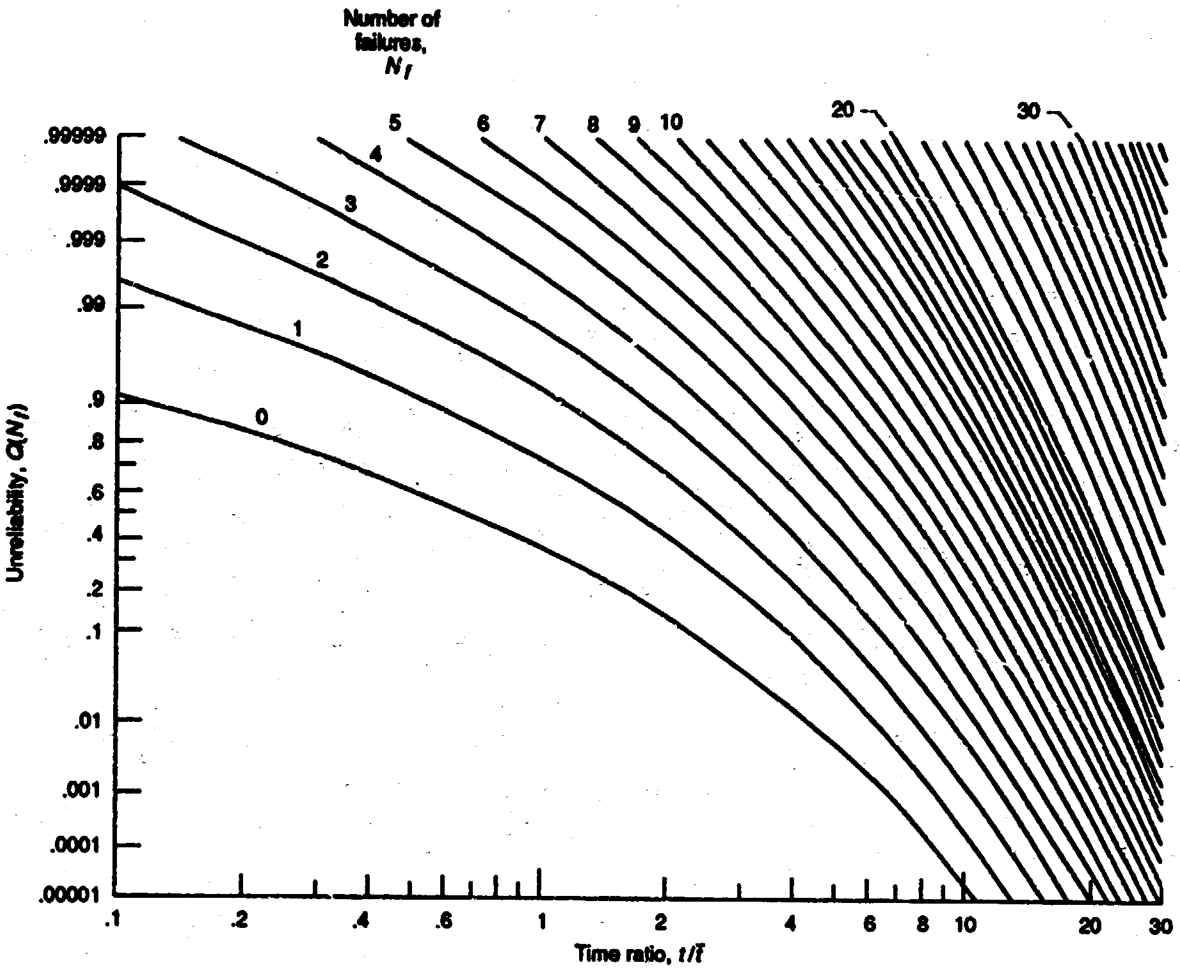


Figure C-10.—Poisson unreliability sum.

for this purpose. Several nonsubjective sampling procedures are described here:

(1) Random sampling—Each item in the population has an equal and independent chance of being selected as a sample. A random-digits table, see figure C-11, has been developed to facilitate drawing random samples. This table has been constructed to make the 10 digits from 0 to 9 equally likely to appear at any location in the table. Adjacent columns of numbers can be combined to get various-sized random numbers.

(2) Stratified sampling—Similar items in a population are grouped or stratified, and a random sample is selected from each group.

(3) Cluster sampling—Items in a population are partitioned into clusters, and a random sample is selected from each cluster.

(4) Double sampling—A random sample is selected; then, depending on what is learned, some action is taken or a second sample is drawn. After the second random sample is drawn, action is taken on the basis of data obtained from the combination of both samples.

(5) Sequential sampling—Random samples are selected and studied one at a time. A decision on whether to take action or to continue sampling is made after each observation on the basis of all data available at that selection.

As an illustration of when to use various sampling methods consider example 15.

Example 15: Describe how a sample should be selected for three cases:

(1) Invoices numbered from 6721 to 8966 consecutively. A random sampling procedure could be used in this case based on the four-digit table given in figure C-11. Using the given invoice numbers, start at the top of the left column and proceed down each column selecting random digits until the desired sample size is obtained. Disregard numbers outside the range of interest.

6433	2582	0820	1460	6606	7143	9158	5114	9491	8063
3465	7348	5774	3821	6216	2148	1221	5895	7942	9971
9601	9189	0141	1377	3467	7971	0811	8309	0504	4606
2364	3260	1430	9505	3146	4815	9732	3447	7705	4532
7304	9292	4580	8160	7144	8073	8476	1896	6661	1285
3764	5460	6385	9045	7170	5831	4668	9386	3979	1116
0251	3139	4201	0578	2172	6876	4347	4288	1514	9985
2031	0919	7613	1535	1610	7491	3255	4014	3614	5599
6398	1374	1904	7490	3941	0284	5817	1630	4629	6773
0911	3930	0324	8151	3365	6685	0566	5047	8471	6166
5052	5023	3045	3433	6365	7310	5073	5416	2332	0922
9225	3984	4659	4642	7260	1383	7625	7512	8547	7343
3100	7916	9757	8869	5307	2691	0786	2701	0102	5745
4598	0065	4257	6557	4638	8418	7398	9790	5074	8018
5956	7285	0480	1411	7766	3377	5023	0227	8047	1887
9360	1041	2094	4212	2623	2384	6422	5374	0651	8673
8796	9974	1913	8309	4943	9423	9143	4683	4436	8413
7071	8254	6825	3020	9000	4673	6129	0176	3670	4836
7336	4451	5863	6559	5344	0714	1856	0451	7855	5998
1660	0222	2005	0215	2370	2687	3039	7953	1960	6679
7506	1020	8718	9665	1892	8245	7249	6023	4602	4227
5000	8237	6203	6829	5325	5784	8720	5053	6347	1112
4255	6894	8093	9191	5011	0452	6199	0009	8086	5170
5764	9837	6780	7490	5412	4869	6950	4183	8671	4008
3609	1368	9129	7113	3099	1887	0544	6415	9148	4381
7218	5939	4932	5465	6648	6365	4179	9266	9803	5572
6854	5911	1495	4940	4630	4514	0942	7218	7382	2145
4403	4263	4755	5451	8251	2652	6207	4841	3528	7665
2978	4381	2205	9638	6946	7126	9039	9194	6676	4396
1072	2292	4428	4934	8183	7385	3236	7748	4488	1351
6488	6568	9530	8316	7709	9022	8041	5564	6667	5329
9263	7756	6300	6793	7769	3099	3606	2468	2574	5230
0357	3493	0385	4451	4313	3024	8243	4920	3523	9644
5372	9351	8393	6023	2811	1744	2306	7083	4330	7278
6570	2866	7565	7871	9490	9050	4454	3475	8319	2972
8596	8251	0336	8119	1966	9115	4202	7785	5269	5941
4177	0092	4207	7386	9891	1149	3429	7062	4622	8415
8438	4892	2089	5509	2054	9024	1213	5791	2543	7863
5820	6287	7484	0339	8585	0968	3675	2440	4000	5148
7721	3804	9520	6184	9152	1853	8640	3601	5606	7218

Figure C-11.—Random digits table.

(2) Printed circuit assemblies to compare the effectiveness of different soldering methods. If boards are all of the same type, a cluster sampling procedure could be used here. Group the boards by soldering methods; select x joints from each cluster to compare the effectiveness of different soldering methods.

(3) Residual gases in a vacuum vessel to determine the partial pressure of gases at various tank locations. A stratified sampling procedure could be used in this case. Stratify the tank near existing feedthroughs into x sections; an appropriate mass run could be taken from each section at various ionizer distances from the tank walls. Analysis would tell how the partial pressures varied with ionizer depth at the feedthrough locations.

Sample size.—A completely general equation for determining sample size n is given by

$$Q(t) = 1 - R(t_t) = \frac{N_f}{n}$$

where

N_f desired number of time-to-failure points

n sample size

t_t test truncation time

This equation can be used with any of the reliability functions given in figure C-1.

As an illustration of how these equations can be applied to electrical parts, consider example 16, which is derived from example 1.

Example 16: Tantalum capacitors with a failure rate of 1×10^{-3} failure/hour are to be tested to failure. In a

1000-hour test what sample size should be used to get 25 time-to-failure data points?

Solution 16: The truncated exponential reliability function is given by

$$R(t_t) = e^{-t_t/1000} = 0.37$$

Solving the general sample size equation for n and substituting values gives

$$n = \frac{N_f}{1 - R(t_t)} = \frac{25}{0.63} = 39.6$$

Rounding off to the nearest whole unit gives $n = 40$ pieces. This means that 40 capacitors tested for 1000 hours should give 24 time-to-failure data points.

Accelerated Life Testing

Life testing to define the time duration during which a device performs satisfactorily is an important measurement in reliability testing because it is a measure of the reliability of a device. The life that a device will exhibit is very much dependent on the stresses it is subjected to. The same devices in field application are frequently subjected to different stresses at varying times. It should be recognized then that life testing involves the following environmental factors:

(1) The use stresses may influence the device's life and failure rate functions.

(2) The field stresses could be multidimensional.

(3) In the multidimensional stress space there is an interdependence among the stress effects.

(4) Because most devices operate over a range in a multidimensional stress space, life performance may vary.

Testing objects to failure under multidimensional stress conditions is usually not practical. Even if it were, if the system were properly designed, the waiting time to failure would be quite long and therefore unrealistic. It has been shown that time-to-failure data are important to reliability testing, and now they appear difficult to obtain. These are some of the reasons why many are turning to accelerated life testing, such as compressed-time testing, advanced-stress testing, or optimum life estimates:

(1) Compressed-time testing—If a device is expected to operate once in a given time period on a repeated cycle, life testing of this device may be accelerated by reducing the operating time cycle. The multidimensional stress condition need not be changed. The stresses are being applied at a faster rate to accelerate device deterioration. Care should be taken not to accelerate the repetition rate beyond conditions that allow the device to operate in accordance with specifications. Such acceleration would move the device into a multidimensional stress region that does not exist in field conditions and would yield biased information. As an illustration of compressed time testing, consider example 17.

Example 17: The stepping motor in example 3 was being pulsed for life testing. How could this life test be accelerated?

The power supply providing the stepping pulses may have been stepping at the rate of one pulse per 10 seconds, resulting in a test time of 10^7 seconds. These motors had a frequency response allowing for 10 pulses per second. Increasing the pulse stepping rate up to the frequency response limit yields comparable time-to-failure data in 10^5 seconds, a savings in time of two orders of magnitude.

(2) Advanced-stress testing—If a device is expected to operate in a defined multidimensional stress region, life testing of this device may be accelerated by changing the multidimensional stress boundary. Usually the changes will be toward increased stresses because this tends to reduce time to failure. There are two basic reasons why advanced stress testing is used:

(a) To save time
(b) To see how a device performs under these stress conditions

Care should be exercised in changing stress boundaries to be sure that unrealistic conditions leading to wrong conclusions are not imposed on the device. A thorough study of the failure mechanisms should be made to ensure that proposed changes will not introduce new mechanisms that are not normally encountered. If an item has a certain failure density distribution in the rated multidimensional stress region, changing the stress boundaries should not change the failure density distribution. Some guidelines for planning advanced-stress tests are as follows:

(a) Define the multidimensional stress region for an item; nominal values should be centrally located.

(b) Study the failure mechanisms applicable to this item.

(c) On the basis of guidelines (a) and (b) decide which stresses can be advanced without changing the failure mechanisms.

(d) Specify multiple stress tests to establish trends; one point should be on the outer surface of the multidimensional region.

(e) Be sure that the specimen size at each stress level is adequate to identify the failure density function and that it has not changed from level to level.

(f) Pay attention to the types of failures that occur at various stress levels to be sure that new failure mechanisms are not being introduced.

(g) Decide whether new techniques being developed for advanced-stress testing apply to this item. Several popular techniques are described here:

(i) Sensitivity testing—Test an item at the boundary stress for a given time. If failure occurs, reduce stress by a fixed amount and retest for the same time. If no failure occurs, increase stress by a fixed amount and retest for the same time. Repeat this process until 25 failures occur. This technique is used to define endurance limits for items.

(ii) Least-of-N testing—Cluster items in groups, subject each cluster to a specified stress for a given time. Stop at the first failure at each stress level. Examine failed items to ensure conformance to expected failure mechanisms.

(iii) Progressive-stress testing—Test an item by starting at the central region in stress space and linearly accelerating stress with time until failure occurs. Observe both the failure stress level and the rate of increasing stress. Vary the rate of increasing stress and observe its effect on the failure stress magnitude. Examine failed items to ensure conformance to expected failure mechanics.

As an illustration of advanced-stress testing, consider example 18.

Example 18: A power-conditioning supply was being life tested at nominal conditions with an associated electric rocket. The nominal electrical, thermal, vibration, shock, and vacuum stresses resulted in fairly long waiting periods to failure. Changing the multidimensional stress conditions by a factor of 1.25 to 2, which is usually done during development testing, tended to identify design deficiencies with shorter waiting periods without affecting the failure mechanism.

(3) Optimum life estimate—One remaining calculation for nonreplacement failure or time-truncated life test is the optimum estimate of mean time between failures $\bar{t}$. It has been shown (ref. C–1) that $\bar{t}$ given by the time sum divided by the number of failures should be modified by a censorship factor and a truncation time factor. The censorship factor K is caused by wearout failures, operator error, manufacturing errors, etc. The correction equation for $\bar{t}$ is given by (ref. C–1)

$$\bar{t} = \frac{\sum_{i=1}^{N_f} t_i + (n - N_f)t_r}{N_f - K}$$

where

N_f number of failures

K censorship factor

As an illustration consider example 19.

Example 19: The tantalum capacitor tested in example 1 could have been stopped when 10 capacitors (580 part-hours) out of 100 had failed at a testing time of 100 hours. What is an optimistic value for $\bar{t}$?

Solution 19: Inspection of the 10 failed capacitors showed that two units failed owing to manufacturing errors. Therefore, $N_f = 10$, $K = 2$, $n = 100$ capacitors, $t_r = 100$ hours, and the sum of $t_i = 580$ hours. Substituting these values into the $\bar{t}$ correction equation gives

$$\bar{t} = \frac{580 + (100 - 10)100}{10 - 2} = 1197 \text{ hours}$$

This is an optimistic estimate for the mean time between failures, but it certainly is fair and reasonable to make these types of corrections.

Accept/Reject Decisions With Sequential Testing

A critical milestone occurs in product manufacturing at delivery time. An ethical producer is concerned about shipping a product lot that does not meet specifications. The consumer is concerned about spending money to purchase a product that does not meet specifications. A test method that permits each to have an opportunity to obtain data for decision making is required.

Sequential testing constraints.—If α is the producer's risk and β is the consumer's risk, two delivery time constants valid for small risks have been defined and are given as

$$A = \frac{1 - \beta}{\alpha}$$

$$B = \frac{\beta}{1 - \alpha}$$

Let P_1 be the probability that N_f failures will occur in time t for a specified minimum acceptable $\bar{t}_1$, and let P_0 be the probability that N_f failures will occur in time t for an arbitrarily chosen upper value $\bar{t}_0$. Test rules using these four constants have been defined for each condition (refs. C–1 and C–5):

(1) Accept if $P_1/P_0 \le B$.
(2) Reject if $P_1/P_0 \ge A$.
(3) Continue testing if $B < P_1/P_0 < A$.

Exponential parameter decision making.—As an illustration of how these testing constraints can be implemented for the exponential distribution, consider example 20.

Example 20: A purchased quantity of 100 000 tantalum capacitors has been received. Negotiations prior to placement of the order had established that $\alpha = \beta = 0.1$, $\bar{t}_1 = 1000$ hours, and $\bar{t}_0 = 2000$ hours and that the sequential reliability test should be truncated in 48 hours.

(1) Calculate A and B.

(2) Write the expressions for P_0 and P_1.

(3) How many units should be placed on test?

(4) Plot a sequential reliability control graph to facilitate decision making at each failure time.

Solution 20:

(1) The delivery time constants are obtained by substituting values into the defining equations.

$$A = \frac{1 - 0.1}{0.1} = 9$$

$$B = \frac{0.1}{1 - 0.1} = 0.111$$

(2) Using binomial distribution from figure C-1 and substituting values gives $P_0(N_f)$ and $P_1(N_f)$ as

$$P_0(N_f) = \left(\frac{t}{2000}\right)^{N_f} \frac{e^{-t/2000}}{N_f!}$$

$$P_1(N_f) = \left(\frac{t}{1000}\right)^{N_f} \frac{e^{-t/1000}}{N_f!}$$

(3) Delivery constant B defines the acceptance criteria for P_1/P_0. Using this constraint and substituting for P_1 and P_0 gives

$$B = \frac{P_1(N_f)}{P_0(N_f)} = 2^{N_f}\, e^{-t/2000}$$

The minimum testing time without failure $t(0)_{\min}$ is given by

$$0.111 = (2)^0\, e^{-t(0)_{\min}/200}$$

Solving for $t(0)_{\min}$ gives

$$t(0)_{\min} = 2.20 \times 2000 = 4400 \text{ unit-hours}$$

The minimum number of capacitors to be life tested for 48 hours is given by

$$n_{\min} = \frac{4400 \text{ unit-hours}}{48 \text{ hours}} = 91.7$$

To ensure good results, choose a sample size n that is more than twice $n_{\min}$, for this problem use $n = 200$ units. The required minimum testing time for 200 units is given by

$$t(0)_{\min} = \frac{4400 \text{ unit-hours}}{200 \text{ units}} = 22.0 \text{ hours}$$

The test can be stopped and an accept/reject decision made at t_t, where t_t is given by

$$t_t = 48 \text{ hours} \times 20 \text{ units} = 9.6 \times 10^3 \text{ unit-hours}$$

(4) The tantalum capacitor reliability chart is constructed by using five points in the (N_f, t) plane; three of these points have already been calculated and are given by

$$t(0)_{\min} = 4400,\ N_f = 0$$

$$t_t = 9.6 \times 10^3,\ N_f = 0$$

$$t = 0,\ N_f = 0$$

The remaining two points are calculated by using the test inequality given by

$$B < p(N_f) < A$$

In general terms the ratio $p(N_f)$ is given by

$$p(N_f) = \left(\frac{\bar{t}_0}{\bar{t}_1}\right)^{N_f} e^{-(1/\bar{t}_1 - 1/\bar{t}_0)t}$$

Taking natural logarithms of the inequality and substituting gives

$$\ln B < N_f \ln\left(\frac{\bar{t}_0}{\bar{t}_1}\right) - \left(\frac{1}{\bar{t}_1} - \frac{1}{\bar{t}_0}\right) t < \ln A$$

Adding $(1/\bar{t}_1 - 1/\bar{t}_0)t$ to each term gives

$$\ln B + \left(\frac{1}{\bar{t}_1} - \frac{1}{\bar{t}_0}\right)t < N_f \ln\left(\frac{\bar{t}_0}{\bar{t}_1}\right) < \ln A + \left(\frac{1}{\bar{t}_1} - \frac{1}{\bar{t}_0}\right)t$$

Dividing all terms by $\ln(\bar{t}_0/\bar{t}_1)$ gives

$$\frac{\ln B}{\ln\left(\frac{\bar{t}_0}{\bar{t}_1}\right)} + \left[\frac{\frac{1}{\bar{t}_1} - \frac{1}{\bar{t}_0}}{\ln\left(\frac{\bar{t}_0}{\bar{t}_1}\right)}\right] t < N_f$$

$$< \frac{\ln A}{\ln\left(\frac{\bar{t}_0}{\bar{t}_1}\right)} + \left[\frac{\frac{1}{\bar{t}_1} - \frac{1}{\bar{t}_0}}{\ln\left(\frac{\bar{t}_0}{\bar{t}_1}\right)}\right] t$$

The inequality is now in the form given by

$$a + bt < N_f < c + bt$$

The constants a and c for this problem for zero failures are given by

$$a = \frac{\ln B}{\ln\left(\frac{t_0}{t_1}\right)} = \frac{-2.2}{0.69} = -3.18, \quad N_f = 0$$

$$c = \frac{\ln A}{\ln\left(\frac{t_0}{t_1}\right)} = \frac{2.2}{0.69} = 3.18, \quad N_f = 0$$

Because these boundary constraints are straight lines in the form

$$N_f = bt + (a \text{ or } c)$$

the slope b is given by

$$b = \frac{\left(\frac{1}{t_1} - \frac{1}{t_0}\right)}{\ln\left(\frac{t_0}{t_1}\right)} = \frac{5\times10^{-4}}{0.69} = 7.22\times10^{-4}$$

Figure C-12 shows the resulting tantalum capacitor reliability chart. The tantalum capacitor acceptance reliability test results in an "accept," "continue to test," or "reject" decision depending on the failure performance of the capacitors as a function of operating time in unit-hours as zoned in figure C-12.

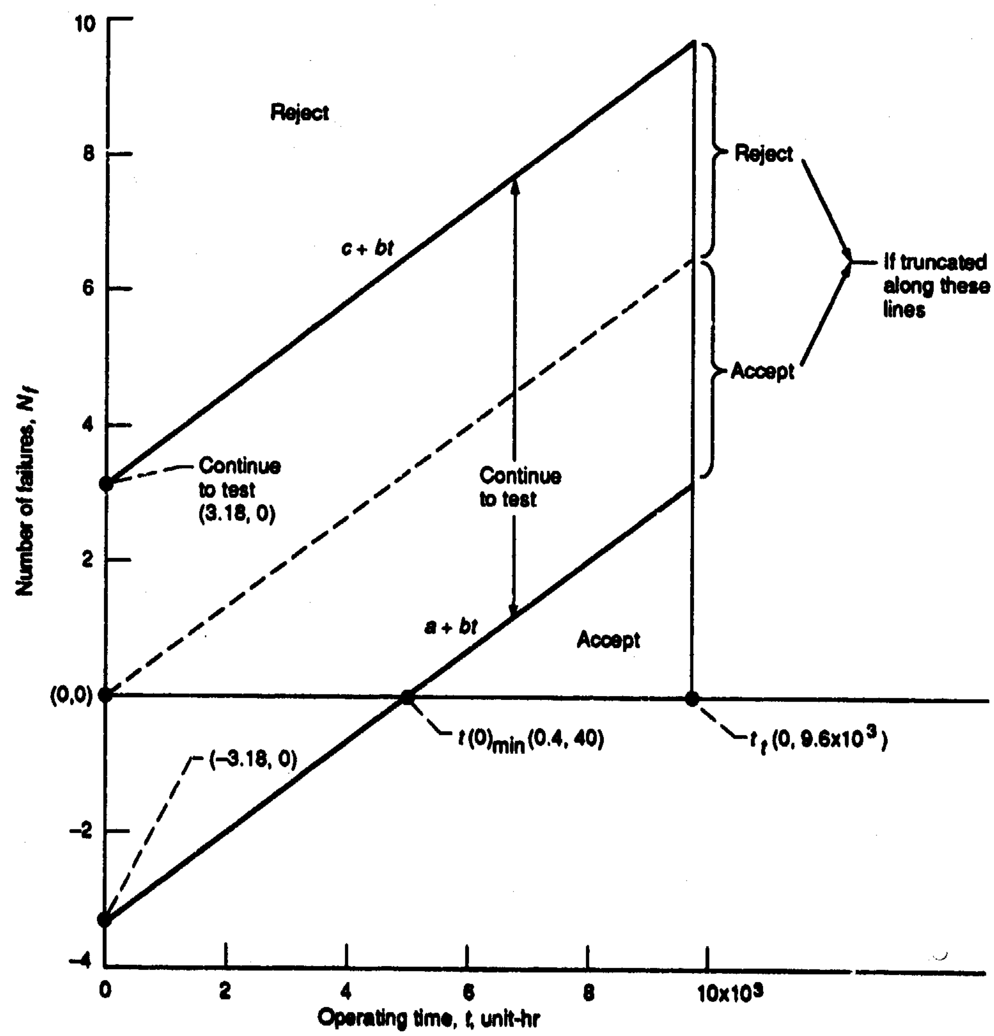


Figure C-12.—Tantalum capacitor reliability chart.

Binomial parameters decision making.—For the binomial frequency function the procedure to set up a sequential reliability test is similar to the Poisson methodology. Because the unreliability, or number of defectives, is given by $1 - R$ for an effectiveness of R, then $P_1(N_f)$ is given in binomial form by

$$P_1(N_f) = (1 - R_1)^{N_f}(R_1)^{n-N_f}$$

where

n	$N_s + N_f$
N_s	number of successful trials
N_f	number of failed trials
R_0, R_1	chosen reliability values at some time t, $R_0 > R_1$

The ratio $P_1(N_f)/P_0(N_f)$ is given by

$$\frac{P_1(N_f)}{P_0(N_f)} = \frac{(1 - R_1)^{N_f}(R_1)^{n-N_f}}{(1 - R_0)^{N_f}(R_0)^{n-N_f}}$$

Following the steps given in example 20, give four of the points in the (N_f, t) plane.

$$N(0)_{\min} = \frac{\ln B}{\ln\left(\frac{R_1}{R_0}\right)}, \quad N_f = 0$$

The test can be stopped and an accept/reject decision made at the number of test truncation trials N_r; N_r is given by

$$N_r = t_r N_c, \quad N_f = 0$$

where N_c is the number of units chosen for testing.

$$n = 0, \quad N_f = 0$$

$$a = \frac{\ln B}{\ln \frac{R_0(1 - R_1)}{R_1(1 - R_0)}}, \quad N_f = 0$$

$$c = \frac{\ln A}{\ln \frac{R_0(1 - R_1)}{R_1(1 - R_0)}}, \quad N_f = 0$$

The slope b is given by

$$b = \frac{\ln\left(\frac{R_0}{R_1}\right)}{\ln \frac{R_0(1 - R_1)}{R_1(1 - R_0)}}$$

The inequality equation for these conditions is given by

$$a + bn < N_f < c + bn$$

Accept/reject charts at delivery milestones when based on reliability sequential testing methods provide a rigorous mathematical method for deciding whether or not to accept or reject an order of components. The actual reliability value for these components is not known, nor is it wise to consider reliability assessment at this critical milestone.

Subsample f chart.—The chief advantages of a subsample f chart are (1) it reduces reliability acceptance testing costs, (2) it provides for product improvements, (3) it determines if statistical control exists, and (4) it determines the mean time to repair.

Example 21: A power supply has the following data:

(1) Acceptable reliability level, r_1, 0.01 failure/hour; producer's reliability risk, R_α, 10 percent; specified mean time to repair, 3.0 hours

(2) Lot tolerance fractional reliability deviation, r_2, 0.005 failure/hour; consumer's reliability risk, R_β, 10 percent

The product test data are given in table C–9. Use figure C–13 to analyze these data; then answer the following questions:

(1) What is a suitable time sample and rejection number for meeting the 80-percent confidence level selected by management?

(2) What are the subsample sizes and rejection numbers?

(3) What are the confidence levels for the various rejection numbers?

(4) What are the control limits on the mean time to repair?

(5) Plot these data on a subsample f chart.

(6) What should be done with the manufactured units?

Solution 21: Given the product data, follow these steps:

(1) Calculate the confidence level γ, the ratio of acceptable reliability level to lot tolerance fractional reliability deviation k, and the mean time between failures m:

$$\gamma = 1 - (R_\alpha + R_\beta) = 1 - (0.1 + 0.1) = 0.80, \text{ or } 80 \text{ percent}$$

$$k = \frac{r_2}{r_1} = \frac{0.005}{0.001} = 5$$

$$m = \frac{1}{r_1} = \frac{1}{1 \times 10^{-3}} = 1000 \text{ hours}$$

TABLE C-9.—POWER SUPPLY PROBLEM DATA

Sample serial number	Number of failures	Reason for failure	Repair time, hr
1	1	A1A-2VR3 zener shorted	1.2
	1	Ground wire broke	1.4
	2	A1A2-VR3 zener shorted; A1A2-Q2 transistor shorted	5.5, 7.3
	0	In a 250-hr test no failure occurred	-------
2	0	In a 250-hr test no failure occurred	-------
	1	A3A1-C3 capacitor leaked	9.5
3	1	A3A1-C3 capacitor leaked	9.0
	0	In a 250-hr test no failure occurred	-------
4	1	A7A1-VR1 unsoldered joint	.5
		A3A1-C3 capacitor leaked	9.5
5	0	In a 250-hr test no failure occurred	-------

Looking up Z_α in a normal curve area table (table 3 in ref. C-3) for $R_\alpha = 0.1$ shows that $Z_\alpha = -1.28$. The value of K^2 when $k = 5$ and $\gamma = 0.80$ is obtained from figure 11-1 in reference C-3, where $K^2 = 1.05$. The equation for t is thus $t = mK^2 = (1000)(1.05) = 1050$ hours $\cong$ 1000 hours. The rejection number R for a time sample of 1000 hours and a confidence level $\gamma = 0.80$ is given by

$$R_{1000(0.80)} = K^2 + Z_\alpha K + 0.5$$

$$= 1.05 + (1.28)\ 1.025 + 0.5 = 2.86 \approx 3$$

(2) Recalculate the subsample for $\gamma = 0.50$ and $k = 5$: From figure 11-1 in reference C-3, $K^2 = 0.29$. Therefore,

$$t = mK^2 = (1000)(0.29) = 290 \text{ hours} \approx 250 \text{ hours}$$

Looking up Z_α in table 3 in reference C-3 for

$$R_\alpha = \frac{1-\gamma}{2} = \frac{0.5}{2} = 0.25$$

shows that $Z_\alpha = -0.68$. Recalculate the rejection number as

$$R_{250(0.50)} = K^2 + Z_\alpha K + 0.5$$

$$= 0.29 + (0.68)\ 0.54 + 0.5$$

$$= 1.16 \approx 1 \text{ failure}$$

TABLE C-10.—SUBSAMPLE DATA

t	K^2	γ, percent	R_α	Z_α	$R_t(\gamma)$
250	0.25	0.46	0.27	0.61	1
500	.50	.63	.185	.89	2
750	.75	.73	.133	1.11	2
1000	1.0	.78	.11	1.22	3

(3) Calculate K^2 for each value of t shown in table C-10 as

$$K^2 = \frac{t}{m} = \frac{250}{1000} = 0.25 \qquad \text{for } k = 5;\ m = 1000 \text{ hours}$$

Look up in figure 11-1 in reference C-3 the confidence level γ values shown in table C-10. Calculate R_α for each confidence level. (The calculated values are shown in table C-10.)

$$R_\alpha = \frac{1-\gamma}{2} = \frac{1-0.46}{2} = 0.27$$

Look up Z_α for each confidence level in table 3 in reference C-3 (the values are tabulated in table C-10). Recalculate the rejection numbers $R_t(\gamma)$ for each subsample (the values are listed in table C-10).

$$R_t(\gamma) = K^2 + Z_\alpha K + 0.5$$

$$R_{250(0.46)} = 0.25 + (0.61)\ 0.5 + 0.5 = 1.05 \approx 1$$

$$R_{500(0.63)} = 0.50 + (0.89)\ 0.71 + 0.5 = 1.63 \approx 2$$

$$R_{750(0.73)} = 0.75 + (1.11)\ 0.87 + 0.5 = 2.21 \approx 2$$

$$R_{1000(0.78)} = 1.00 + (1.22)\ 1 + 0.5 = 2.72 \approx 3$$

(4) Find the control limits on the mean time to repair for the data given in table C-9.

$$\mathrm{UCL}_\phi = \frac{2f\phi}{\chi^2_{8(0.90)}} = \frac{2\times4\times3}{3.49} = 6.88 \text{ hours}$$

$$\mathrm{LCL}_\phi = \frac{2f\phi}{\chi^2_{8(0.10)}} = \frac{2\times4\times3}{13.4} = 1.79 \text{ hours}$$

where f is the average number of failures and ϕ denotes mean time to repair. These control limits are shown in figure C-13 for the repair time process. The lower control limit in this case

SUBSAMPLE f CHART

General Identification

Chart Number 1 ARL=.081 fph

LTRFD=.005 fph k=5

Time Sample 1,000 hrs.

Subsample size 150 hrs.

MTTR= MAPH=

Item Name

Item Drawing Number

Subsample No.	1	2	3	4		5	6	7	8		9	10	11	12
Sub-Time Sample, Hrs.	250	250	250	250		250	250	250	250		250	250	250	250
Cumulative Rejection No.	1	2	2	3		1	2	2	3		1	2	2	3
Cumulative Failures	1	2	4	4		0	1	2	2		1	2	2	2
Sample unit S. N.	1	1	1	1		2	2	3	3		4	4	5	5

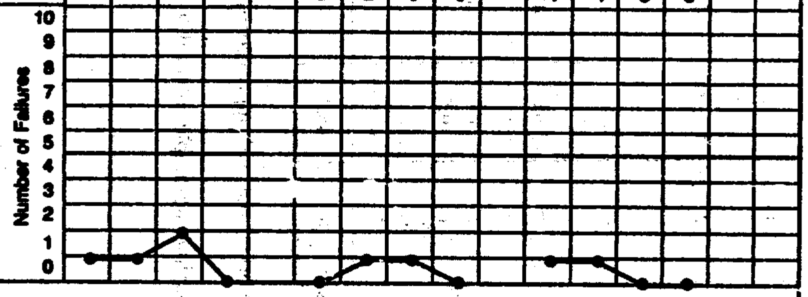
R ( ) ——

R ( ) ——

R_{1000} (0.80) ——

R_{550} (0.83) ——

R_{values} ——



Fail. No.	Reason for the Failure	1	2	3	4		5	6	7	8		9	10	11	12
1	A1A2-VR3 zener shorted	1		1											
2	Ground wire broke		2												
3	A1A2-Q2 transistor shorted			3											
4	A3A1-C3 capacitor leaked							4	4				4		
5	A7A1-VR1 unsoldered joint											5			
6															
7															
8															
9															
10															

Maintenance Time Analysis

UCL=6.8 hrs (•)

Specified MTTR=3.0 hrs. (□)

LCL=1.8 hrs. (•)

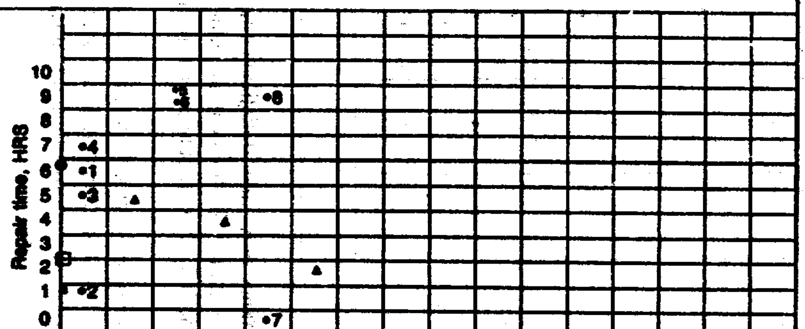


Figure C-13.—Completed subsample *f* chart for problem 22.

TABLE C-11.—POWER SUPPLY ANALYZED DATA
[Sample size, 250 hr.]

Time sample	Sample serial number	Subsample number	Reason for failure	Number of failures	Repair time, hr	Mean time to repair, hr
1	1	1	A1A2-VR3 zener shorted	1	1.2	---
		2	Ground wire broke	1	1.4	---
		3	A1A2-VR3 zener shorted; AHV2-Q2 transistor shorted	2	5.5, 7.3	5.1
		4	No failures occurred	0	-------	---
2	2	5	No failures occurred	0	-------	---
		6	A3A1-C3 capacitor leaked	1	9.5	---
	3	7	A3A1-C3 capacitor leaked	1	9.0	4.6
		8	No failures occurred	0	-------	---
3	4	9	A7A1-VR1 unsoldered joint	1	0.5	---
		10	A3A1-C3 capacitor leaked	1	9.5	---
	5	11	No failures occurred	0	-------	---
		12	No failures occurred	0	-------	---
			Totals	8	48.9	---

has no importance other than statistical completeness because any value less than 1.79 hours is an indication of a better maintenance activity than what has been specified—a desirable condition.

The completed subsample f chart is shown in figure C-13. Table C-11 shows the tabulated data calculated to solve this problem. During the various subsample intervals some useful conclusions can be drawn.

(1) During subsample interval 1 to 4 failures

$$\sum_{i=1}^{4} f_i \geq R$$

reject serial number 1, request an engineering investigation, and repair and retest serial number 1 later.

(2) During subsample interval 5 to 8 failures

$$\sum_{i=5}^{8} f_i \leq R$$

ship serial numbers 2 and 3 after all failures have been reviewed, the cause identified, and appropriate corrective action worked out and approved by an engineering review board.

(3) During subsample interval 9 to 12 failures

$$\sum_{i=9}^{12} f_i \leq R$$

ship serial numbers 4 and 5 after all failures have been reviewed, properly closed out, and approved by the engineering review board.

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Reliability Training Answers

Chapter	Answers
1	1 (C), 2 (B), 3 (C), 4 (C)
2	1a (C), 1b (B), 2a (C), 2b (B), 3a (C), 3b (A), 4a (B), 4b (C), 5ai (B), 5aii (C), 5aiii (B), 5b (C), 6a (C), 6b (B), 7a (B), 7b (C), 8a (C), 8b (C), 9 (D), 10 (A), 11 (B), 12 (C), 13 (C), 14 (C), 15 (D), 16 (E), 17 (D), 18 (F)
3	1a (B), 1b (B), 1c (C), 2a (A), 2b (C), 2c (A), 3a (B), 3b (A), 3c (B), 4 (C), 5a (B), 5b (B), 6 (C), 7a (A), 7b (B), 7c (B), 7d (C), 7e (A), 8 (B), 9a (B), 9b (C), 10a (C), 10b (C), 10c (A)
4	1a (B), 1b (B), 2a (A), 2b (A), 3 (C), 4a (B), 4b (B)
5	1 (C), 2 (B), 3a (C), 3b (A), 3c (C), 4a (C), 4b (B), 4c (A), 5a (C), 5b (A), 6a (C), 6b (C), 6c (A), 7a (B), 7b (C), 7c (C), 7d (C), 8a (A), 8b (C), 8c (B), 8d (C), 8e (B), 8f (B)
6	1a (B), 1b (C), 1c (A), 2a (C), 2b (B), 2c (A), 2d (C), 3a (B), 3b (C), 3ci (B), 3cii (A)
7	1 (C), 2 (B), 3 (D), 4 (A), 5 (B), 6 (C), 7 (B), 8 (C)
8	1 (B), 2 (A), 3 (C), 4a (C), 4b (B), 4c (F), 5 (A), 6a (C), 6b (B), 7 (A), 8a (B), 8b (A)
9	1 (D), 2 (D), 3 (G), 4 (B), 5 (A), 6 (E), 7 (B), 8 (D), 9 (A), 10 (C), 11 (B), 12 (F), 13 (E), 14a (C), 14b (C), 15 (C), 16 (B), 17 (E), 18 (A), 19a (C), 19b (B), 19c (A)

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE
June 1992

3. REPORT TYPE AND DATES COVERED
Reference Publication

4. TITLE AND SUBTITLE
Reliability Training

5. FUNDING NUMBERS
WU-572-10-00

6. AUTHOR(S)
Vincent R. Lalli and Henry A. Malec, Editors

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

8. PERFORMING ORGANIZATION REPORT NUMBER
E-5456

9. SPONSORING/MONITORING AGENCY NAMES(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, D.C. 20546-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA RP-1253

11. SUPPLEMENTARY NOTES
This manual was edited by Vincent R. Lalli, NASA Lewis Research Center, and Henry A. Malec, Siemens Stromberg-Carlson, Albuquerque, NM 87123-2840. Other contributors were Richard L. Dillard, Martin Marietta Corp., Orlando, FL 32855; Kam L. Wong, Hughes Aircraft Company, El Segundo, CA 90245; Frank J. Barber and Frank J. Barina, NASA Lewis Research Center. Responsible person, Vincent R. Lalli, (216) 433-2354. For sale by the National Technical Information Service, Springfield, VA 22161.

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Unclassified - Unlimited
Subject Category 15

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The theme of this manual is failure physics—the study of how products, hardware, software, and systems fail and what can be done about it. The intent is to impart useful information, to extend the limits of production capability, and to assist in achieving low-cost reliable products. In a broader sense the manual should do more. It should underscore the urgent need for mature attitudes toward reliability. Five of the chapters were originally presented as a classroom course to over 1000 Martin Marietta engineers and technicians. Another four chapters and three appendixes have been added. We begin with a view of reliability from the years 1940 to 2000. Chapter 2 starts the training material with a review of mathematics and a description of what elements contribute to product failures. The remaining chapters elucidate basic reliability theory and the disciplines that allow us to control and eliminate failures.

14. SUBJECT TERMS
Derating; Design; Tradeoffs; Environmental engineering; Failure analysis; Failure modes; Human engineering; Life cycle cost; Maintainability; Quality control; Reliability; Repairing; Service life; Sneak circuit analysis; Program verification; Statistical distributions; Systems engineering

15. NUMBER OF PAGES
232

16. PRICE CODE
A11

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	